UNIT 5

CIRCLES

Unit outcomes

After Completing this unit, you should be able to:

- have a better understanding of circles.
- realize the relationship between lines and circles.
- apply basic facts about central and inscribed angles and angles formed by intersecting chords to compute their measures.

Introduction

In the previous grades you had learnt about circle and its parts like its center, radius and diameter. Now in this unit you will learn about the positional relationship of a circle and lines followed by chords and angles formed inside a circle and how to compute their degree measures of such angles.

5.1 Further On Circles

Activity 5.1

Discussed with your teacher orally.

Define and show by drawing the following key terms:

a. circles
b. radius
c. diameter
d. chord
e. circumference of a circle

Now in this lesson you will discuss more about parts of a circle i.e **minor arc** and **major arc**, **sector** and **segment of a circle**, **tangent** and **secant of a circle** and **center of a circle** by construction.

Parts of a circle

Group Work 5.1

1. a. Draw a circle of radius 4cm.
   b. Draw a diameter in your circle. The diameter divides the circle in to two semicircles.
   c. Colour the two semicircles indifferent colours.
   d. Draw a minor arc in your circle and label your minor arc.
   e. Draw a major arc in your circle and label your major arc.
2. A circle has a diameter of 6cm.
   a. write down the length of the radius of the circle.
   b. Draw the circle.
   c. Draw a chord in the circle.

Definition 5.1: The set of points on a circle (part of a circle) contained in one of the two half-planes determined by the line through any two distinct points of a circle is called **an arc of a circle.**

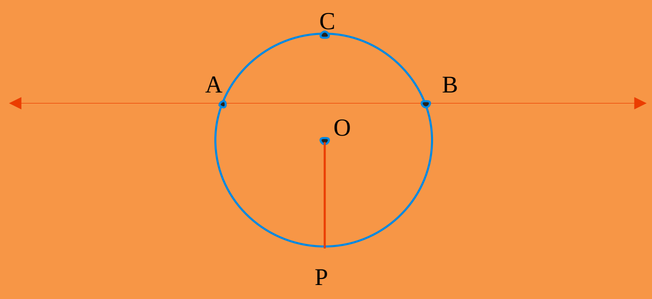


Figure 5.1 Circle

The center of the circle is O and PO is the radius. The part of the circle determined by the line through points A and B is an arc of the circle. In Figure 5.1 above arc ACB is denoted by $\overset{\frown}{ACB}$ or arc APB is denoted by $\overset{\frown}{APB}$.

A. Classification of Arcs

i. Semi-circle: Is half of a circle whose end points are the end points of a diameter of the circle and measure is 180^0.

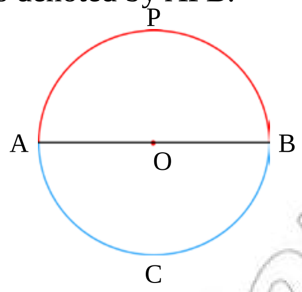


Figure 5.2 $\overset{\frown}{APB}$ and $\overset{\frown}{ACB}$ are a semi-circles.

ii. Minor arc: is the part of a circle which is less than a semi-circle.

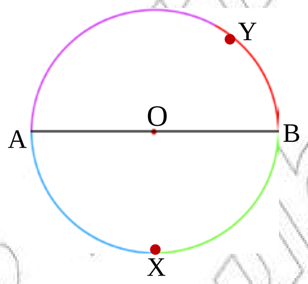


Figure 5.3 $\overset{\frown}{AX}$, $\overset{\frown}{BY}$, $\overset{\frown}{AY}$ and $\overset{\frown}{BX}$ are a minor arcs

iii. Major arc: is the part of a circle which is greater than a semi-circle.

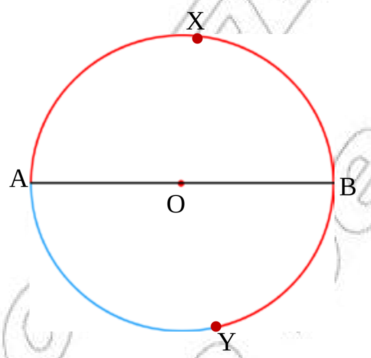


Fig. 5.4 $\overset{\frown}{AXBY}$ and $\overset{\frown}{BYAX}$ are major arcs.

Example 1: In Figure 5.5 below determine whether the arc is a minor arc, a major arc or a semicircle of a circle O with diameters $\overline{AD}$ and $\overline{BE}$.

a. $\overset{\frown}{AFB}$
b. $\overset{\frown}{ABD}$
c. $\overset{\frown}{BED}$
d. $\overset{\frown}{CAE}$
e. $\overset{\frown}{CDE}$
f. $\overset{\frown}{BCD}$
g. $\overset{\frown}{AED}$
h. $\overset{\frown}{ABC}$

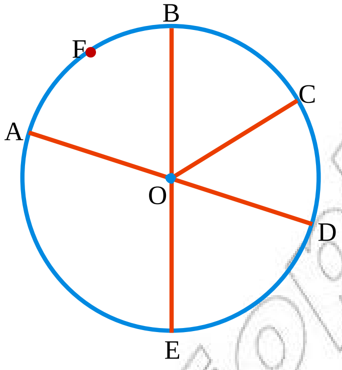


Figure 5.5 Circle

Solution:

a. minor arc
b. semi-circle
c. major arc
d. major arc
e. minor arc
f. minor arc
g. semi-circle
h. minor arc

B. Sector and segments of a circle

Definition 5.2: A sector of a circle is the region bounded by two radii and an intercepted arc of the circle.

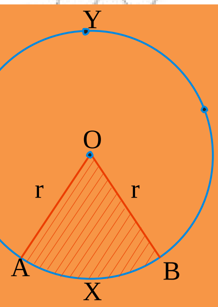


Figure 5.6 Circle

In Figure 5.6 above, the shaded region AOB and the unshaded region AYB are sectors of the circle.

Definition 5.3: *A segment* of a circle is the region bounded by a chord and the arc of the circle.

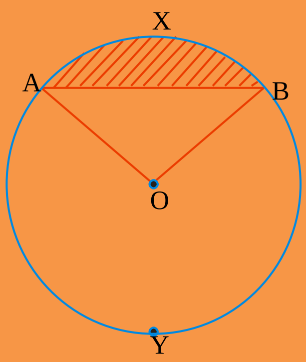


Figure 5.7 Circle

In Figure 5.7 above the shaded region AXB and the unshaded region AYB are segments of the circle.

C. Positional relations between a circle and a line

A circle and a line may be related in one of the following three ways.

1. The line may not intersect the circle at all.

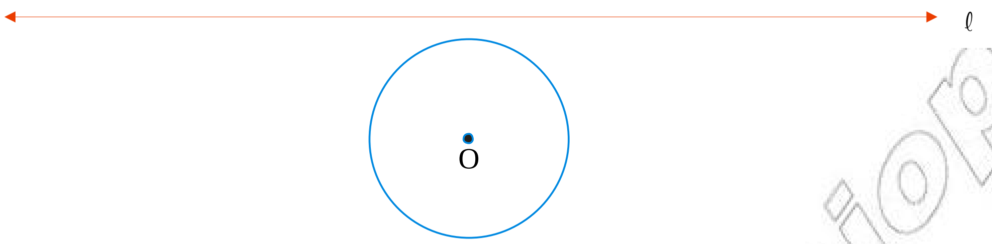


Figure 5.8 Circle

2. The line may intersect the circle at exactly one point.

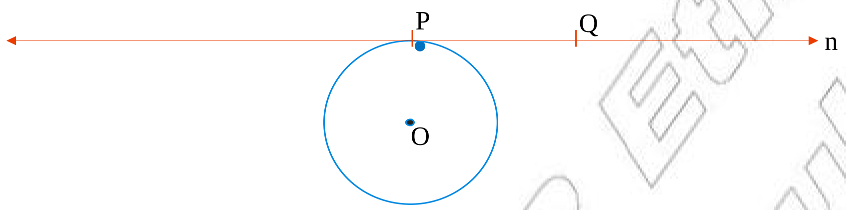


Figure 5.9 Circle

3. The line may intersect the circle at two points

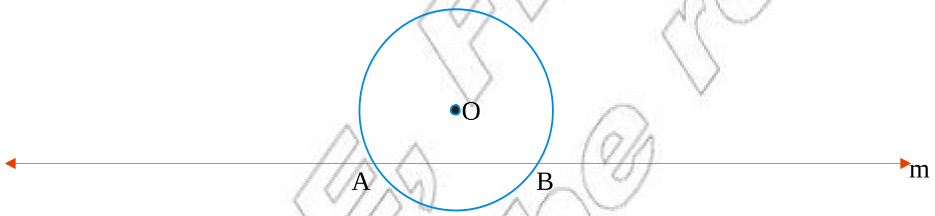


Figure 5.10 Circle

Definition 5.4: a. If a line intersect a circle at exactly one point, then the line is called a tangent of the circle.

b. The point at which it intersect the circles is called point of tangency.

c. If a line intersect a circle at two points then the line is called a secant of the circle.

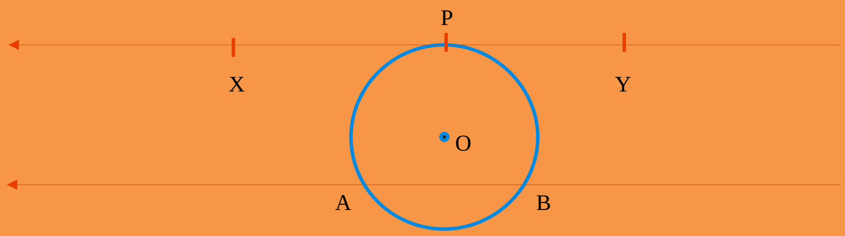


Figure 5.11 Circle

In Figure 5.11 above P is the point of **tangency**, $\overleftrightarrow{XY}$ is a tangent to the circle O and $\overleftrightarrow{AB}$ is a **secant** line to the circle O.

D. Construction

To find the center of a circle by construction the following steps is important:

Step i : Draw a circle by using coins

Figure 5.12 circle

Step ii : Draw a chord $\overline{AB}$

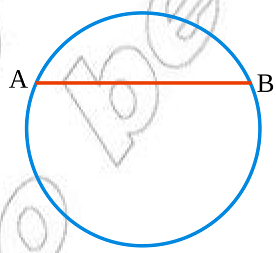
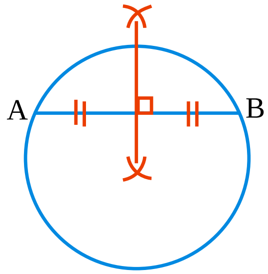


Figure 5.13 circle

Step iii: Construct the perpendicular bisector of $\overline{AB}$



Figurer 5.14 circle

Step iv: Draw another chord $\overline{CD}$.

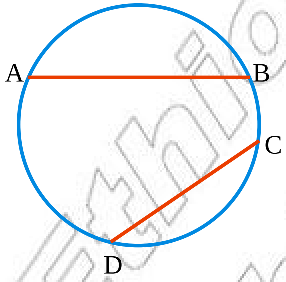


Figure 5.15 circle

Step v: Construct the perpendicular bisector of $\overline{CD}$.

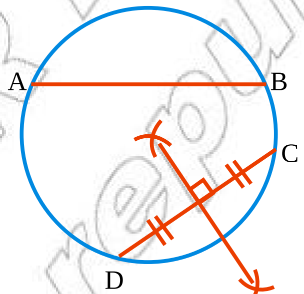


Figure 5.16 circle

Step vi: The perpendicular bisectors of $\overline{AB}$ and $\overline{CD}$ intersect at O, the centre of the circle.

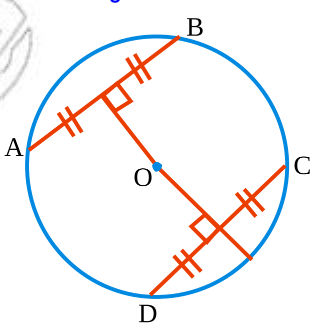


Figure 5.17 The required circle

Exercise 5A

1. Write true for the correct statements and false for the incorrect ones of each of the following.
   a. A secant of a circle contains chord of the circle.
   b. A secant of a circle always contains diameter of the circle.
   c. A tangent to a circle contains an interior point of the circle.
   d. A tangent to a circle can pass through the center of the circle.
2. In Figure 5.18 below A is an interior point of circle O. B is on the circle and C is an exterior point. Write correct for the true statements and false for the incorrect ones of each of the following.
   a. You can draw a secant line through point C.
   b. You can draw a secant line through point B.
   c. You can draw a tangent line through point A.
   d. You can draw tangent line through point C.

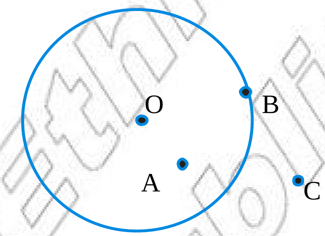


Figure 5.18 Circle

3. Consider the following Figure 5.19 to complete the blank space.
   a. ______ is tangent to circle O.
   b. ______ is secant to circle O.
   c. ______ is tangent to circle Q.
   d.______ is secant to circle Q.
   e. ______ is the common chord to circle O and Q.

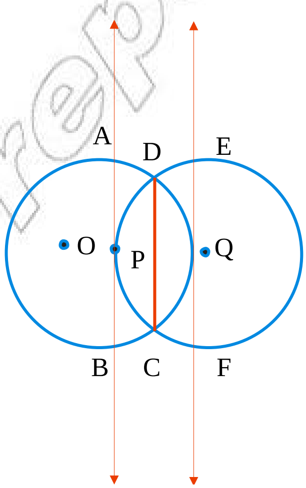


Figure 5.19 Circle

5.2 Angles in the Circle

Now in this lesson you will discuss more about central angle, inscribed angle, Angles formed by two intersecting chords and cyclic quadrilaterals.

5.2.1 Central Angle and Inscribed Angle

Group Work 5.2

1. **What is central angle?**
2. **What is inscribed angle?**
3. **Explain the relationship between the measure of the inscribed angle and measure of the arc subtends it.**
4. **In the given Figure 5.20 below m(∠CAO)=30° and m(∠CBO)=40°. Find m(∠ACB) and m(∠AOB).**

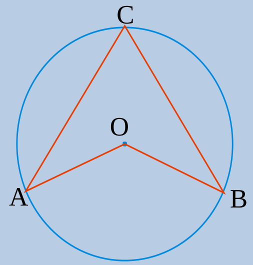


Figure 5.20

5. **If in Figure 5.21 arc BD is two times the arc AC, find ∠ BAD.**

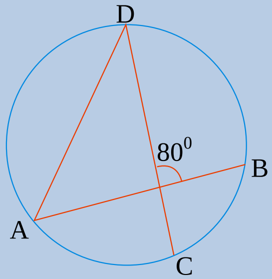


Figure 5.21

6. **O is the center of the circle. The straight line AOB is parallel to DC. Calculate the values of a, b and c.**

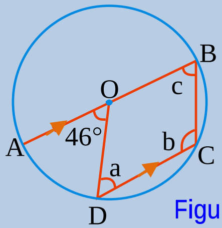


Figure 5.22

Definition 5.5: A Central angle of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.

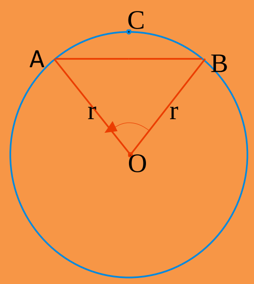


Figure 5.23 ∠AOB is a central

Note: 1. $\overset{\frown}{ACB}$ is said to be intercepted by ∠AOB and ∠AOB is said to be subtended by $\overset{\frown}{ACB}$.

2. The chord $\overline{AB}$ is said to subtend $\overset{\frown}{ACB}$ and $\overset{\frown}{ACB}$ is said to be subtended by chord $\overline{AB}$.

3. Chord $\overline{AB}$ subtends ∠AOB.

4. The measure of the central∠AOB is equal to the measure of the intercepted $\overset{\frown}{ACB}$. i.e m (∠AOB) = $\overset{\frown}{ACB}$.

Fact:- If the measure of the central angle is double or halved, the length of the intercepted arc is also doubled or halved. Thus you can say that the length of an arc is directly proportional to the measure of the central angle subtended by it. Hence you can use this fact to determine the degree measure of an arc by the central angle under consideration.

Definition 5.6: An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

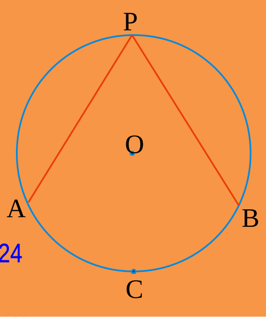


Figure 5.24

In Figure 5.24 above ∠APB is inscribed angle of circle O. we say ∠APB is inscribed in $\overset{\frown}{ACB}$ and $\overset{\frown}{ACB}$ subtends ∠APB.

Note: In Figure 5.25 below, the relationship between the measure of the central angle and inscribed angle by the same arc is given as follows:

1. The measure of the inscribed angle is half of the measure of central angle.
2. The measure of the inscribed angle is half of the measure of the arc subtends it.

$m(\angle ABC) = \frac{1}{2}\ m(\angle AOC)$

$m\ (\angle ABC) = \frac{1}{2}\ m(\overset{\frown}{ADC})$

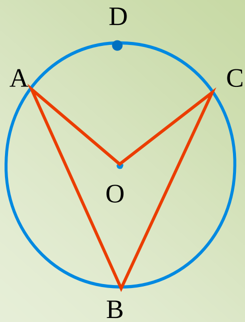


Figure 5.25

3. In Figure 5.26 to the right the relationship of inscribed angles subtended by the same arc is i.e m (∠ABE)=m(∠ACE)=m(∠ADE)

$= \frac{1}{2}\ m\ (\overset{\frown}{AXE})$

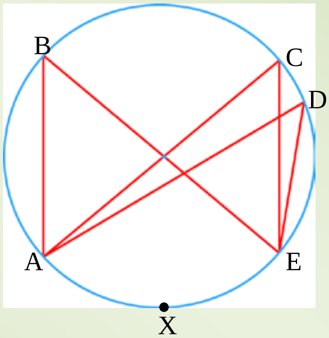


Figure. 5.26

Examples 2: In Figure 5.27 to the right, O is the center of the circle. If $m(\angle AOC)=52^0$, find $m\ (\angle ABC)$ and $m\ (\overset{\frown}{AC})$.

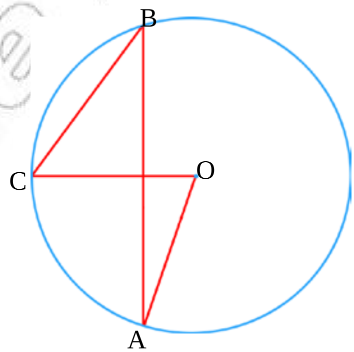


Figure 5.27

Solution:

$m\ (\angle AOC) = m\ (\overset{\frown}{AC}) = 52^0$ and

$m\ (\angle ABC) = \frac{1}{2} m\ (\overset{\frown}{AC})$

$= \frac{1}{2}(52^0)$

$= 26^0$

Examples 3: In Figure 5.28 to the right, O is the center of the circle, m (∠ABC) = 65^0, and m (∠AOE) =70^0, find m (∠CFE).

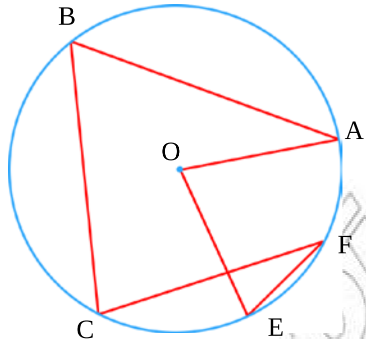


Figure 5.28

Solution:

m(∠AOE)=70^0 .... Given

m(∠ABC)=65^0 .... Given

m(∠ABC)= $\frac{1}{2} m(\overparen{AFEC})$

$65^0 = \frac{1}{2} m(\overparen{AFEC})$

m ($\overparen{AFEC}$)=130^0

m (∠AOE) = m ($\overparen{AFE}$)=70°

Thus m ($\overparen{EC}$) = m($\overparen{AFEC}$) – m ($\overparen{AFE}$)

$=130^0 - 70^0$

$=60^0$

Therefore, m (∠CFE) = $\frac{1}{2} m(\overparen{EC})$

$= \frac{1}{2}(60^0)$

$= 30^0$

Examples 4: In Figure 5.29 to the right, O is the center of the circle,

m (∠AQB) = 35^0

Find a. m (∠AOB)

b. m (∠APB)

c. m (∠ARB)

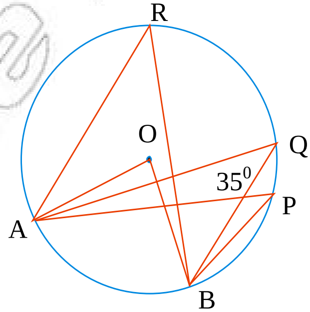


Figure 5.29

Solution:

$m(\angle AOB) = m(\overset{\frown}{AB})$

a. $m(\angle AQB) = \frac{1}{2}(m\angle AOB)$

$\Rightarrow m(\angle AOB) = 2m(\angle AQB)$

$= 2(35^0)$

$= 70^0$

b. $m(\angle APB) = \frac{1}{2} m(\angle AOB)$

$= \frac{1}{2}(70^0) = 35^0$

c. $m(\angle ARB) = \frac{1}{2} m(\angle AOB)$

$= \frac{1}{2}(70^0) = 35^0$

Examples 5: In Figure 5.30 to the right, find the values of the variables.

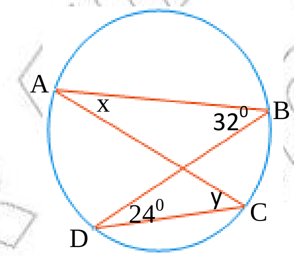


Figure 5.30

Solution:

$m(\angle ABD) = \frac{1}{2} m(\overset{\frown}{AD})$

$\Rightarrow m(\overset{\frown}{AD}) = 2m(\angle ABD)$

$= 2 \times 32^0$

$= 64^0$

$m(\angle ACD) = \frac{1}{2} m(\overset{\frown}{AD})$

$y = \frac{1}{2}(64^0)$

$y = 32^0$

$m(\angle BDC) = \frac{1}{2} m(\overset{\frown}{BC})$

$\Rightarrow m(\overset{\frown}{BC}) = 2m(\angle BDC)$

$= 2 \times 24^0$

$= 48^0$

$m(\angle BAC) = \frac{1}{2} m(\overset{\frown}{BC})$

$x = \frac{1}{2}(48^0)$

$x = 24^0$

5.2.2 Theorems on Angles in A Circle

You are already familiar with central angles and inscribed angles of a circle. Under this sub-section you will see some interesting result in connection with central and inscribed angles of a circle.

It is well know that the measure of a central angle is equal to the measure of the intercepted arc. But, a central angle is not the only kind of angle that can intercept an arc.

Theorem 5.1: The measure of an inscribed angle is equal to one half of the measure of its intercepted arc.

This important theorem proved in three cases. But here you can consider only the first case.

Proof: Given an inscribed angle ABC with sides $\overleftrightarrow{BC}$ passing through the center O.

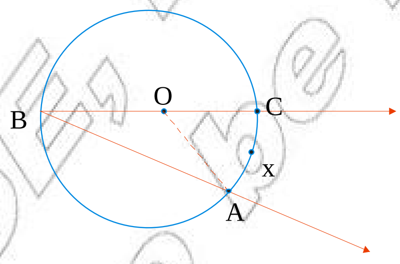


Figure 5.31

We want to show that: $m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AXC})$

Statements	Reasons
1. Draw $\overline{OA}$	1. Through two points there is exactly one line.
2. ΔBOA is isosceles	2. $\overline{OB} \equiv \overline{OA}$ (radii).
3. $\angle OBA \cong \angle OAB$	3. Base angles of isosceles triangle
4. $m(\angle OBA)+m(\angle OAB)=m(\angle AOC)$	4. $\angle AOC$ is supplementary to $\angle OBA$, $m(\angle OAB)$, + $m(\angle OBA) + m(\angle BOA) = 180^o$.
5. $m(\angle ABO)= \frac{1}{2}m(\angle AOC)$	5. since $\angle BAO \cong \angle ABO$.
6. $m(\overset{\frown}{AXC})=m(\angle AOC)$	6. central angle AOC intercepts $\overset{\frown}{AXC}$.
7. $m(\angle ABC)=m(\angle ABO)$	7. Naming the same angle.
8. $m(\angle ABC) = \frac{1}{2}m(\overset{\frown}{AXC})$	8. Substitution in step 5 .

Example 6: In Figure 5.32 below, O is the centre of a circle.
$m(\angle QPT)=54^0$ and $m(\angle TSQ)=21^0$.
Find: $m(\angle ROS)$.

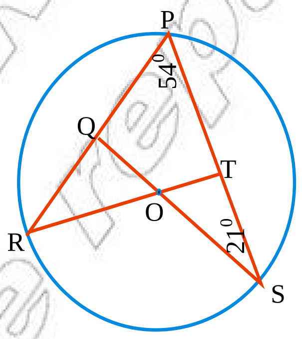


Figure 5.32

Solution:

$m(\angle RPS) = \frac{1}{2}m(\overset{\frown}{RS})$

$\Rightarrow m(\overset{\frown}{RS}) = 2m(\angle RPS)$
$= 2(54^0)$
$= 108^0$

Thus $m(\angle ROS)=m(\overset{\frown}{RS})=108°$.

Theorem 5.2: In a circle, inscribed angles subtended by the same arc are congruent.

Proof: Given circle O, inscribed angles B and D subtended by the same arc AC.

We want to show that: m(∠ABC)=m(∠ADC)

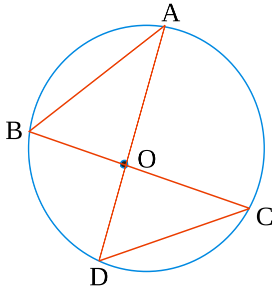


Figure 5.33

Statements	Reasons
1. m(∠ABC)= $\frac{1}{2}m(\overset{\frown}{AC})$	1. Theorem 5.1
2. m(∠ADC) = $\frac{1}{2}m(\overset{\frown}{AC})$	2. Theorem 5.1
3. m(∠ABC)=m(∠ADC)	3. Substitution

Examples 7: In Figure 5.34 to the right, m(∠CPD) =120^0, m(∠PCD) = 30^0 find m(∠A).

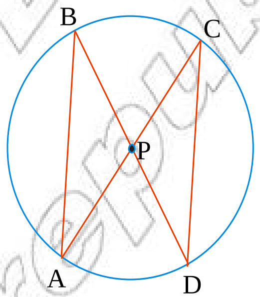


Figure 5.34

Solution:

m(∠CPD)=120° .... Given

m(∠PCD)=30°............ Given

m(∠CPD)+m(∠PCD)+m(CDP)=180° … why?

120°+30°+m (∠CDP)=180°

m(∠CDP)=180°-150°

=30°

Therefore, m(∠CDP)=30°.

Hence m(∠CDB)=m(∠CAB)=30°.................. Theorem 5.2

Exercise 5B

1. In Figure 5.35 to the right, O is the center of the circle. If m(∠ABC)=30^0, $\overline{CB} // \overline{OA}$ and $\overline{CO}$ *and* $\overline{AB}$ intersect at D, find m(∠ADC).

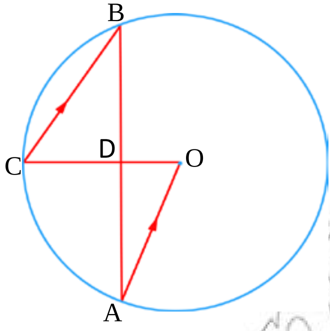


Figure 5.35

2. In Figure 5.36 to the right, if m($\overset{\frown}{AHB}$) = 100°
and m($\overset{\frown}{DIC}$) = 80° m(∠BAC) = 50°
and $\overleftrightarrow{FG}$ is tangent to the circles at C,
then find each of the following.
   a. m(∠BDC)
   b. m(∠ACD)
   c. m(∠AEB)

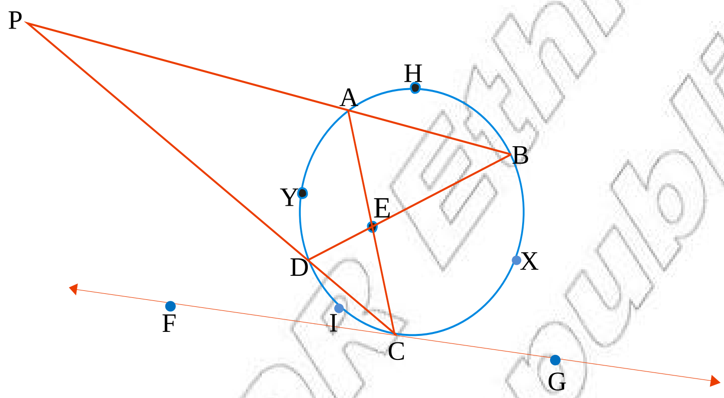


Figure 5.36

3. In Figure 5.37 below, O is the centre of the circle $\overrightarrow{PA}$ *and* $\overrightarrow{PB}$ are tangents to the circle at A and B, respectively. If m(∠ACB)=115^0 ,then find:
   a. m($\overset{\frown}{AB}$)
   b. m($\overset{\frown}{ACB}$)

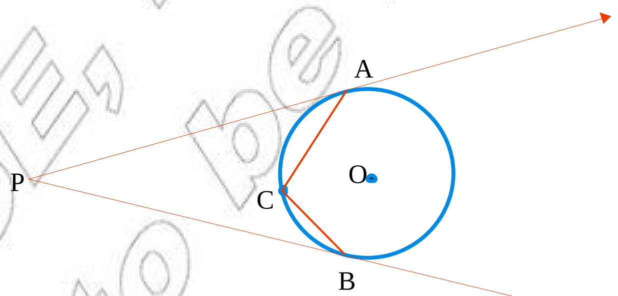


Figure 5.37

4. In Figure 5.38 to the right, O is the center of the circle. If m(∠B)=140^0, What is m(∠AMC)?

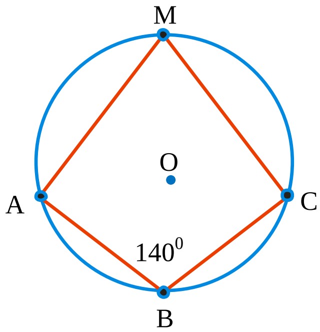


Figure 5.38

Challenge Problems

5. In Figure 5.39 to the right, O is the center of the circle, m(∠ABC) = 80^0 and m(∠AED) = 20^0 then what is m(∠DOC)?

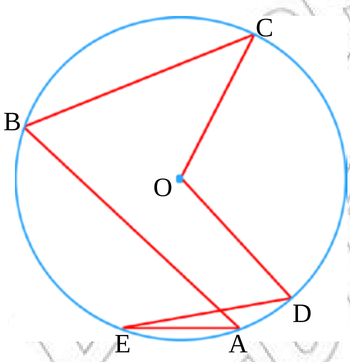


Figure 5.39

6. In Figure 5.40 to the right, O is the center of the circle and m $(\overset{\frown}{BYC})$ = 40^0, m$(\overset{\frown}{AXD})$=120^0 and m(∠ADB) =50^0. What is m(∠DOC) and m$(\overset{\frown}{DC})$?

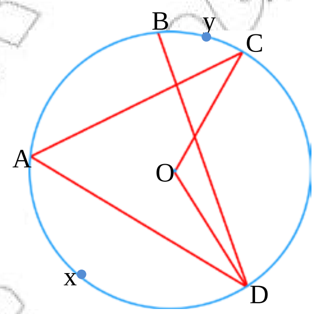


Figure 5.40

7. In Figure 5.41 given to the right , O is the center of the circle and m(∠BOC) = 120^0. What is m(∠ADC)?

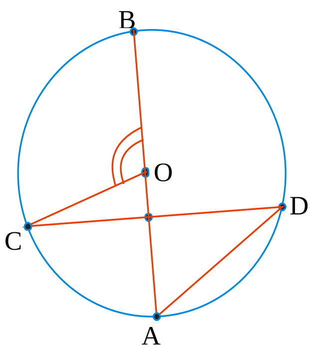


Figure 5.41

5.2.3 Angles Formed by Two Intersecting Chords

Activity 5.2

Discuss with your friends.

1. In Figure 5.42 given to the right, find m(∠x).

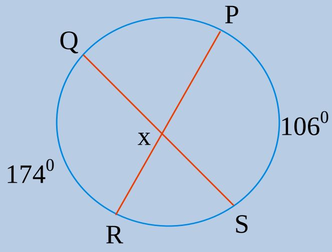


Figure 5.42

2. In Figure 5.43 given to the right, can you derived a formula m(∠a) and m(∠b)

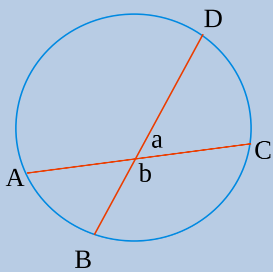


Figure 5.43

Theorem 5.3: The measure of an angle formed by two chords intersecting inside a circle is half the sum of the measures of the arcs subtending the angle and its vertical opposite angle.

Proof: Given $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersecting at P inside a circle

We want to show that:

$$m(\angle BPD) = \frac{1}{2}\, m(\overset{\frown}{AYC}) + \frac{1}{2}\, m(\overset{\frown}{DXB})$$

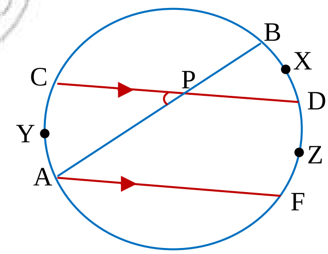


Figure 5.44

Statements	Reasons
1. Draw a line through A such that $\overline{AF} \parallel \overline{CD}$	1. construction
2. m(∠ BPD) = m (∠BAF)	2. corresponding angles formed by two parallel lines and a transversal line
3. m (∠ BAF) = $\frac{1}{2}$ m ($\widehat{BDF}$)	3. Theorem 5.1
4. m (∠ $\widehat{AYC}$) = m ($\widehat{DZF}$)	4. Why ?
5. m (∠ BPD) = $\frac{1}{2}$ m ($\widehat{BDF}$)	5. Why?
6. m(∠BPD) = $\frac{1}{2}$ m ($\widehat{BXD}$)+ m $\frac{1}{2}$ ($\widehat{DZF}$)	6. Why?
7. m (∠BPD)= $\frac{1}{2}$ m ($\widehat{AYC}$) + $\frac{1}{2}$ m ($\widehat{BXD}$)	7. Substitution

Examples 8: In Figure 5.45 given to the right, find the value of β, if m ($\widehat{AB}$) = 82° and m ($\widehat{DC}$) = 46°

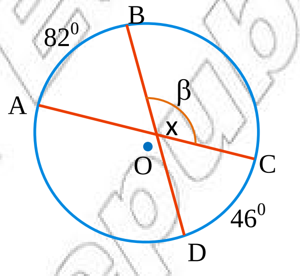


Figure 5.45

Solution:

To find β, simply by theorem 5.3

m (∠AXB) = $\frac{1}{2}$ ($\widehat{AB}$ + $\widehat{DC}$)

= $\frac{1}{2}$ (82° + 46°)

= $\frac{1}{2}$ (128°) = 64°

m (∠AXC) = 180°. . . . . . (Why)?

m (∠AXC) = m (∠AXB) + m (∠BXC)

180° = 64° + m(∠BXC)

116° = m(∠BXC)

Therefore, β = 116°

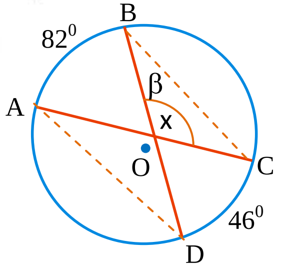


Figure 5.46

Examples 9: In Figure 5.47 given to the right, find the value of a and b.

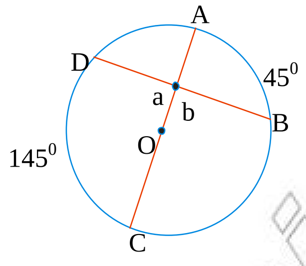


Figure 5.47

Solution:

$m(\angle a) = \frac{1}{2}[m(\overset{\frown}{DC}) + m(\overset{\frown}{AB})]$ .... Theorem 5.3

$= \frac{1}{2}(145^0 + 45^0)$

$= \frac{1}{2}(190^0)$

$= 95^0$ and

$m(\angle a) + m(\angle b) = 180^0$ .... Angle sum theorem

$95^0 + m(\angle b) = 180^0$

$m(\angle b) = 85^0$

Exercise 5C

1. Find the values of the variables in Figure 5.48 to the right.

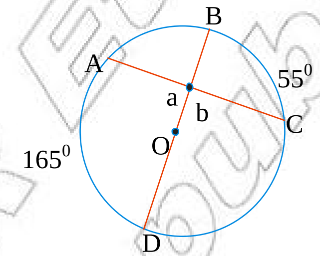


Figure 5.48

2. Find the values of the variables in Figure 5.49 below.

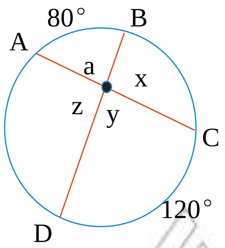


Figure 5.49

3. Find the m $(\overset{\frown}{AB})$ and m $(\overset{\frown}{DC})$ in Figure 5.50 to the right.

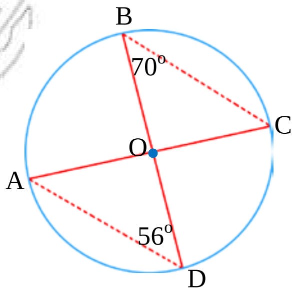


Figure 5.50

5.2.4 Cyclic Quadrilaterals

Group Work 5.3

Discuss with your friends/ partners.

1. What is a cyclic quadrilateral?
2. Based on Figure 5.51 answer the following questions.
   a. What is the sum of the measure angles A and C?
   b. What is the sum of the measure angles B and D?
   c. Is ABCD is a cyclic quadrilateral?
3. Find the sizes of the other three angles in the cyclic quadrilateral, if AB // DC.

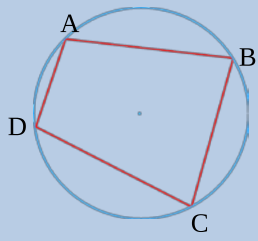


Figure 5.51

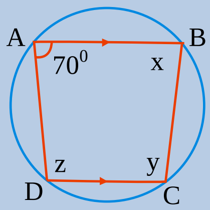


Figure 5.52

4. Angle ∠ECD =80^0. Explain why AEDB is a cyclic quadrilateral. Calculate the size of angle ∠EDA.

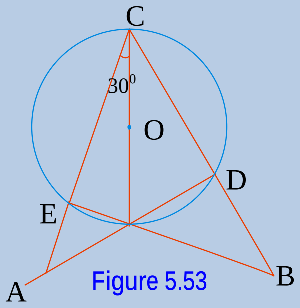


Figure 5.53

5. In Figure 5.54 below ABOD is acyclic quadrilateral and O is the center of the circle. Find x ,y, z and w, if DO // AB.

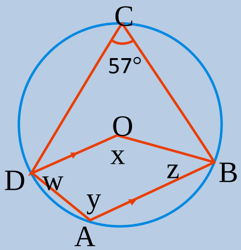


Figure 5.54

Draw a circle and mark four points A, B, C and D on it. Draw quadrilateral ABCD as shown in Figure 5.55. This quadrilateral has been given a special name called **cyclic quadrilateral.**

Definition 5.7: A quadrilateral inscribed in a circles is called cyclic quadrilateral.

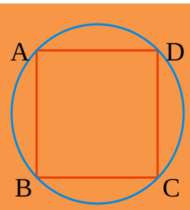


Figure 5.55 ABCD is cyclic quadrilateral

Property of Cyclic Quadrilateral

i. In Figure 5.55 above $m(\angle A)+m(\angle c)=180°$.

ii. Similarly $m(\angle B)+m(\angle D)=180°$.

Theorem 5.4: In cyclic quadrilateral, opposite angles are supplementary.

Given: ABCD is a quadrilateral inscribed in a circle.

We want to show that:

i. $m(\angle A)+m(\angle C)=180^0$

ii. $m(\angle B)+m(\angle D)=180^0$

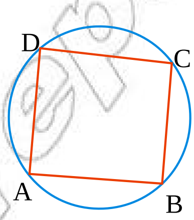


Figure 5.56

Proof

Statements	Reasons
1. ∠DAB and ∠DCB are opposite angles	1. Given
2. $m(\angle DAB)=\frac{1}{2}m(\overset{\frown}{DCB})$ and $m(\angle DCB)=\frac{1}{2}m(\overset{\frown}{DAB})$	2. Theorem 5.1

3. $m(\angle DAB)+m(\angle DCB)=\frac{1}{2}[m(\overset{\frown}{DCB})+m(\overset{\frown}{DAB})]$	3. By addition property
4. $m(\angle DAB)+m(\angle DCB)=\frac{1}{2}(360^0)$	4. Degree measure of a circle
5. $m(\angle A)+m(\angle C)=180^0$	5. Supplementary

? **Can you prove similarly $m(\angle B)+m(\angle D)=180^0$?**

Examples 10: In Figure 5.57 to the right, ABCD is a cyclic quadrilateral E is on $\overline{CD}$. If $m(\angle C)=110^0$, find x and y.

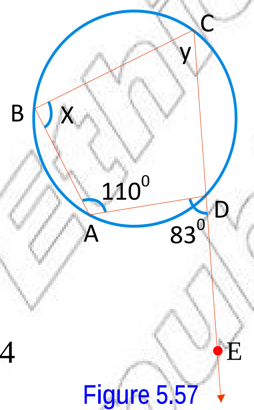


Figure 5.57

Solution:

$m(\angle BAD + m(\angle BCD)=180^0$ ....Theorem 5.4

$m(\angle y) + 110^0 = 180^0$

Therefore, $y=70^0$

$m(\angle ADE) + m(\angle CDA) = 180^0$ ....straight angle and $\overrightarrow{CE}$ is a ray.

$83^0 + m(\angle CDA) = 180^0$

Therefore, $m(\angle CDA)=97^0$

$m(\angle CBA) + m(\angle CDA) = 180^0$ ....Theorem 5.4

$m(\angle x) + 97^0=180^0$

Therefore, $x=83^0$

Examples 11: ABCD is an inscribed quadrilateral as shown in Figure 5.58. Find the $m(\angle BAD)$ and $m(\angle BCD)$.

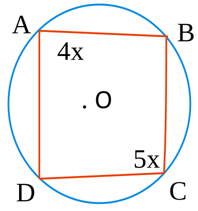


Figure 5.58

Solution:

$m(\angle DAB) + m(\angle DCB) = 180^0$ .... Theorem 5.4

$4x+5x = 180^0$ .... Substitution

$9x = 180^0$

$x = 20^0$

Hence $m(\angle DAB) = 4(20^0) = 80^0$ and $m(\angle DCB) = 5(20^0) = 100^0$.

Exercise 5D

1. In Figure 5.59 to the right, A, B, C, D and E are points on the circle. If m(<A)=100^0, find:
   a. $m(\angle C)$
   b. $m(\angle D)$

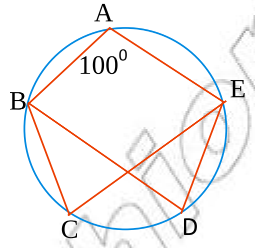


Figure 5.59

2. In Figure 5.60 to the right, ABCD is a cyclic trapezium where $\overline{AB} // \overline{CD}$. If $m(\angle A) = 95^0$, then find the measure of the other three angles.

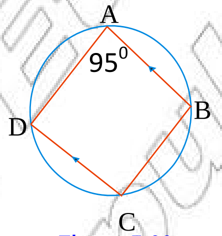


Figure 5.60

3. Consider the quadrilateral ABCD. Is it a cyclic quadrilateral?

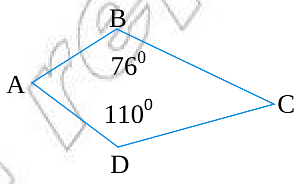


Figure 5.61

4. In Figure 5.62 to the right, ABCD is an inscribed quadrilateral. Find the measure of $\angle BAD$ and $\angle BCD$.

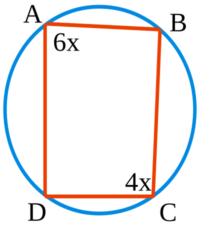


Figure 5.62

Challenge Problems

5. In Figure 5.63 to the right, find
   a. m(∠ABC)
   b. m(∠ADC)
   c. m(∠PAB)

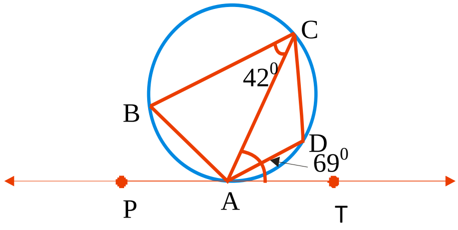


Figure 5.63

Summary for Unit 5

1. *A circle is the set of all points on a plane that are equidistant from a given point, called the* ***centre of the circle.***
2. *A* ***Chord*** *of a circle is a segment whose end points are on the circle.*
3. *A* ***diameter*** *of a circle is any chord that passes through the center and denoted by d.*
4. *A* ***radius*** *of a circle is a segment that has the center as one end point and a point on the circle as the other end point, and denoted by r.*
5. *The perimeter of a circle is called its* ***circumference.***
6. ***An arc*** *is part of the circumference of a circle.*

   Arcs are classified in the following three ways. .

   a. ***Semi-circle****: an arc whose end points are also end points of a diameter of a circle.*

   b. ***Minor arc:*** *is the part of a circle less than a semi circle.*

   c. ***Major arc:*** *is the part of a circle greater than a semi-circle.*
7. ***A sector*** *of a circle is the region bounded by two radii and an arc of the circle.*
8. ***A segment:*** *of a circle is the region bounded by a chord and the arc of the circle.*
9. ***A central angle*** *of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.*
10. ***An inscribed angle*** *is an angle whose vertex is on a circle and whose sides contain chords of the circle.*

11. *A quadrilateral inscribed in a circles is called* ***cyclic quadrilateral.***
12. *The measure of an inscribed angle is equal to one half of the measure of its intercepted arc.*
13. *The measure of an angle formed by two chords intersecting inside a circle is half the sum of the measure of the arcs subtending the angle and its vertical opposite angle.*
14. *The sum of the opposite angles of cyclic quadrilateral is* ***supplementary***

Miscellaneous Exercise 5

I. Write true for the correct statements and false for the incorrect ones.

1. Opposite angles of an inscribed quadrilateral are supplementary.
2. A central angle is not measured by its intercepted arc.
3. An angle inscribed in the same or equal arcs are equal.
4. A tangent to a circle can pass through the center of the circle.
5. If the measure of the central angle is double, then the length of the intercepted arc is also double.

II. Choose the correct answer from the given four alternatives

6. In Figure 5.64 to the right, O is the center of the circle. What is the value of x?

   a. 36^0 c. 10^0

   b. 60^0 d. 18^0

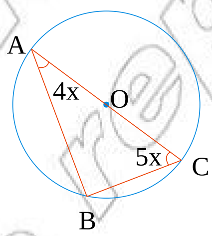


Figure 5.64

7. In Figure 5.65 below, $\overline{OA}$ and $\overline{OB}$ are radii of circle O. Which of the following statement is true?

   a. AB = OA
   b. AB>OA
   c. AB<OA
   d. None

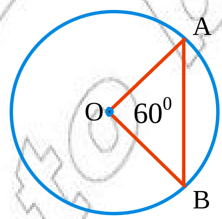


Figure 5.65

8. The measure of the opposite angles of a cyclic quadrilateral are in the ratio 2:3. What is the measure of the largest of these angles?

   a. 27^0 b. 120^0 c. 60^0 d. 108^0

III. Workout problems

9. In Figure 5.66 to the right, lines $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$ are parallel and $m(\overset{\frown}{AB}) = 100^0$ and $m(\overset{\frown}{CD}) = 80^0$. What is m(∠AED)?

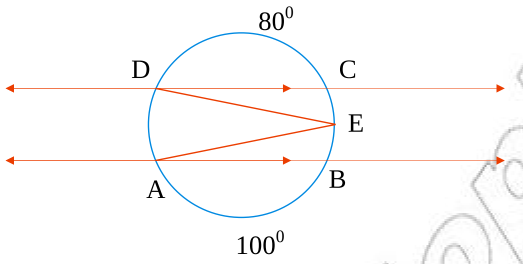


Figure 5.66

10. In Figure 5.67 below, find the value of the measure ∠a.

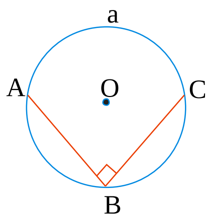


Figure 5.67

11. Construct the circle through A, B and C where AB=9cm, AC = 4cm and BC = 4cm.

12. In Figure 5.68 given below;
   a. If m (∠AOC) =140^0, find m(∠ABC) and m(∠ADC).
   b. If m(∠ABC) = 60^0, find m(∠AOC) and m(∠ADC).
   c. If m(∠AOC) = 200^0 find m(∠ABC).
   d. If m(∠ABC) = 80^0, find m(∠OAC).
   e. If m(∠OCA) = 20^0, find m (∠ADC).

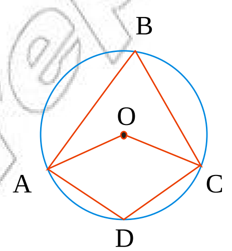


Figure 5.68

13. ABCD is a quadrilateral inscribed in a circle BC = CD. AB is parallel to DC and m (∠DBC) = 50^0. Find m (∠ADB).

14. O is the center of the circle.
   Calculate the size of angle ∠QSR

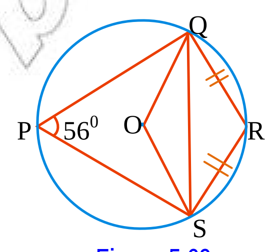


Figure 5.69

15. For the given circle O is its center and the two secant lines m and n are parallel. Find x.

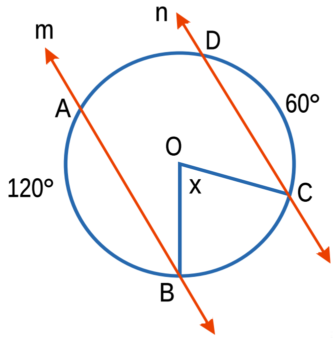


Figure 5.70

16. In Figure 5.71 below $\overline{PT}$ is a chord and O is the center of the circle. Calculate the size of m (∠TPO).

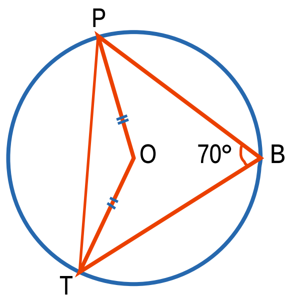


Figure 5.71

17. O is the center in Figure 5.72 to the right, Angle ABC = 158°.

Find (a) reflex angle AOC (c) angle ADC

(b) angle AOC

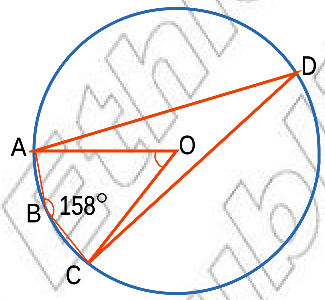


Figure 5.72

18. Find all the unknown angles in the figure in which $\overline{AB}$ // $\overline{DC}$ and angle ACD = 24°.

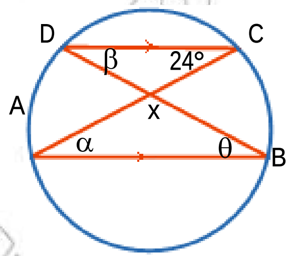


Figure 5.73

19. Given that angle BFC = 58°.

Angle BXG = 48° and angle CBF = 22°.

Find (i) ∠BGX (iv) ∠BCG

(ii) ∠BGF (iv) ∠BFG

(iii) ∠BCF

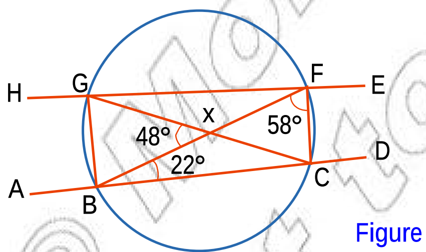


Figure 5.74