Current electricity

Unit 3

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3.1 Electric current

By the end of this section you should be able to:

- Define electric current and its SI unit.
- Explain the flow of electric charges in a metallic conductor.
- Calculate the number of electrons that pass a point in a given length of time when the current in the wire is known.

KEY WORDS

ampere *SI unit of electric current*

Electric current and its SI unit

An electric current is a flow of charge. Comparing it with water, a small current is like a trickle passing through a pipe; a really large current is like a river in flood.

The rate of flow of electric charge – that is, the electric current – is measured in amperes (A). The **ampere** is one of the fundamental units of the SI system. This means that the size of the ampere is not fixed in terms of other units: we simply compare currents to a 'standard ampere'.

Household appliances, such as toasters and kettles, run on a current of a few amperes. An ampere is quite a sizeable flow of charge; however, especially in electronic circuits, we also often deal in milliamperes (mA, thousandths of an ampere, 10^{-3} A) or even microamperes (μA, millionths of an ampere, 10^{-6} A).

If charge flows at a rate of one ampere, and continues to flow like that for a second, then the total amount of charge that has passed is one coulomb. This is how the size of the coulomb is fixed: in terms of the ampere and the second.

In practice it is probably best to picture coulombs of charge flowing at a rate of so many amperes. A current of 3 A, for example, is a flow rate of 3 coulombs of charge every second (3 C/s). With that

Worked example 3.1

120 C of charge passes in 1 minute. What is the current?

$$I = \frac{Q}{t} = \frac{120\ \text{C}}{60\ \text{s}} = 2\ \text{C/s} = 2\ \text{A}$$

KEY WORDS

galvanoscope *an instrument for detecting the presence of an electric current*

electron *a negatively charged particle that orbits round the nucleus of an atom*

ion *an atom bearing unequal numbers of electrons and protons*

conduction electrons *electrons in the conduction band of a solid, free to move under the influence of an electric field*

insulator *a material that resists the flow of electric charge*

current, therefore, it should be obvious that in 10 seconds a total of 30 C of charge will pass, or that to supply 12 C of charge the current must flow for 4 seconds.

You are strongly advised to understand the previous paragraph, rather than just remember. When you need the formula linking amperes, coulombs and seconds, however, it is:

$Q = It$ Units
Q coulombs (C)
I amperes (A)
t seconds (S)

In the formula, Q stands for the quantity of charge that passes when a current I flows for a time t.

Worked example 3.2

How long will a current of 5 A take to pass 100 C of charge?

$$Q = It, \text{ so } t = \frac{Q}{I} = \frac{100\ \text{C}}{5\ \text{A}} = 20\ \text{s}$$

Activity 3.1: Making a current tester

You can use a compass needle to make a simple **galvanoscope** (an instrument that detects the presence of an electric current). Simply wrap a few turns of thin insulated copper wire around a compass, connect the ends as shown in Figure 3.1 to a 1.5 V cell and a 1.5 V light bulb. When the wires are connected as shown, the bulb will light and the compass needle will move around in one direction. If you want to investigate this further, disconnect the cell, turn it round and reconnect it the other way round. The needle should move in the opposite direction.

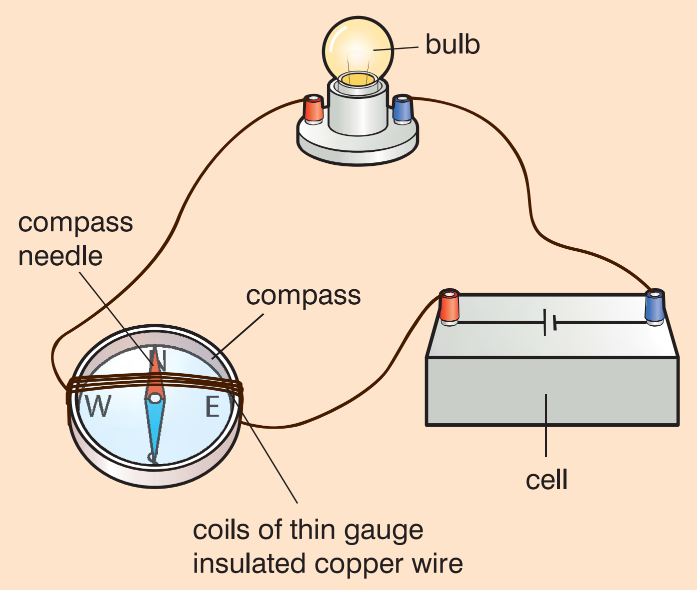


***Figure 3.1** Current tester.*

What are the charges that flow round a circuit?

To answer this question we must first look in rather more detail at the structure of an atom. All its positive charges are located in the central part, the nucleus. Each chemical element has a different number of positive charges in its nucleus, from one to nearly 100. Take copper as an example: it has 29 positive charges in the nucleus.

This means that an uncharged copper atom must have 29 negative electrons as well. They orbit round the nucleus, and each one has its own path. The first two electrons orbit in the innermost shell, the next eight fill the second shell and the following 18 just complete the third shell. That leaves a solitary electron as the first member of a new fourth shell.

It is this electron that accounts for the behaviour of copper as a conductor. It is comparatively easy to remove this electron, changing an uncharged copper atom into what we call a copper **ion** with an overall single positive charge (Figure 3.2).

In a copper wire the atoms are packed close together just as in any other solid. More precisely, what are packed together are the positive ions; the complete atoms except for those single outer electrons. They are there as well, so the metal as a whole is uncharged.

In each copper atom 28 of the electrons are still firmly bound in orbit around their nucleus, fixed in its place in the solid. The 29th electrons we call the **conduction electrons**. They remain trapped within the metal as a whole, but otherwise are free to drift about inside it.

They are the charges that move when an electric current flows down the wire. Being negative they will be repelled from the cell's negative terminal and attracted to the positive one (Figure 3.3).

All metals have these extra one or two outer electrons, so they will all conduct electricity. With most other elements, every electron without exception is tightly bound to its own nucleus, so most other elements are **insulators**.

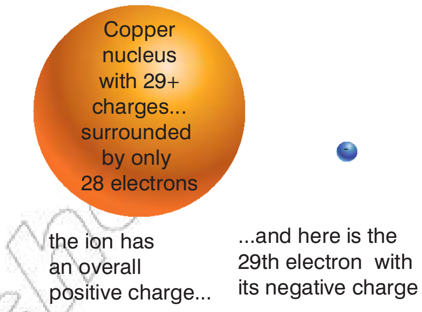


Figure 3.2 *The copper atom can be changed into a positive copper ion plus a negative electron.*

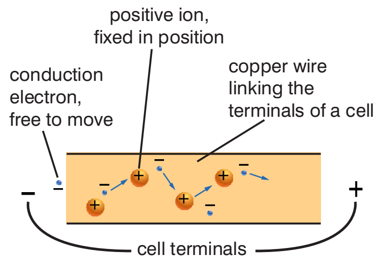


Figure 3.3 *The unattached electrons in the copper wire move when it is connected to the terminals of a cell.*

Conventional current

Electric currents have been investigated since the first cell was invented around 1800. It soon became obvious that charges were flowing round the circuit, but there was no way of telling whether they were positive charges going one way or negative charges going the other.

It was agreed to picture the flow as positive charges repelled from the positive plate, and this was referred to as conventional current.

When the electron was discovered in 1895, it was realised that the guess had been wrong. However, so firmly fixed was the idea that even today we still mark on our circuits the 'conventional current' flowing from the positive pole of the cell to the negative.

You must understand, however, that the electron flow is really negative charge going the opposite way.

Activity 3.2: Testing conductivity

Not all materials have conduction electrons in their structure, so not all materials will conduct electricity. Use your simple galvanoscope (from Activity 3.1) to test which materials conduct electricity and which do not.

Connect your galvanoscope to a copper rod as shown in Figure 3.4a. Observe that the compass needle moves, showing that the rod conducts electricity, allowing an electric current to flow. Connect your galvanoscope to a glass or plastic rod, as in Figure 3.4b. Observe that the compass needle does not move, showing that the rod does not allow an electric current to flow. Repeat this for a selection of different materials of the same size, and rank your materials in order of the size of the deflection of the compass needle.

You will see that some materials conduct electricity better than others, while some materials do not conduct electricity at all.

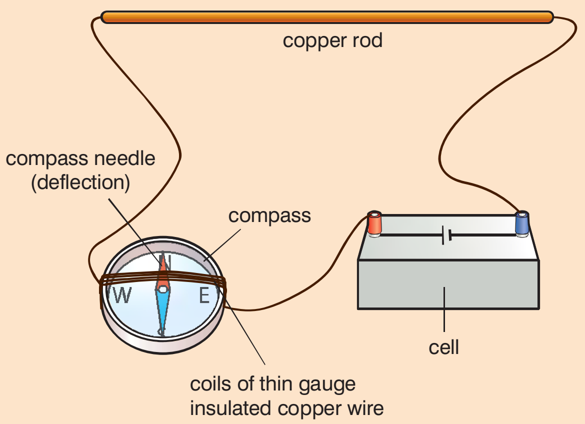


Figure 3.4a

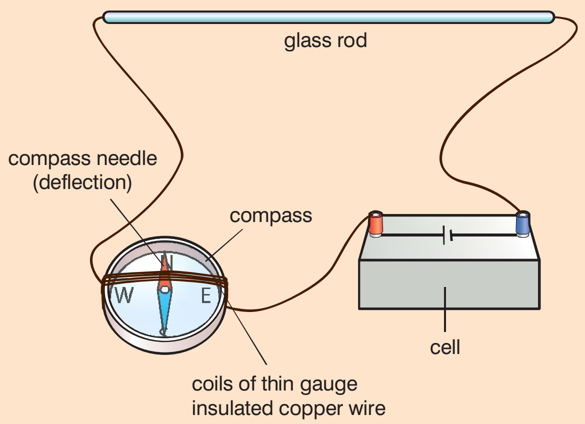


Figure 3.4b

KEY WORDS

drift speed *the average speed that an electron attains in an electric field*

Calculating the number of electrons that pass a point at a given length of time when the current in the wire is known

The wire shown in Figure 3.5 carries a current of I amperes. There are n free electrons per cubic metre of wire. Each electron carries a charge of e coulombs, the cross-sectional area of the wire is A square metres and the average **drift speed** of the electrons in this material is v metres per second.

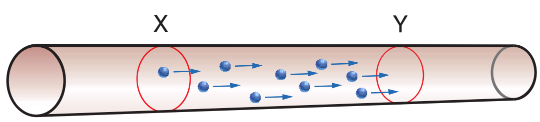


Figure 3.5

Assuming that it takes t seconds for an electron to pass from X to Y (and using the formula distance = speed × time), distance X–Y is vt metres. The volume of wire between X and Y is therefore Avt cubic metres.

The number of electrons between X and Y is therefore $nAvt$.

As each electron carries a charge of *e* coulombs, the total charge passing point Y in *t* seconds is *nAevt*.

The current, *I*, is the charge per second

$$I = \frac{nAevt}{t}$$

or $I = nAev$

As the charge carried by an electron is known to be 1.6×10^{-19} C, the number of electrons passing through a wire can be calculated if the current in the wire is known.

Worked example 3.3

A current of 5 A is flowing through a 2 mm diameter wire. The drift speed of electrons in the wire is 10^{-5} m/s and the charge on the electron is 1.6×10^{-19} C. Calculate the number of electrons passing a point in the wire in a second.

The cross sectional area (*A*) of the wire is πr^2.

$$A = \pi \times \frac{d}{2} \times \frac{d}{2} = \pi \times 0.001 \times 0.001 = \pi \times 10^{-6}\ m^2.$$

Using $n = \frac{I}{Aev}$

$$\text{number of electrons} = \frac{5}{\pi \times 10^{-6} \times 1.6 \times 10^{-19} \times 10^{-5}}$$

$= 9.947 \times 10^{29}$

Study of electric charges in a metallic conductor

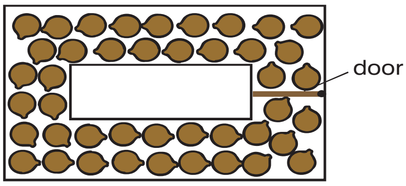


Figure 3.6

In Activities 3.1 and 3.2 you detected an electric current. Where does this electric current keep coming from and where does it keep going to? To start investigating the answer to this, look at Figure 3.6 which shows the top view of a corridor that ends up where it started. The whole place is filled with people who form a kind of endless queue. The door is shut, which means that nobody in the queue can move forward. Once the door is opened (once the circuit is switched on), all the people in that corridor can immediately start moving. They can keep circulating round and round the corridor, thus forming a continual current.

Of course when the door was opened the people filling the corridor found themselves free to move round the circuit, but they did not have to do so. Likewise charges will not circulate to give a current unless there is something that keeps 'pumping' them round. That is the job of the cell, as we shall see below.

KEY WORDS

electrochemical cell *a device capable of deriving electrical energy from chemical reactions*

electrodes *conductors used to make contact with part of a circuit*

electrolyte *a solution that conducts electricity*

electromotive force *a source of energy causing current to flow in an electrical circuit*

e.m.f. *electromotive force*

dry cell *electrochemical cell containing electrolyte in the form of a paste*

polarisation *the formation of a film of hydrogen gas on the positive plate of a dry cell*

Electric energy from chemicals

In Activities 3.1 and 3.2 a flow of electric current is detected by a galvanoscope. What causes this current to flow around the circuit?

An electrochemical cell is one device that makes current flow round a circuit. Such cells have two **electrodes** made of two different **conductors**, an **electrolyte** solution (a solution that conducts electricity) which reacts with the electrodes, and a conductive wire through which electrons can flow.

Activity 3.3: Make your own electrochemical cell

Roll a lemon, orange, grapefruit, or other citrus on a firm surface to break the internal membranes.

Insert two rods – one copper, one zinc – into the fruit and connect as shown in Figure 3.7. Observe the size of the deflection of the needle on the instrument.

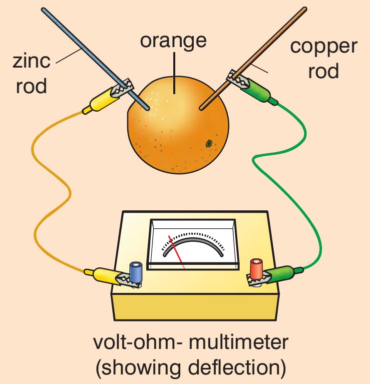


Figure 3.7

The electrodes in this cell are the copper rod and the zinc rod. The electrolyte here is the juice inside the orange.

Repeat the experiment using rods made from different materials and observe the deflection of the needle on the instrument. Do some pairs of materials produce a greater deflection than others?

The volt

A volt is defined as the potential difference (p.d.) required to produce a current of 1 A in a circuit with a resistance of 1 ohm (or Ω). It is named after the Italian scientist Alessandro Volta (1745–1827).

Some cells are more powerful than others. An ordinary torch cell (usually referred to as a battery) is rated at 1.5 volts. We call this the **electromotive force** of the cell, usually shortened to **e.m.f**. It is the cell that pumps the charge round the circuit, and for the time being it is sufficient to think of the e.m.f. as its 'electrical pumping strength'. All other things being equal, the greater the e.m.f. of the cell the greater the current that it will drive round a circuit. You will have got the right idea when you do not like to hear people talking about volts flowing round a circuit. Volts do not flow. They cause coulombs of charge to flow round a circuit at a rate of amperes.

Sources of electricity

There are many different types of cell. Figure 3.8 illustrates the principles of one common form, which we call a **dry cell**. Experiment shows that the carbon plate is the positive one, the zinc is the negative, and the e.m.f. is very close to 1.5 V.

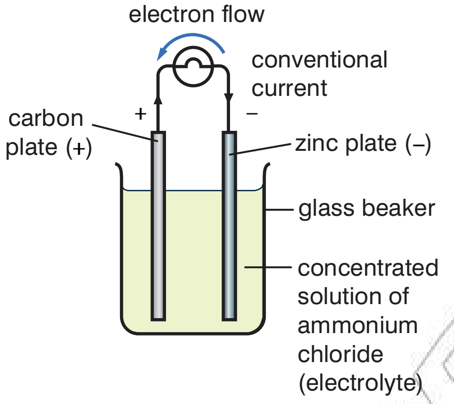


Figure 3.8 *The principle of a common sort of cell.*

Such a cell is about as portable as a glass of water, and we now see how it has been converted into the tough flashlight cell (commonly known as a battery) we are familiar with.

The negative zinc plate is shaped to form the casing of the cell (Figure 3.9). The carbon runs down the centre in the form of a rod (with a brass cap on the top, because carbon is easily damaged). The concentrated solution of ammonium chloride that has to go into the space between them is made much less runny by forming it into a paste or jelly. It is for this reason that the design is called a dry cell.

This leaves the complete cell as shown in Figure 3.10.

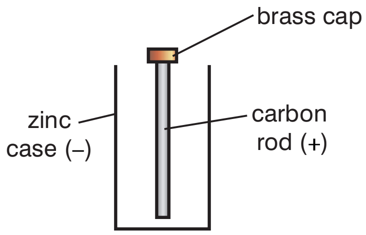


Figure 3.9 *The zinc casing forms the negative plate.*

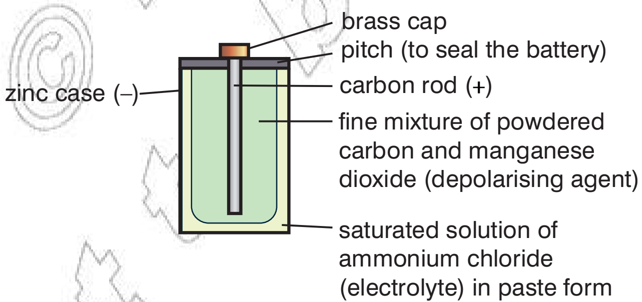


Figure 3.10 *The complete dry cell.*

The positive plate is both the carbon rod down the centre and the thick layer of powdered carbon that surrounds it.

Mixed in with the powdered carbon is manganese dioxide, another chemical in powder form. While the cell is giving a current it suffers from **polarisation**: the formation of a film of hydrogen gas on the positive plate which 'clogs' the cell up. The manganese dioxide is there to deal with this problem by supplying oxygen to convert this unwanted hydrogen to water. Sometimes the mixture of black powders is held in place round the central carbon rod with a cloth bag, but in other batteries it is just the 'stiffness' of the ammonium chloride paste that keeps it there.

KEY WORDS

primary cells *electrochemical cells that transform chemical energy directly to electrical energy*

secondary cells *electrochemical cells that have to be charged up by passing an electric current through them*

As the cell runs down, the zinc casing gets thinner and thinner, and when it gets very old the ammonium chloride may seep out from holes in the casing as a white corrosive paste.

There is nothing unique about carbon, zinc and ammonium chloride solution: any combination of two different metals (or one metal and carbon) placed in a solution of an electrolyte will produce a voltage. Other combinations are available, such as the alkaline cell, which uses potassium hydroxide for the electrolyte.

Primary cells and secondary cells

Cells that produce their voltage directly from chemical energy stored in the ingredients which make them up are known as **primary cells**. When all their chemicals have reacted, you have to buy a new one. The dry cell is one of these.

There are also **secondary cells**, which have first to be charged up by forcing a current 'backwards' through them. The commonest types of these are the increasingly popular rechargeable cells and the lead–acid accumulators that make up the battery in a car.

Activity 3.4: The direction of current flow

Part A

Connect the cell, motor and switch (open) as shown in Figure 3.11 on the edge of a table or bench. Clamp the motor to the edge of the table. Attach weights to the spindle of the motor, to hang over the edge of the table or bench. Close the switch. The motor will turn and the weights will move. Observe the movement of the weights; one weight will rise and one weight will fall.

Part B

Attach the cell the opposite way round. Close the switch and observe the movement of the motor and the weights. (x rises, y falls)

You will see that the motor turns in the opposite direction in Part B, causing the weight which rose in Part A to fall in Part B, and the one which fell in Part A to rise in Part B.

This demonstrates that the electrons in the circuit flowed in one direction in Part A and in the opposite direction in Part B.

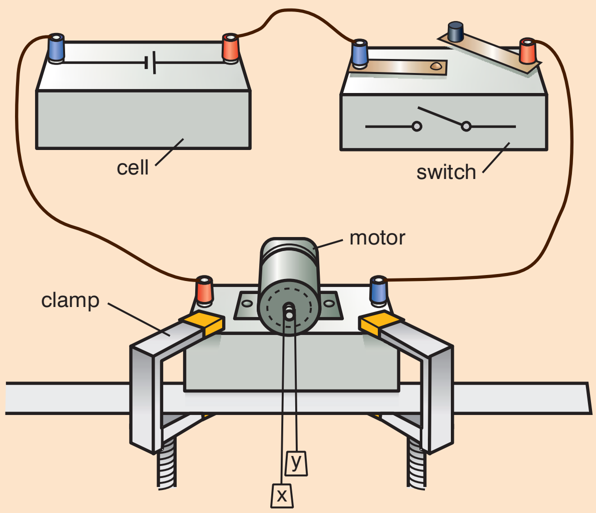


Figure 3.11

The 'standard ampere' – a note to turn back to when you have covered more physics

You may be puzzled as to how anyone can keep a standard ampere somewhere. That is not possible, of course, but we do have a way of defining it. (There is no need for you to remember the details.)

Two parallel wires each carry the same current (see Figure 3.12) and each current produces a magnetic field.

The magnetic field produced by one current-carrying wire acts on the other current-carrying wire to cause the **motor effect**, which pushes the wires apart.

The bigger the current in the wires, the greater the force pushing them apart.

If the wires are 1 metre apart and the push reaches 2×10^{-7} newtons on each metre length, then we say that each of the currents is one ampere.

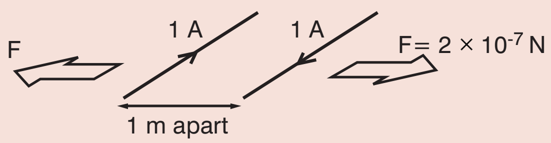


Figure 3.12

KEY WORDS

motor effect *when a current-carrying wire passes at right angles through a magnetic field, a force is exerted on the wire*

thermoelectricity *electricity produced when two metals of different temperatures are joined in a circuit*

Electric energy from heat

We have seen how electric energy can come from chemicals and we now see that it can also come from heat; this is called **thermoelectricity**.

If two different metals are joined in a circuit, and if one junction between the metals is hotter than the other, a very small voltage (a few millivolts, mV) is generated. This effect is known as the Seebeck effect after the German physicist Thomas Seebeck (1770–1831) who discovered it.

The circuit must be made from two different metals, any two, although some pairs may be better than others.

The thermocouple thermometer

The Seebeck effect is used in the thermocouple thermometer to measure temperature. It consists of a circuit made from any two different metals and includes a sensitive meter to measure small currents (Figure 3.13). If one of the junctions between the metals is hotter than the other, a small voltage is generated, which drives a tiny current round the circuit. The larger the temperature difference, the greater the current.

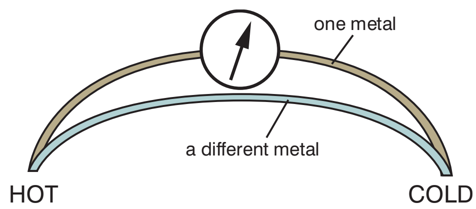


Figure 3.13 *The thermocouple thermometer.*

When using a thermocouple thermometer, it is usual to hold one junction at 0 °C by keeping it immersed in melting ice. The other junction acts as the probe to investigate the temperature to be measured, as in Figure 3.14.

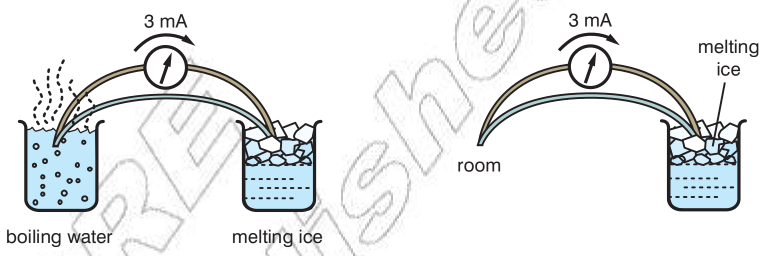


Figure 3.14 The thermocouple thermometer in use.

In the first part of Figure 3.14, a temperature difference of 100 °C between boiling water and melting ice shows a current of 10 mA (milliamperes, thousandths of an ampere).

Each 1 mA therefore indicates a difference of 10 °C.

Thus when the probe is at room temperature and a current of 3 mA is measured, the junction at room temperature must be 30 °C hotter than the cold junction, which is at 0 °C. Room temperature is therefore 30 °C.

Worked example 3.4

One junction of a thermocouple thermometer is immersed in melting ice and the other in boiling water. A current of 20 mA is recorded. The thermocouple thermometer is then used to measure the temperature of a liquid. One junction is immersed in melting ice and the other in the liquid whose temperature is to be measured. A current of 5.5 mA is recorded. What is the temperature of the liquid?

In the first, calibration, measurement, 20 mA represents a temperature difference of 100 °C. Therefore 1 mA represents a temperature difference of 5 °C. In the temperature measurement, the temperature difference between the junctions is $5.5 \times 5 = 27.5$ °C.

The junction immersed in melting ice is at 0 °C. The temperature of the liquid being measured is therefore

$0 + 27.5 = 27.5$ °C.

Activity 3.5: Making a thermocouple thermometer

Create thermocouple junctions at both ends of a section of iron wire by twisting the ends together with copper wires.

Place one copper–iron junction in a beaker with ice water and leave the other junction outside. The two remaining ends of the copper wires should be connected to a sensitive galvanometer.

Heat the exposed junction with a Bunsen burner or match and record the current.

Does the current increase or decrease if the heat source is removed? Is the change in current immediate? Discuss these questions.

Summary

- Current is a flow of electric charge.
- Coulombs of charge flow at a rate of amperes.
- We need a voltage supply to cause the charge to circulate round the circuit.
- A primary cell uses the chemicals in it to supply electrical energy; a secondary cell has to be charged up first.

Review questions

1. Charge flows along a wire at rate of 3 A. How many coulombs of charge will pass a given point in the circuit in:
   a) 1 s
   b) 12 s
   c) 2 minutes?
2. A very sensitive ammeter records a current of 30 μA. How long will it take before 6 μC of charge has flowed through it?
3. In an experiment to copperplate a coin, a current of 100 mA is passed through a solution of copper sulphate. If 30 C of charge must pass before the coin has enough copper deposited on it, for how long must you leave the current switched on?
4. a) A single electron has a charge of 1.6×10^{-19} C. How many electrons must pass round the circuit of question 5 to achieve the 30 C?
   b) For every two electrons that arrive at the coin which is being plated, one copper atom is deposited. If the mass of a copper atom is 1.1×10^{-25} kg, what is the total mass of the copper which now plates the coin?
5. What name do we give to the unit 'ampere second' (A s)? What physical quantity would be measured in such a unit?
6. You have two batteries, a large one and a tiny one, each consisting of a single dry cell. In terms of their performance, what would you expect to be the same for the two batteries and what would you expect to be different about them?
7. What is the difference between a primary cell and a secondary cell?
8. What temperature does the thermocouple in Figure 3.15 indicate for:
   a) air
   b) liquid B
   c) liquid C?

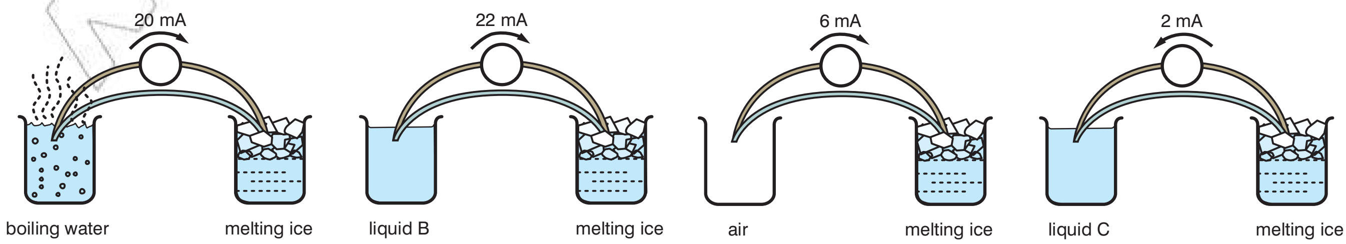


Figure 3.15

9. A thermocouple junction will respond to changes in the temperature of its surroundings more quickly than a mercury-in-glass thermometer would. There are two reasons for this:
   a) the junction is smaller than the bulb full of mercury
   b) the junction is not encased in glass.

   Explain why each of these factors helps the thermocouple to respond quickly.
10. A metal wire is uncharged. Explain how it is possible for a current to flow through it.

3.2 Ohm's law and electrical resistance

By the end of this section you should be able to:

- Describe factors affecting the resistance of a conductor.
- Write the relationship between resistance R, resistivity ρ, length l and cross-sectional area A of a conductor.
- Calculate the resistance of a conductor using the formula $R = \rho l/A$.
- Find the relationship between resistivity and conductivity.
- Construct and draw an electric circuit consisting of a source, connecting wires, resistors, a switch and a bulb using their symbols.
- Explain why an ammeter should be connected in series with a resistor in a circuit.
- Explain why a voltmeter should be connected in parallel across a resistor in a circuit.
- Do experiments using an ammeter and a voltmeter to investigate the relationship between current and p.d. for metallic conductors at constant temperature.

Electrical resistance

The size of the current a cell will pump round a circuit depends on two things. One is the electromotive force, or e.m.f., of that cell, measured in volts. As we shall see in Section 3.4, adding a second cell **in series** (the current passes through one cell then the other) with the first makes a battery of cells, which has twice the e.m.f. of a single cell, and the charge will be pumped round the circuit at twice the rate. In other words, the current will double.

The second factor determining the current is the circuit through which the cell must drive the charge. Is it an easy circuit made of thick pieces of a good conductor, or is it a more difficult circuit consisting of thin wire made from a metal that does not conduct electricity so well? It is this second factor we are going to consider now. Does the circuit have a low **resistance**, or does it have a high resistance?

We specify the resistance of a circuit by the number of volts of 'battery power' we would need to get a current of 1 A to flow round it. A low resistance circuit might need only 2 V per ampere (2 V/A), say. This suggests that a 1 V battery would produce a flow rate of 0.5 A round it, or that to get a current of 3 A going you would need to provide an e.m.f. of 6 V. A less easy circuit might have a resistance of 200 V/A; in other words, as many as 200 V would be needed to establish a current of just 1 A. It is important to study the behaviour of resistors in electrical circuits.

KEY WORDS

in series *wiring an electrical circuit so that there is only one path for the current to take between any two points*

resistance *the opposition to a flow of current in an electrical circuit*

Measuring the resistance of a resistor

The principle is simple: apply a voltage across the resistor, and measure the size of the resulting current. A suitable circuit is shown in Figure 3.16 where *R* is the resistor to be measured.

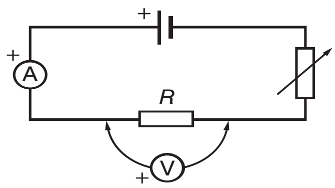


Figure 3.16 *Measuring the resistance of a resistor.*

Figure 3.16 also shows the symbol of a variable resistor (sometimes called a 'rheostat'). By moving a slider or rotating a knob you can alter its resistance. This in turn will alter the total resistance of the whole circuit, and will therefore control the current drawn from the battery which then passes through the ammeter.

In a lighting circuit a variable resistor would act as a dimmer switch to make the bulb fainter or brighter.

The ammeter may be placed anywhere in the circuit since the same current flows all the way round. The voltmeter is not part of the circuit itself; it is placed alongside to measure the drop in voltage between the two ends of *R*. The variable resistor is not essential, but it enables you to alter the voltage drop across *R* and the current through it so as to get check readings.

If you try this activity for yourself, you should choose a resistor to measure that is not significantly warmed by the current you send through it. A length of resistance wire open to the air should do, but avoid using a light bulb. You will find out why in the next section.

Your results should be recorded in a table like that in Figure 3.17.

p.d. *V* across *R* (volts)	current *I* through *R* (amperes)	$R = \frac{V}{I}$ (ohms)
	Average	

Figure 3.17

KEY WORDS

Ohm's law *the current that flows through a conductor is proportional to the potential difference between its ends*

As you repeat the experiment for different values of V and I, you should find that the value for R remains constant. This illustrates **Ohm's law**.

Ohm's law

For a metal wire at a constant temperature, the current that flows through it is proportional to the potential difference (the voltage drop) between its ends.

In other words, if the voltage drop across the wire doubles, charge will flow through it at exactly twice the rate (that is, the current doubles too). Put yet another way, $\frac{V}{I}$ stays constant: the wire's resistance does not change. Notice that this will apply only if the temperature of the conductor does not change.

This can be put into a formula. The current I that is produced when a battery of e.m.f. V is connected to a circuit of resistance R is given by:

$$I = \frac{V}{R}$$

Current (A)	I	ampere
Voltage (V)	V	volt
Resistance (Ω)	R	ohm

To make the current larger you could either increase V by adding another cell or you could reduce the resistance of the circuit.

Worked example 3.6

A 12 V battery is connected to a circuit. If the current is 2 mA, what must the resistance of the circuit be? (Notice here that the current is given as 2 mA. Before it can be entered into the formula it must be converted to 0.002 A (or to 2×10^{-3} A.)

$I = \frac{V}{R}$, so rearranging we get $R = \frac{V}{I} = \frac{12}{0.002} = 6000\ \Omega$

A thousand ohms is one kilohm (rather than 'kiloohm'), so the answer could be expressed as 6 kΩ.

The ohm (Ω)

The unit for resistance is the ohm (Ω).

It is named after Georg Ohm (1787–1854), a German physicist who was one of the first to investigate how currents flowed in circuits.

The abbreviation for ohm should really be a capital 'O', but that could be confused with a zero. Luckily there is a letter in the Greek alphabet called 'omega', which provides the ideal replacement: the abbreviation for ohms, therefore, is 'Ω', which is a capital omega.

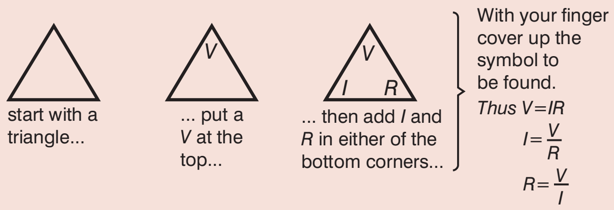
Worked example 3.5

A 12 V car battery is connected to a circuit for which total resistance is 6 Ω. What current will flow?

$$I = \frac{V}{R} = \frac{12}{6} = 2\text{ A}$$

A trick to help you with the calculations

If you have had a problem with changing $I = \frac{V}{R}$ into $R = \frac{V}{I}$, there is a method which will help. The diagram shows you how.



Provided you remember that V goes at the top, you will not go wrong.

Worked example 3.7

What voltage battery would be needed to send a current of 3 A round a circuit for which the total resistance is 4 Ω?

Here $I = 3$ A and $R = 4\ \Omega$.

$V = IR = 3 \times 4 = 12$ V

Factors affecting the resistance of a conductor

Effect of heat on resistance

If you display the current and voltage readings measured when studying a resistor at constant temperature (recorded in a table such as Figure 3.17) in the form of a graph of the current through the resistor plotted against the voltage drop applied across it, you should get a straight line (Figure 3.18). Double the potential difference (p.d.) across the wire, and you will double the current flowing through it.

However, if you use a light bulb as your resistor and take a range of readings such that the filament of the bulb is varied from not even red-hot to brilliantly white-hot, the graph of your readings will form a curve (Figure 3.19). As you increase the voltage and the lamp glows more brightly, the current does not rise as rapidly as expected.

As the filament becomes white hot, its temperature increases by at least several hundred degrees. The resistance of metals rises with temperature, and that is why a hot bulb does not conduct as well as a cool one.

Instead of taking current and voltage readings, it is useful to measure the resistance of a light bulb at as wide a range of currents as possible. At one extreme, readings should be taken when the current is so small that the filament is not even glowing feebly red; at the other extreme the bulb should be brighter than normal.

Use these readings to plot a graph of resistance against current, as in Figure 3.20 on the next page. If the temperature had not changed, the resistance would have been constant (line (a)). As it is, the resistance rises markedly (line (b)).

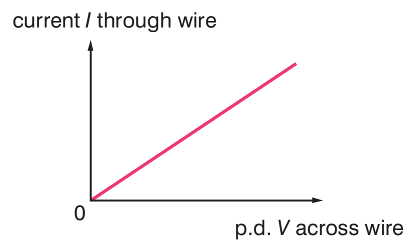


Figure 3.18 *A graph of current against voltage drop for a resistor, R.*

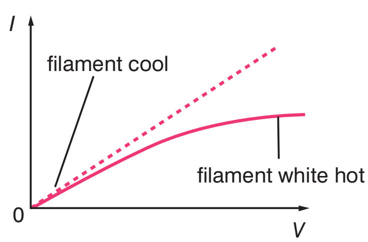


Figure 3.19 *As the temperature of the filament rises, so does its resistance.*

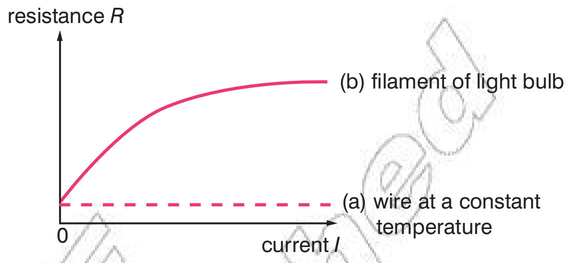


***Figure 3.20** Resistance–current graph.*

Worked example 3.8

A copper resistor (α = 4.3×10^{-3} K^{-1}) whose value is 10.0 Ω at 0 °C is warmed from that temperature up to 150 °C. Work out its resistance when hot.

Increase in resistance
$= \alpha R_0 \Delta T$

$= 4.3 \times 10^{-3} \times 10 \times 150$
$= 6.45\ \Omega$

Resistance when hot
$= 10 + 6.45 = 16.45\ \Omega$

Notice that the line will not pass through the origin. If you continue the curve back so it approaches or equals zero, you will have the resistance of the bulb when it is cold (that is, at room temperature). At its working temperature the resistance of the bulb may well have risen to ten times that value.

The resistance of a metal increases in an approximate straight line if you plot it against its temperature. The increase in the value of the resistance as it warms up depends on:

- The rise in the wire's temperature, ΔT. The greater the rise, the bigger the increase.
- How many ohms of resistance it possesses. Since this will vary with its temperature, we work on the basis of its resistance at 0 °C and call it R_0. The greater the number of ohms at the start, the greater the increase in resistance will be.
- The metal it is made from.

Putting these together, we can say:

the increase in resistance = $\alpha R_0 \Delta T$

Alpha (α) is a constant, which varies from one metal to another. We call it the metal's temperature coefficient of resistance. The units of α have to be K^{-1} ($kelvin^{-1}$) in order to give an answer that is in ohms. You will probably be working in degrees Celsius rather than kelvin, but there is no need to worry, remember that a rise in temperature of 10 °C is exactly the same as a rise of 10 K.

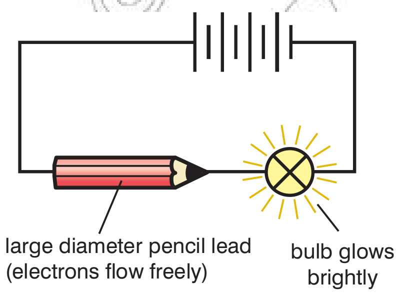


***Figure 3.21** A large diameter pencil lead, bright bulb*

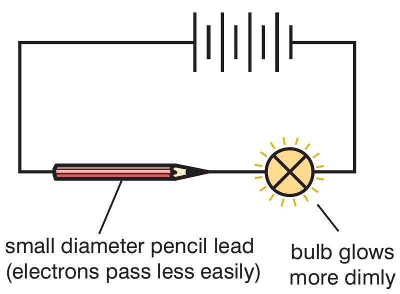


***Figure 3.22** A small diameter pencil lead, dim bulb*

Effect of length and diameter of resistor on resistance

Connect a light bulb (1 to 2 V) in series with a pencil lead (pencil 'lead' is made of carbon, an element frequently used in resistors) and a 6 V battery (four 1.5 V cells connected in series). Turn off the room lights and observe the brightness of the bulb.

Move one wire contact along the *length* of the pencil lead and observe the change in the intensity of light. The light should become brighter as the resistor becomes shorter, as there is less resistance impeding the circuit.

Repeat the activities with a pencil lead of different *diameter* and observe the changes in the intensity of light. The light should become brighter when the diameter of the resistor is larger as it is easier for the current to flow.

The relationship between resistance R, resistivity ρ, length l and cross-sectional area A of a conductor

The resistance of a metal wire at a given temperature is determined by three factors:

- Its length l, in metres – the resistance is proportional to l, so if the length doubles so does the resistance.
- Its area of cross-section A, in m^2 – the resistance is inversely proportional to A, so a wire with twice the cross-sectional area will have only half the resistance.
- The material from which the wire is made – copper, for example, is a better conductor than iron.

Thus the resistance R of a wire can be expressed in the form:

$$R = \frac{\rho l}{A}$$

The symbol ρ (the Greek letter rho) is a constant, the value of which depends on the material from which the wire is made – the value for iron will be greater than the value for copper. We call ρ the resistivity of the material: it is defined by the equation above.

The units of resistivity are Ω m – ohms multiplied by metres. To see this, consider the units of the right-hand side. They will be Ω m × m divided by m^2, which works out correctly to give the resistance in ohms.

A tip for you

When you come to tackle a question on this topic, especially in an exam, you may have a moment of doubt: is the expression $R = \frac{\rho l}{A}$, or is it $R = \frac{\rho A}{l}$?

If you remember that the units of ρ are Ω m, you can quickly check that the second expression would give units for R of Ω m^2, which is wrong.

Worked example 3.9

What is the resistance of a copper cable that has a cross-sectional area of 1 cm^2 and a length of 2 km? The resistivity of copper is 2×10^{-8} Ω m.

Be careful over the units.

$l = 2 \text{ km} = 2 \times 10^3 \text{ m}$

$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$ (since there are 100 × 100 centimetre squares in a metre square)

$$R = \frac{\rho l}{A}$$

Putting in the values, we get

$$R = \frac{2 \times 10^{-8} \times 2 \times 10^3}{1 \times 10^{-4}} = 0.4\ \Omega$$

The relationship between resistivity and conductivity

The resistivities of most metals are in the range 10^{-7} to 10^{-8} Ω m. Those with larger resistivities conduct electricity less well.

The resistivity of an insulator such as dry polythene may be as high as 10^{15} Ω m.

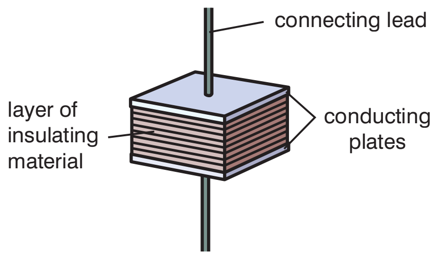


Figure 3.23 *The capacitor.*

Conductors and insulators are two extremes, but there are only a few materials in between these extremes: germanium at room temperature, for instance, may display a resistivity of around 0.001 Ω m. We call such materials **semiconductors**.

The capacitor (Figure 3.23) is an electrical component made of a combination of resistor and insulator. Many circuits use the capacitor to store and release charge. While there are many types of capacitor, they all have the same basic features – two conducting plates separated by a layer of insulating material (see page 71).

KEY WORDS

semiconductors *materials that have an electrical conductivity between that of a conductor and an insulator*

ammeter *an instrument used to measure the electric current in a circuit*

series circuit *an electrical circuit in which the current passes through each point in turn*

voltmeter *an instrument used to measure potential difference in a circuit*

potential difference *the difference in electrical charge between two points in a circuit measured in volts*

Constructing and drawing electric circuits

An electric current is a flow of charge. A circuit is made of conducting materials, with a device such as a cell, which keeps pumping the charges round the circuit.

Figure 3.22a shows the symbol for a cell. We often refer to it as a battery, though strictly speaking this is incorrect: 'battery' is the term reserved for a whole collection of cells acting together. The plus and minus signs are not usually marked on the drawing, so you must remember which is which. Remember too that **conventional current flows round the circuit from positive to negative**.

Figure 3.22b represents a battery of four cells. Notice that in order for them all to be pumping the charge the same way, the '+' terminal of one cell has to be joined to the '–' terminal of the next one.

Figure 3.24 *a) The symbol for a cell; b) a battery of four cells.*

The symbols for a cell, a light bulb, an ammeter and a switch are shown in the circuit in Figure 3.25. The **ammeter** is an instrument that measures the size of a current. It is fitted into the circuit so the current to be measured flows through it. In order that the ammeter does not reduce the current it has been put in to measure, it is essential that the instrument allows the current to flow through it freely (has a low resistance).

It is important that you connect the ammeter the correct way around, otherwise the current would try to twist it backwards. The terminal on it marked '+' needs to be joined to the positive side of the battery, as shown in Figure 3.25.

(Sometimes the terminals are colour-coded. The '+' terminal will be red, and the '–' terminal black.)

Figure 3.26 shows the symbols for a range of electrical components. If you have access to equipment, test what you have learnt so far by using some of these components to make circuits such as that shown in Figure 3.25.

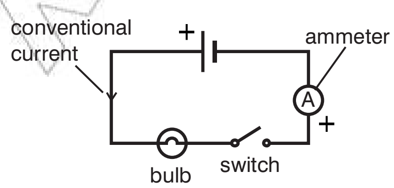


Figure 3.25 *A simple circuit.*

Try constructing the circuit shown in Figure 3.16 (page 93). Use the ammeter and **voltmeter** to make measurements, varying the current using the variable resistor. Record your measurements and discuss your findings with the class.

If you have access to other equipment – such as resistors, variable resistors, ammeter, voltmeter, capacitors – use these and observe what happens when you construct a circuit. Using the symbols for each component, draw a circuit diagram.

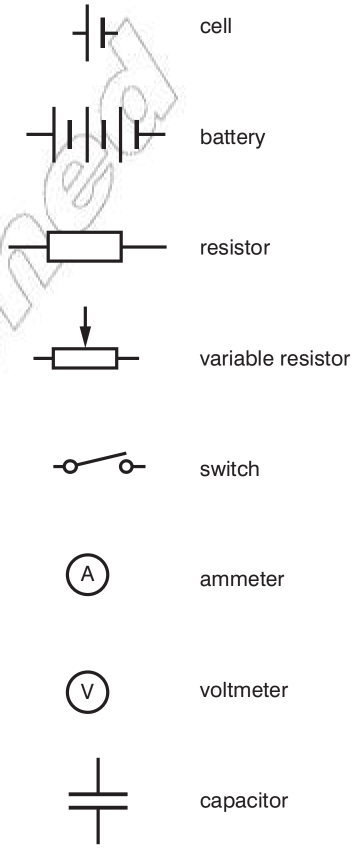


Figure 3.26 *Some circuit symbols.*

The correct position for an ammeter in a circuit

Do you think the current would be the same both before and after the light bulb in Figure 3.25? We will investigate this for the slightly more complex circuit of Figure 3.27 where there are two bulbs.

We call a circuit like this a **series circuit**: the current has to pass first through one bulb and then through the other. Notice again the way the three ammeters have been connected, with their '+' terminals nearer the positive side of the battery.

The result may surprise you:

the current has the same value all the way round a series circuit.

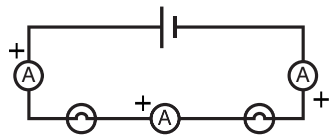


Figure 3.27 *Testing the current before and after it passes through the light bulbs.*

The correct position for a voltmeter in a circuit

Consider the circuit of Figure 3.28, which shows two resistors in series in a circuit. What is the voltmeter doing? You may think of the voltmeter as being connected in parallel (see Section 3.3 for a description of parallel circuits) with the first resistor. This is true, but it is probably not the best way of looking at it.

Unlike an ammeter, the voltmeter is not part of the circuit. If you wish to measure a voltage drop, you place the voltmeter on the bench nearby. One lead comes from the voltmeter to see what conditions are like at one point in the circuit, while the other lead goes to a second point in the circuit to find out what things are like there: the voltmeter then indicates the difference between the two points. We call it the **potential difference** (p.d.) between the ends of the resistor, and measure it in volts. Sometimes we speak of it as the voltage drop (or just the voltage) across the resistor.

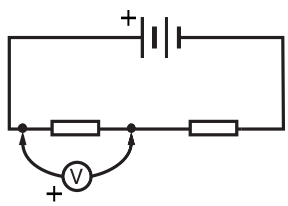


Figure 3.28 *A voltmeter connected in a circuit.*

The voltmeter shows the difference in something – but the difference in what? It might help you to think of a comparison with heat: a temperature difference makes thermal energy flow, and a potential difference makes charge flow.

Ideally a voltmeter should draw no current from the circuit it is investigating. It must therefore have a very high resistance.

Summary

- In a series circuit, the current is the same all the way round.
- You calculate current using $I = \frac{V}{R}$.
- You can measure the value of a resistor with an ammeter and voltmeter by applying Ohm's law.
- The resistance of a metal increases as its temperature rises. Its temperature coefficient of resistance (α) provides a numerical measurement of this.
- You can work out the resistance of a wire from $R = \frac{\rho l}{A}$, where ρ is the resistivity of the material of the wire.

Review questions

1. A battery of e.m.f. 6 V is connected to a circuit with a total resistance of 12 Ω. What current would you expect to flow?
2. A flashlight bulb has '2.4 V 0.3 A' marked on it. This means 'If you connect it to a 2.4 V battery a current of 0.3 A will be driven through it; this will make it white-hot'.
   a) Estimate the resistance of that flashlight bulb when it is lit up.
   b) If the bulb was connected to the mains (either 110 V or 220 V) instead, would the current initially be 0.3 A, more than 0.3 A or less than 0.3 A? Give your reason.
   c) What would happen next? Explain.
3. The resistivity of iron is 1.0×10^{-7} Ω m. Find the resistance of a 12 km length of a railway line with a cross-sectional area of 200 cm^2.
4. A tungsten resistor ($\alpha = 5.8 \times 10^{-3}$ K^{-1}) with a value of 10.0 Ω at 0 °C is warmed from that temperature up to 150 °C. Work out its resistance when hot.
5. Figure 3.29 shows a circuit with a cell, a variable resistor and a lamp.
   a) Draw a circuit diagram of this arrangement.
   b) Redraw the circuit diagram adding an ammeter to measure the current in the circuit, and a voltmeter to measure the p.d. across the lamp.
   c) Describe how you would use this circuit to measure the current through the lamp for various p.d.s across it and describe the shape of a graph of p.d. against current that would be obtained.
   d) What would you change in this circuit to demonstrate Ohm's law?

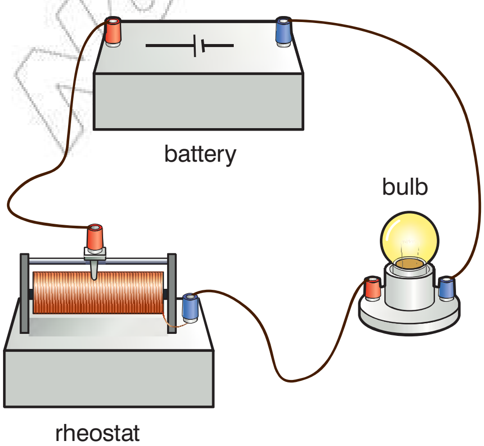


Figure 3.29

6. A student designs the circuit shown in Figure 3.30 in order to try to test Ohm's law.
   a) What is wrong with this circuit?
   b) Draw the circuit diagram the student should have used.
7. What is the difference between a cell and a battery?

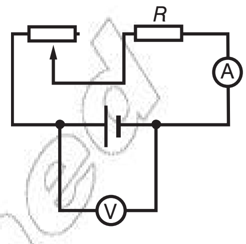


Figure 3.30

3.3 Combinations of resistors

By the end of this section you should be able to:

- Identify combinations of resistors in series, parallel and series–parallel connection.
- Derive an expression for the effective resistance of resistors connected in series.
- Derive an expression for the effective resistance of resistors connected in parallel.
- Calculate the effective resistance of resistors connected in series.
- Calculate the effective resistance of resistors connected in parallel.
- Calculate the current through each resistor in simple series, parallel and series–parallel combinations.
- Calculate the voltage drop across each resistor in simple series, parallel and series–parallel connections.

KEY WORDS

resistors *electrical components that limit or regulate the flow of electrical current in a circuit*

Resistors in different combinations

We shall now look at **resistors** connected in electric circuits in different ways – in series (Figure 3.31) and in parallel (Figure 3.34). In both cases, it might help to think of an electrical circuit as a large crowd of people trying to get into a stadium.

In Figure 3.29 overleaf, two resistors – 1 Ω and 3 Ω – are connected in series. Think of a line of people trying to get into a stadium, they have to queue up to get through one gate (1 ohm) and then through a narrower one (3 ohms) on their way into the stadium.

In Figure 3.35 on page 104 three resistors – 2 Ω, 3 Ω and 6 Ω – are connected in parallel. Think of this as the queue for the stadium again. In this case, the stadium has three (differently sized) gates for people to use, and they can enter the stadium much more easily.

Resistors connected in parallel are thus seen to have a lower total resistance than resistors connected in series.

In practice, electrical circuits can be quite complicated and can consist of resistors and other electrical components connected both in series and parallel.

Activity 3.6: Series and parallel circuits

Do these activities in groups.

Connect one bulb in series with a battery and note its brightness. Now connect a second and third bulb in series with the bulb.

Is there any change in the brightness of the first bulb when the second and third bulbs are added?

Once all bulbs are lit, remove one of the bulbs. What happens to the brightness of the others?

Connect one bulb to a battery and note its brightness. Now connect a second and third bulb in parallel with this bulb.

Is there any change in brightness of the first bulb when the second or third ones are added?

Once all bulbs are lit, remove one. Is there any change in the brightness of the remaining bulbs?

Series combination

Look at the circuit in Figure 3.31. There are two resistors (1 Ω and 3 Ω) in series. Each resistor offers a different resistance to the current flow and there is therefore a different voltage drop across each resistor.

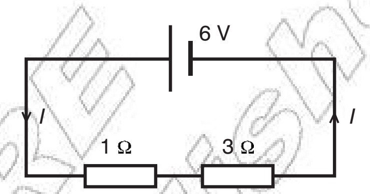


Figure 3.31 *Resistors in series in a circuit.*

With resistors in series, it is easy to work out the resistance of the circuit as a whole. You simply add up the separate values. In this case the circuit has a resistance of 4 Ω. (Be sure you could then work out that a current of 1.5 A will be drained from the battery, as $I = \frac{V}{R} = \frac{6}{4}$ from Ohm's law.)

For a series circuit, the total resistance $R = R_1 + R_2$

where R_1 and R_2 are the resistances of the separate components.

Worked example 3.10

Consider the circuit shown in Figure 3.32.

a) Calculate the total resistance of the whole circuit.

b) Calculate the current in the circuit.

Since the circuit is a series one, the same current must flow through all three resistors. Consider the 3.5 Ω resistor.

c) Calculate the voltage drop necessary to send 2 A through 3.5 Ω.

d) Calculate the p.d. across the other two resistors.

You will need to use the relationship $R = \frac{V}{I}$ in the form of $V = IR$ and $I = \frac{V}{R}$.

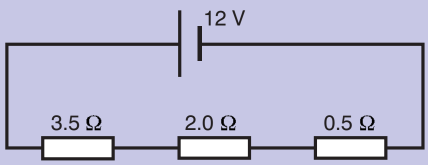


Figure 3.32

a) Total resistance in the circuit (series circuit)

= 3.5 + 2 + 0.5 = 6 Ω

b) Voltage supplied to the circuit is 12 V

Resistance in the circuit is 6 Ω

From Ohm's Law, $I = \frac{V}{R}$

$I = \frac{21}{6} = 2$ A

c) Resistance = 3.5 Ω

Current = 2 A

$V = IR$

$V = 2 \times 3.5 = 7$ V

The voltage drop across the 3.5 Ω resistor is 7 V.

d) For the 2 Ω resistor:

resistance = 2 Ω, current = 2 A

$V = IR = 2 \times 2 = 4$ V

For the 0.5 Ω resistor:

resistance = 0.5 Ω, current = 2 A

$V = IR = 0.5 \times 2 = 1$ V

Worked example 3.11

Calculate the size of resistor X in Figure 3.33.

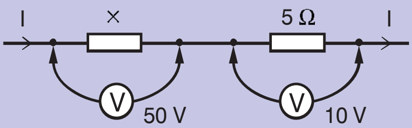


Figure 3.33

First look at the 5 Ω resistor. From $I = \frac{V}{R}$, a p.d. of 10 V across it must mean that a current I is flowing through it, given by:

$$I = \frac{V}{R} = \frac{10}{5} = 2 \text{ A}$$

Now look at resistor 'X'. Since they are in series the same 2 A is flowing through it. To achieve this there is a p.d. of 50 V across it.

$$R = \frac{V}{I} = \frac{50}{2} = 25 \ \Omega$$

Parallel combination

It is less obvious how to work out the overall resistance of a circuit that consists of two resistors, not in series but in parallel (Figure 3.34).

If the two resistors are R_1 and R_2, then they behave like a single resistor R the value of which is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

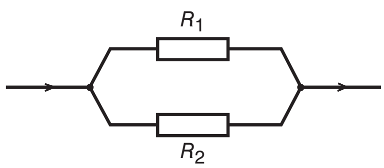


Figure 3.34 *Resistors in parallel.*

Worked example 3.12

A circuit consists of a 2 Ω resistor and a 3 Ω resistor in parallel. A 3 V battery is connected to the circuit. How big a current is drawn from it?

We must first calculate the effective resistance of the circuit using the formula for resistors in parallel.

Here,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

That gives us 'one over *R*', so to find *R* we must turn it over.

$$R = \frac{6}{5} = 1.2\ \Omega$$

The question now becomes 'What current will a 3 V battery send round a 1.2 Ω circuit?'

$$I = \frac{V}{R} = \frac{3}{1.2} = 2.5\ \text{A}$$

The voltage drop across resistors in parallel

Look at the circuit of Figure 3.35. Remembering that there is no voltage drop down a conducting lead, you should be able to see that the left-hand end of all three resistors is at +6 V and their right-hand ends are at 0 V.

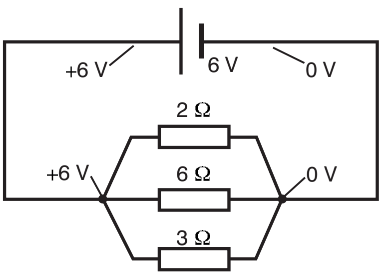


Figure 3.35

That illustrates what happens in this case:

if resistors are connected in parallel, they all have the same voltage drop across them.

This provides us with a means to work out the current in each branch of the circuit. Take the 2 Ω resistor as an example. The full 6 V of the battery is dropped across it, so the current *I* through it is given by:

$$I = \frac{V}{R} = \frac{6}{2} = 3\ \text{A}$$

Worked example 3.13

For the circuit shown in Figure 3.36:

a) calculate the voltage drop across each resistor

b) calculate the current through each resistor

c) calculate the effective resistance of the circuit.

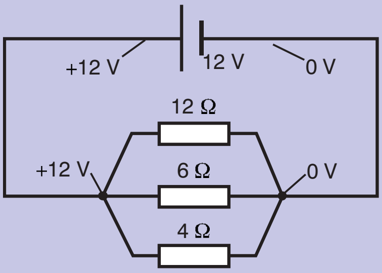


Figure 3.36

a) As the resistors are connected in parallel, the voltage drop will be the same across each, and will be equal to the full 12 V of the battery.

b) The voltage across the 12 Ω resistor is 12 V,

$I = \frac{V}{R} = \frac{12}{12} = 1$ A

The voltage across the 6 Ω resistor is 12 V,

$I = \frac{V}{R} = \frac{12}{6} = 2$ A

the voltage across the 4 Ω resistor is 12 V,

$I = \frac{V}{R} = \frac{12}{4} = 3$ A

c) Total resistance of the circuit is R.

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

$$= \frac{1 + 2 + 3}{12} = \frac{6}{12} = \frac{1}{2}$$

$R = 2\ \Omega$

Worked example 3.14

Two resistors of 10 Ω and 15 Ω are connected. What is their combined resistance if they are connected:

a) in series

b) in parallel?

a) In series

$R = R_1 + R_2$

$= 10 + 15 = 25\ \Omega$

b) In parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{10} + \frac{1}{15}$$

$$= \frac{3 + 2}{30} = \frac{5}{30}$$

$$R = \frac{30}{50} = 6\ \Omega$$

Summary

- Resistors in series simply add.
- Resistors in parallel add by a ‘ $\frac{I}{R}$ ’ formula. Two resistors in parallel will conduct better than either one on its own.
- The resistance of the whole circuit includes the resistance within the battery itself, and this may not always be negligible.

- Where the circuit branches in a parallel circuit, the sum of the currents approaching the junction equals the sum of the currents leaving it.
- In a series circuit, the separate voltage drops (potential differences) across all the resistors add up to the voltage of the battery.
- In a parallel circuit each branch will have the same voltage drop across it.

Review questions

1. What should ammeters (a), (b) and (c) read in Figure 3.37?

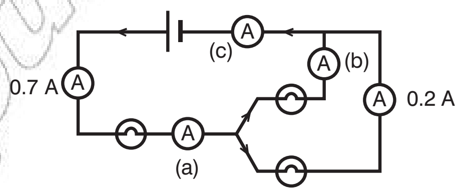


Figure 3.37

Which bulb should be the brightest? Why?

2. Give the missing ammeter readings a and b in Figure 3.38. Suggest why more current flows through some bulbs than through others.

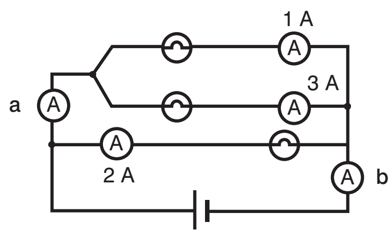


Figure 3.38

3. Give the missing ammeter readings c and d in Figure 3.39. Assume all the bulbs are identical ones. How would you expect the brightness of the different bulbs to compare?

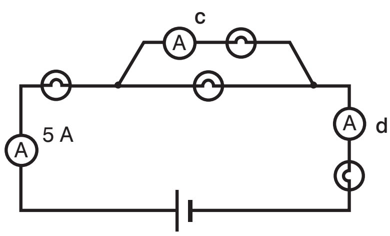


Figure 3.39

4. Draw a circuit diagram to show a battery of two cells with three light bulbs connected in parallel. Include two switches: one must turn one of the bulbs on and off, and the other must do the same for the other two bulbs together.
5. For the circuit diagram in Figure 3.40:
   a) What is the effective resistance of the two 4 Ω resistors in parallel?
   b) What is the total resistance of the circuit?
   c) Work out the current *I*.

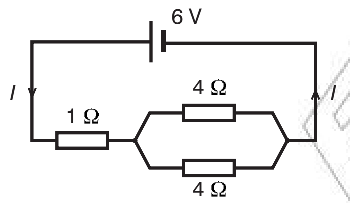


Figure 3.40

6. What single resistor could be placed in the box to replace the two shown in Figure 3.41?

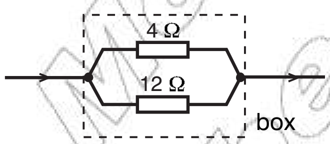


Figure 3.41

7. Calculate the effective resistance of a 25 Ω resistor and a 100 Ω resistor in parallel. What current will be drawn from a 50 V supply connected to the circuit?
8. a) Work out the total resistance of the circuit in Figure 3.42.
   b) What current will be drawn from the battery?

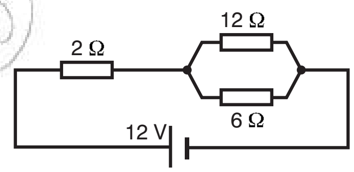


Figure 3.42

10. In the circuit of Figure 3.43 the battery has negligible internal resistance. Calculate:
    a) the total resistance in the circuit
    b) the current flowing in the circuit
    c) the potential difference across the 50 Ω resistor.

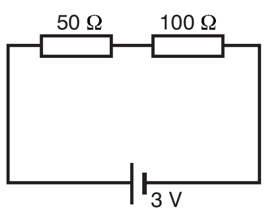


Figure 3.43

3.4 E.m.f. and internal resistance of a cell

By the end of this section you should be able to:

- Define the electromotive force (e.m.f.) of a cell.
- Distinguish between e.m.f. and terminal p.d. of a cell.
- Write the relationship between e.m.f., p.d., current and internal resistance in a circuit.
- Use the equation $V = E - Ir$ to solve problems in a circuit.
- Identify cell combinations in series and parallel.
- Compare the e.m.f. of combinations of cells in series and parallel.

The electromotive force of a cell

As we saw in Section 3.1, a cell pumps charge round a circuit. Its effectiveness at doing this is called its electromotive force (e.m.f.), in volts. A voltmeter is an instrument that can measure the e.m.f. of a cell: it has two leads going to it, and one is connected to each terminal of the cell.

Volts do not flow round a circuit. Volts cause coulombs of charge to flow at a rate of amperes. If we know the resistance *R* of a circuit, we can predict the current *I* that a battery will send round it, from $I = \frac{V}{R}$.

The resistance inside a cell

Suppose you short-circuit a cell. This means that you join its two terminals by a circuit that effectively has no resistance – a short piece of very thick copper wire, for instance. The cell has an e.m.f. *V*, but the circuit apparently has no resistance *R*. What happens then? Does the current increase without limit?

The point we are forgetting is that the cell has to pump the charge round the whole circuit, and that includes the part within the cell as well as the external circuit. Internal resistance varies between different cells, but it is what finally sets a limit to the current a cell can supply.

A 1.5 V torch cell (represented in Figure 3.45 by the dotted line) typically has an internal resistance of up to an ohm. This means that even if you short-circuit the cell, there is still an ohm of resistance. The biggest current it can deliver is given by $I = \frac{V}{R} = \frac{1.5}{1} = 1.5$ A.

So far, we have taken it for granted that if you double the e.m.f. in a circuit, you will double the current. This should be checked experimentally, but it needs some careful thought in order to give it a fair test.

The obvious thing to do is to add cells one at a time to a circuit, measure the current and see if it increases accordingly. A fair test

Activity 3.7: Measuring electromotive force and terminal voltage of a cell

Set up a circuit consisting of a dry cell, a bulb, a switch and a voltmeter. Connect the voltmeter as shown in Figure 3.44.

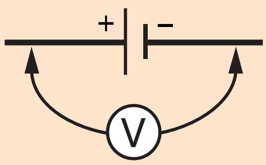


Figure 3.44

Take the reading in the voltmeter while the switch is on.

Then another reading while the switch is off.

Compare the two readings.

Which one is larger? Which reading is the e.m.f.? Discuss with your group.

requires that you change only one thing at a time, which means that you must not change the resistance of the circuit.

There are two problems here:

1. Adding another cell adds a bit more resistance to the circuit. All you can do about that is to choose cells that have a very low internal resistance (such as the lead–acid cells that make up a car battery) or to use a circuit for which the resistance is so high that another ohm or two makes virtually no difference.
2. The resistance of a metal wire changes as it heats up. This rules out circuits containing light bulbs, because as you add more batteries their temperature changes from less than red-hot to strongly white-hot: a change of perhaps 1000 degrees!

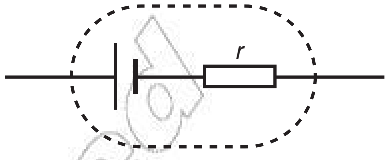


Figure 3.45 *Representation of a cell indicating its internal resistance r.*

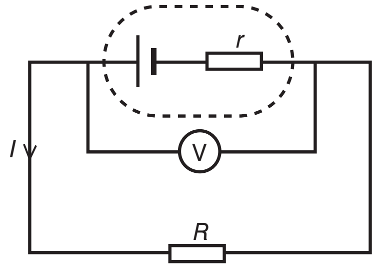


Figure 3.46

The difference between e.m.f. and terminal p.d. of a cell

A high resistance voltmeter connected across the terminals of a cell can give a good reading of the cell's e.m.f. However, if the cell is then connected in a circuit where the current is high, the reading on the voltmeter drops. This drop is caused by the cell's internal resistance.

Figure 3.46 shows the cell from Figure 3.45 connected in a circuit.

If the e.m.f. of this cell is E and it is connected to a resistor R, the potential difference across its terminals (V) will now be less than E because some of its energy is used to drive the current (I) through the internal resistance (r) in the cell. If R decreases, the current (I) increases and the terminal p.d. (V) of the cell will decrease.

The relationship between e.m.f., current and internal resistance in a circuit

The relationship between a cell's e.m.f. (E) and internal resistance (r) and the current (I) and resistance (R) in a circuit is given by the expression:

E = p.d. across R + p.d. across r

$E = IR + Ir$

$E = I(R + r)$

As $V = IR$ (Ohm's law) this can also be written as $E = V + Ir$, or $V = E - Ir$.

Worked example 3.15

In the circuit shown in Figure 3.46, the cell's e.m.f. is 10 V and its internal resistance is 2 Ω. Find the p.d. across the terminals of the cell when it is connected to a 3 Ω resistor.

First find the current in the circuit:

$E = I(R + r)$

$$I = \frac{E}{(R + r)}$$

$$= \frac{10}{(3 + 2)} = 2\text{ A}$$

The p.d. across the cell terminals is equal to the p.d. across the resistor (3 Ω). Find the p.d. across the resistor:

$V = IR = 2 \times 3 = 6$ V

Therefore the p.d. across the terminals of the cell is 6 V.

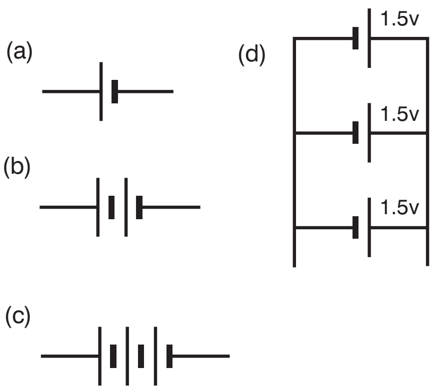


Figure 3.47 *Combining cells.*

Combinations of cells in series and parallel

In Figure 3.47, (a), (b) and (c) show cells connected in series, (d) shows cells connected in parallel. Each cell has an e.m.f. of 1.5 V.

The total e.m.f. of cells connected in series is the sum of the e.m.f. of each cell: $E = E_1 + E_2 + \dots + E_n$. Equipment requiring high power uses cells arranged in series.

The e.m.f. of cells connected in parallel is the e.m.f. of an individual cell: $E = E_1 = E_2 = \dots = E_n$. The current supplied by a parallel arrangement of cells can be maintained for far longer than that supplied by a single cell, so equipment requiring a steady corrent for long periods will use cells arranged in parallel.

(a) 1 cell	total e.m.f. = 1.5 V
(b) 2 cells in series	total e.m.f. = 1.5 + 1.5 = 3 V
(c) 3 cells in series	total e.m.f. = 1.5 + 1.5 + 1.5 = 4.5 V
(d) 3 cells connected in parallel	total e.m.f. = 1.5 V

Activity 3.8: Connecting cells together

For this activity you can use either the fruit cells made in Activity 3.3 or purchased 1.5 V cells. Connect a small light bulb in a circuit with one cell. Observe its brightness. Now take another cell and connect it in the circuit as shown in Figure 3.48a. Observe the brightness of the bulb. You can add further cells in this way if you wish.

The cells in Figure 3.48a are connected in series. Now connect two cells together in parallel, as shown in Figure 3.48b. Now connect these cells to a light bulb as shown in Figure 3.48c. Observe its brightness.

How does the brightness of the bulb in the circuit with one cell compare with its brightness in the circuit with two cells in series?

How does the brightness of the bulb in the circuit with the cells in parallel compare to the brightness when the cells are in series?

You can see that the cells are connected negative to negative, positive to positive in Figure 3.48b. See what happens if you connect them negative to positive and positive to negative before connecting them in a circuit. What do you find?

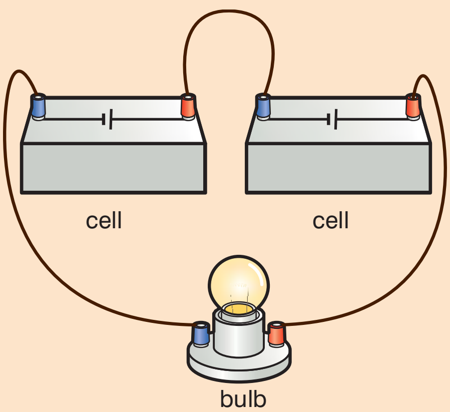


Figure 3.48a

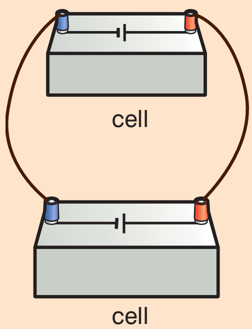


Figure 3.48b

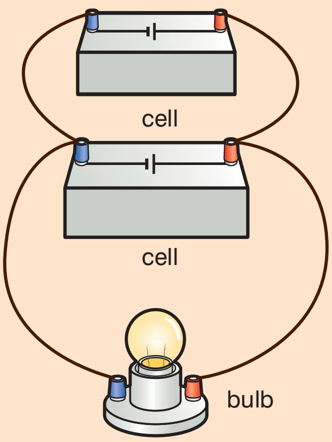


Figure 3.48c

Review questions

1. Imagine you have a brother two years younger than you are. Explain to him why:
   a) the '+' terminal of one cell in a circuit should be connected to the '–' terminal of a second cell, but
   b) the '+' terminal of an ammeter needs to be connected to the '+' terminal of the battery.
2. A circuit consists of three resistors in series: one of 3 Ω, one of 7 Ω and one of 10 Ω. A battery of e.m.f. 12 V and negligible internal resistance is connected to the circuit. What size current will it supply?
3. A battery has an e.m.f. of 3 V and an internal resistance of 1 Ω. What current will it give if it is connected to a circuit of resistance:
   a) 2 Ω
   b) 4 Ω?

   What current will it give if its terminals are short-circuited?
4. In Figure 3.49, A and B represent the terminals of a battery of e.m.f. 4 V and internal resistance 0.5 Ω. *R* is the total resistance of the circuit to which it is connected.
   a) Explain why a voltmeter connected as shown across *R* would read the same as a voltmeter connected across the terminals of the battery.
   b) Calculate the current which flows round the circuit if the resistance of *R* is 1.5 Ω.
   c) What then is the p.d. between the terminals of the battery?

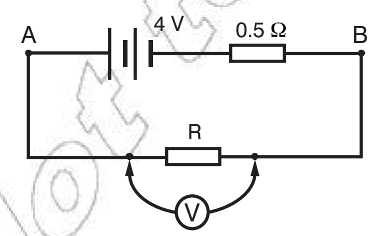


Figure 3.49

5. a) What current is flowing through the 3 Ω resistor in Figure 3.50?
   b) If its internal resistence is 2 Ω , what is the e.m.f. of the battery?

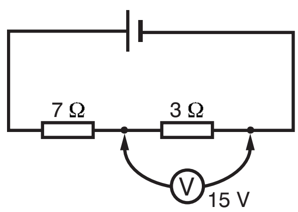


Figure 3.50

3.5 Electric energy and power

By the end of this section you should be able to:

- Define electrical energy and power in an electrical circuit.
- Find the relationship between KWh and joule.
- Use $P = VI = \frac{V^2}{R} = I^2R$ to solve problems in electric circuits.
- Use $W = VIt = I^2Rt = \frac{V^2t}{R}$ to calculate electric energy dissipated in an electric circuit.
- Calculate the cost of electrical energy expressed in KWh.

Worked example 3.16

A 60 W light bulb is switched on for 2 minutes. How many joules of electrical energy does it convert into heat and light in that time?

The power of the light bulb is 60 W; this means that it uses 60 J of energy every second.

It is switched on for 2 minutes = 120 s.

Energy = VIt (and Power = VI)

Total energy supplied = 60 J/s × 120 s = 7200 J

Definitions for electrical energy and power

The word 'power' here means the rate at which energy is being supplied or converted. Power is measured in the units of joules per second (J/s), or watts (W).

The electrical energy produced by a current of I amperes flowing through a p.d. of V volts for a time t seconds is given by:

Energy = VIt J.

Power is the energy produced by an electrical appliance in one second. Thus, if $t = 1$:

Power = $VI \times 1 = VI$ W.

Worked example 3.17

If the potential difference across a working electrical motor is 50 V and the current is 2 A, calculate the power of the motor.

Power = $VI = 50 \times 2 = 100$ W

Consider a resistor that converts all the electrical energy supplied to heat.

The power produced = VI

This can be rewritten (using Ohm's law, $V = IR$) as $IR \times I = I^2R$

Therefore, for a resistor

heat produced per second = power = I^2R.

If V and R are known, then the equation can be written (again using Ohm's law)

power = $VI = V \times \frac{V}{R} = \frac{V^2}{R}$.

Worked example 3.18

Two heating coils A and B produce heat at a rate of 1 kW and 2 kW, respectively, when connected to 250 V mains.

a) Calculate the resistance of each resistor.

b) Find the power they would produce when connected in series to the mains.

a) For the first resistor:

Power = VI

$1000 = 250 \times I$, so $I = \frac{1000}{250} = 4$ A.

So $R = \frac{V}{I} = \frac{250}{4} = 62.5\ \Omega$

For the second resistor:

Power = VI

$2000 = 250 \times I$, so $I = \frac{2000}{250} = 8$ A.

So $R = \frac{V}{I} = \frac{250}{8} = 31.25\ \Omega$

b) If the resistors are wired in series, their total resistance is

$R_1 + R_2 = 62.5 + 31.25 = 93.75\ \Omega$.

using $R = \frac{V}{I}$

$93.75 = \frac{250}{I}$, giving $I = \frac{250}{93.75}$

= 2.67 A.

So power = $VI = 250 \times 2.67 = 666$ W

Notice that this is less than the power produced when either resistor is connected to the mains separately. Can you see why?

Worked example 3.19

Calculate the power of a water heater that draws a current of 10 A from a 220 V supply.

Power = $VI = 10 \times 220 = 2.2$ kW

Cost of electrical energy

Electricity is distributed to homes and businesses and the quantity supplied is measured in kilowatt hours (KWh). A kilowatt hour is the energy used by a 1 kW appliance working for 1 hour. Consumers of electricity are charged for each kWh used.

How many joules are there in 1 kWh?

One kWh = energy transformed by 1 kW (1000 W) for 1 hour (3600 s) = energy when 1000 J are transformed each second for 3600 s = 1000×3600 = 3 600 000 J = 3.6 MJ

Worked example 3.20

How many units (kWh) are used by:

a) a 3 kW electric fire used for 2 hours,

b) a 100 W light bulb used for 15 hours?

a) Number of units = $3 \times 2 = 6$ kWh

b) Number of units = $0.1 \times 15 = 1.5$ kWh

Activity 3.9 Power requirements

Which appliances in your home consume the greatest amount of energy? Is it the refrigerator? The TV set? The electric stove? You can find out by inspecting appliances in your home and determining the number of watts each consumes and multiplying this by the number of hours operated. By law, each electric device must specify power requirements, and these are generally recorded on a small tag located on the appliance or on the power cable connected to it. Inspect all the appliances in your home and record power requirements in a table.

Worked example 3.21

A 1.5 kW electric fire is accidentally left on overnight for eight hours. The cost of a unit of electrical energy is 0.273 Birrs. How much money has been wasted?

The number of units (kWh) = power in kW × time in hours = 1.5 × 8 = 12 kWh

The total cost = 12 × cost of each unit = 12 × 0.273 = 3.28 Birrs

Worked example 3.22

A current of 4 A flows through an electric fire for 1 hour. The supply voltage is 240 V. What energy is transformed by the fire in 1 hour? (1 hour = 60 × 60 seconds)

$W = VIt = 240 \times 4 \times 3600 = 3.456$ megajoules (MJ)

Worked example 3.23

The Ethiopian Power Corporation is distributing power saving lamps to the public free of cost. If an 11 W (0.011 kW) power saving bulb is used in place of the equivalent 60 W (0.06 kW) conventional lamp, and each type of bulb is used for 10 hours a day for four weeks (seven days a week), how much would the customer of the Ethiopian Power Corporation save?
Number of hours = 10 × 7 × 4 = 280
Cost of one unit of electricity is 0.273 Birrs

Conventional lamp
Number of units (kWh) = power in kW × time in hours = 0.06 × 280 = 16.8 kWh
Total cost of using conventional bulb = 16.8 × 0.273 = 4.59 Birrs

Power-saving lamp
Number of units = 0.011 × 280 = 3.08 kWh
Total cost of using power-saving lamp = 3.08 × 0.273
= 0.84 Birrs

The customer would save 3.75 Birrs.

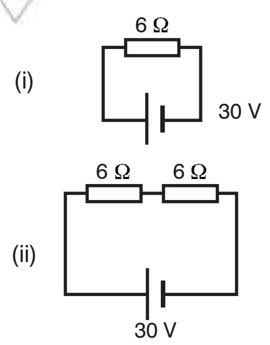


Figure 3.51

Review questions

1. Work out the sources of energy used at home. Can you suggest any economies or improvements?
2. Carry out the same analysis for your school.
3. In the circuit of Figure 3.51(i) calculate:
   a) the current flowing
   b) the number of joules per second (power) of heat produced in the 6 Ω resistor.

Another 6 Ω resistor is added in series with the first (Figure 3.51(ii)). Calculate:

c) the current flowing now

d) the power produced in the two 6 Ω resistors together

e) the power produced in a single 6 Ω resistor.

Comment on the reasons for the difference in the heat produced in a 6 Ω resistor in the two circuits.

3.6 Electric installation and safety rules

By the end of this section you should be able to:

- Understand the dangers of mains electricity.
- Have some awareness of safety features incorporated in mains electrical installations.
- Understand the nature of the generation and supply of electricity in Ethiopia.
- Consider employment prospects in Ethiopia's electricity industry.

KEY WORDS

fuse *protective device for protecting an electrical circuit, containing a wire that melts and breaks when the current exceeds a certain value*

In Ethiopia, while traditional sources of energy, such as wood, are still of great importance, the provision of mains electricity is becoming increasingly significant. Mains electricity is generated, transmitted and distributed by the Ethiopian Electric Power Corporation (EEPCO).

While Ethiopia uses some fossil fuels for electricity generation, the country is fortunate in having access to sources of hydroelectric power at Melka Wakena, Finchaa, Koka, Awash, Tis Abay, Gilgel Gibe and other planned sites, for electricity generation. Ethiopia also has potential for using other renewable energy sources, such as solar, wind and geothermal energy, for generating electricity.

Work is underway to increase the availability of mains electricity in Ethiopia. Mains electricity is much more powerful than the electricity we have studied in this unit, and great care must be taken when using it. Any work being undertaken on mains electrical installations must only be undertaken by those who have qualified through the Electrical Professional Competence Certification Scheme (operated by the Ethiopian Electricity Agency).

Various safety features are incorporated into mains electrical installations; one of these features is the **fuse**, which uses the heating effect studied earlier in this unit. A fuse is a small, thin piece of wire in a glass tube that has metal connectors on each end. It is designed to be used in a circuit to allow a current of a certain size to pass through it. If the current in the circuit increases beyond this point, the fuse heats up and then breaks, causing the current in the circuit to stop. The fuse is a very useful safety device, protecting appliances from surges of current.

KEY WORDS

earthed *circuit connected to the earth, allowing any dangerously large current to be safely discharged*

Electrical safety is also increased if appliances are **earthed**. This means that if there were a fault in an appliance, any dangerous large current would run harmlessly to the earth and cause the fuse to blow, thus preventing injury.

Miniature circuit breakers (MCBs) can now be used in place of fuses in some appliances. They cut off the power if there is a fault that causes the appliance to overheat. They can be re-set when the appliance cools down.

Activity 3.10: Electrical safety

Take five 1.5 V cells, a 2 A fuse and a selection of connecting wires. Connect the circuit shown in Figure 3.52.

Look carefully at the fuse, and while observing the fuse closely, close the switch.

The fuse consists of a very fine length of wire inside a transparent tube with metal connectors at each end.

When the switch is closed, the wire in the tube quickly becomes red and then separates into two pieces. The wire becomes dark again as it cools down. At the end of the experiment, the wire in the fuse has a gap in the centre.

There might be a small lump of metal on one of the broken ends of wire. This is a result of the metal becoming very hot and melting as the current passes through it, before the fuse wire breaks and causes the current to stop.

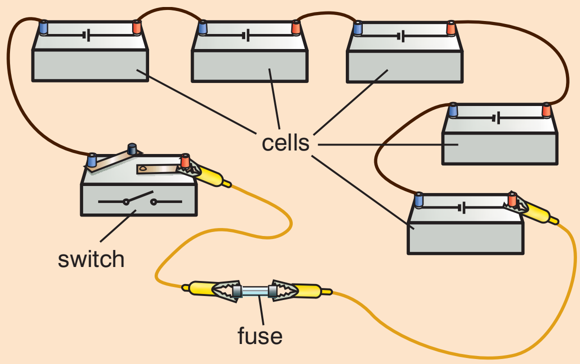


Figure 3.52

Engineering project

New designs for cars employ hybrid motors. Cars use both electricity and gasoline for power. When going down hill, instead of standard brakes, which convert kinetic energy into heat, these cars capture energy by using 'electric brakes' that recharge the battery or convert the energy into rotational energy in a massive fly wheel.

Try designing on paper a hybrid car that captures as much energy as possible. In teams you may wish to build a model of your design, showing innovative ways to save energy.

Earth circuit leakage breakers (ECLBs) are used in place of fuses in domestic electricity supply boards, where they are useful in cutting off the supply very quickly if a fault occurs.

Small, plug-sized ECLBs (also called residual current circuit breakers) are also useful in preventing injury when using electrical equipment outdoors. They will cut off the power supply very quickly if the power cable is cut accidentally.

Review questions

1. For safety, the circuit for a 3 kW metal-bodied electric kettle is both earthed and contains a fuse.
   a) Why is the kettle earthed?
   b) Why does the circuit contain a fuse?
2. In a house you might find i) MCBs, ii) an ECLB.
   a) Say where each of these may be found.
   b) Explain the purpose that each might be serving.

End of unit questions

1. If 36 C of charge pass through a wire in 4 s, what current is it carrying?
2. An appliance has a resistance of 5 Ω and requires a current of 0.5 A for it to work. The only battery available has an e.m.f. of 12 V and negligible internal resistance. What extra resistance will you need to provide to limit the current to its correct value. Draw the circuit you would assemble.
3. A student incorrectly set up the circuit shown in Figure 3.53.
   a) What is wrong with it?
   b) Assume that both the ammeter and the battery have negligible resistance. Starting from one terminal of the battery, what route will all the current take as it flows round the circuit? How much resistance will this circuit have? What will the ammeter read?
   c) What will the voltmeter read?
   d) The student then reconnects the circuit so that the two resistors, the ammeter and the voltmeter are all in series with the battery. What would you expect the ammeter to read now? Give a reason for your answer.

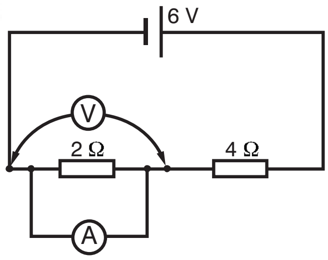


Figure 3.53

4. A 12 V light bulb is put under test, with the following results:

Current through bulb (A)	0.4	0.6	0.8	1.0	1.2	1.4	1.6
p.d. across bulb (V)	1.4	2.6	3.9	5.5	7.4	9.7	12.6

   a) Draw the circuit you would use to obtain these readings.
   b) Copy the table out, and for each set of readings work out the resistance R of the bulb.
   c) Plot a graph of R against I. Why is it not valid to assume the graph goes through the origin?
   d) Use your graph to estimate the resistance of the bulb at room temperature. (Hint: in what circumstance will the bulb not be heated at all?)
5. Constantan is the name of an alloy that is sometimes used for making resistors in the laboratory. Its resistivity is 4.9×10^{-7} Ω m. Calculate the resistance of a 3 m long constantan wire with 1 mm^2 cross-sectional area.
6. Draw a labelled diagram of a dry cell. State the function of each of its parts.
7. The resistivity of copper is 2×10^{-8} Ω m. Work out the resistance of a copper wire of 1 mm^2 cross-sectional area and 3 m long.

8. A battery sends a current of 3 A through a 4 Ω resistor.
   a) What must the e.m.f. of the battery be?
   b) How many coulombs of charge will pass through the resistor in 1 minute?
9. Explain why it is wrong to write the units of resistivity as ohm/metre (Ω/m) with the stroke between the two quantities.
10. Draw a circuit diagram to show a battery of two cells and three light bulbs connected in series. Include a switch to turn the lights on and off. Does it matter where the switch is placed in the circuit?
11. a) What current is flowing round the circuit in Figure 3.54?
    b) How great must the p.d. be across resistor R?
    c) What is the resistance of resistor R?
12. What should be the reading of ammeters A_1 and A_2 in Figure 3.55?
13. These questions refer to the circuit drawn in Figure 3.56.
    a) What are the currents through the 6 Ω branch, through the 3 Ω branch and through the 2 Ω branch?
    b) What is the total current drawn from the battery?
    c) As far as the battery is concerned, what is the total resistance of its whole circuit?
    d) Compare this with the value obtained from

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

14. In Figure 3.57, A and B represent the terminals of a battery of e.m.f. 4 V and internal resistance 0.5 Ω. *R* is the total resistance of the circuit to which it is connected. Calculate the current that flows round the circuit and the p.d. between the terminals of the battery if the resistance *R* is
    a) 7.5 Ω
    b) 0 Ω.

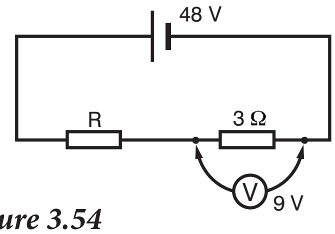


Figure 3.54

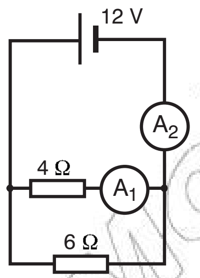


Figure 3.55

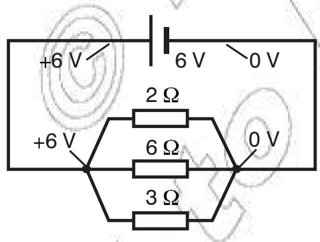


Figure 3.56

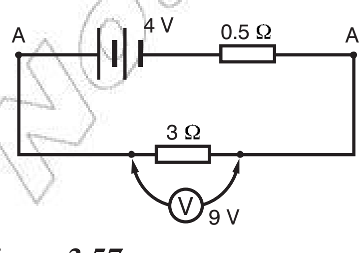


Figure 3.57