UNIT 2

LINEAR EQUATIONS AND INEQUALITIES

Unit outcomes:

After completing this unit, you should be able to:

- solve linear equations using transformation rules.
- solve linear inequalities using transformation rules.

Introduction

Based on your knowledge of working with variables and solving one step of linear equations and inequalities. You will learn more about solving linear equations and inequalities involving more than one steps. When you do this you will apply the rules of equivalent transformations of equations and inequalities appropriately.

2.1. Solving Linear Equations

Group Work 2.1

Discuss with your friends

1. Explain each of the following key terms, and give your own example.
   a. Term, like terms or similar terms.
   b. Coefficient of a term.
   c. Algebraic expressions.
   d. Equation.
   e. Equivalent equation.
2. Give examples of your own for:
   a. like terms or similar term
   b. unlike terms
   c. equation
   d. algebraic expressions
3. What are the numerical coefficients of x^3 and $-y^3$?

Definition 2.1: A constant (a number), a variable or product of a number and variable is called a term.

Example 1: $2, \frac{-3}{2}, x, 3x, -4x^2$ are called terms.

Consider Group A and Group B

Group A
$5x$ and $-20x$
$-80a^2b^2$ and $\frac{-1}{2}a^2b^2$
$6x^2$ and $70x^2$

Group B
$-10a^2b^2$ and $12c^2d^2$
$20xy$ and $abcd$
$5ab$ and $6xy$

? In general how do you see the differences between Group A and Group B? Discuss the differences with your teacher orally.

Definition 2.2: Like terms or similar terms are terms whose variables and exponents of variables are exactly the same but differ only in their numerical coefficients.

Note: Terms that are not like terms are called unlike terms.

Example 2: Terms like $-10a^2$, $170a^2$ and a^2 are like terms. Because they have the same variables with equal exponents but differ only in their numerical coefficients.

Example 3: Terms like -5ab and $7x^2y^2$ are unlike terms. Because they do not have the same variables.

Definition 2.3: In the product of a number and variable, the factor which is a numerical constant of a term is called a numerical coefficient.

Example 4: In each of the following expression, determine the numerical coefficient.

a. 56b b. $\frac{-5}{2}a^2b^2$ c. $\frac{-1}{4}xy$ d. $-x^2$

Solution:

a. The numerical coefficient of 56b is 56.

b. The numerical coefficient of $\frac{-5}{2}a^2b^2$ is $\frac{-5}{2}$.

c. The numerical coefficient of $\frac{-1}{4}xy$ is $\frac{-1}{4}$.

d. The numerical coefficient of $-x^2$ is -1.

Consider Group C and Group D

Group C
2x - 3
5y
a + 2b + 3c
2(ℓ+w)

Group D
2x – 3 = 10
5y = 60
a + 2b + 3c = 100
P = 2(ℓ+w)

? Do you observe the differences between Group A and group B? Discuss the differences with your teacher.

Definition 2.4: An equation is a mathematical statement in which two algebraic expressions are joined by equality sign. Therefore, an equation must contain an equal sign,=.

Example 5. Some examples of equations are:

a. $\frac{5}{2}x-10=40$

b. $4x+10=3\frac{1}{2}$

c. $3\frac{1}{2}x-5\frac{3}{2}=10$

d. $\frac{1}{2}x+\frac{2}{5}x-10x=50$

Note: Algebraic expressions have only one side.

- Algebraic expressions are formed by using numbers, letters (variables) and the basic operations of addition, subtraction, multiplication, and division.

Example 6. Some examples of algebraic expressions are:

a. $2x-4$

b. $\frac{\pi}{2}+\frac{1}{2}|-5x|$

c. $\left(\frac{-6}{5}\right)+\frac{x}{3}+20$

d. $\left(-5\frac{1}{2}\right)\div\frac{\pi}{2}$

e. 215

f. 3x

Exercise 2A

1. State whether each of the following is an equation or an algebraic expression.

   a. 2x+10=5x+60

   b. $|2x+10|$

   c. 10+3.8= 14.78x-10

   d. 9x + 10=5x

2. In each of the following expressions, determine the numerical coefficient.

   a. $\frac{3}{2}x^4$

   b. $-3\frac{1}{2}x^2$

   c. $\frac{-2}{3}x^2y^2$

   d. $\frac{-2}{7}x^5$

3. Identify whether each pair of the following algebraic expressions are like terms or unlike terms.

a. $\frac{3}{5}a^5b^2$ and $\frac{-5}{2}b^2a^5$

b. $3\frac{5}{6}xy$ and $3\frac{5}{6}x^2y^2$

c. -80abc and abc

d. $a^2b^2c^2d^2$ and $a^4b^4c^4d^4$

Challenge Problems

4. $0.0056x+26=100x+3\frac{1}{2}$ is a linear equation. Explain the main reason with your partner.

5. $a^5b^5c^5d^5$ and $-2(a^5b^5c^5d^5)$ are like terms. State the reason with your teacher orally.

2.1.1 Rules of Transformation for Equation

The following are basic rules of equality (=) that are used to get equivalent equations in solving a given equation.

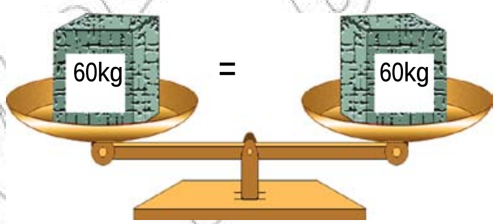
Rule 1: For all rational numbers a, b and c

a. If an equation a =b is true, then a + c = b + c is true for any rational number c.

b. If an equation a =b is true, then a-c =b-c is true for any rational number c.

Addition and subtraction properties of equality indicate that adding or subtracting the same quantity to each side of an equation results in an equivalent equation. This is true because if two quantities are increased or decreased by the same amount, then the resulting quantities will also be equal see Figure 2.1 below.

a)



b)

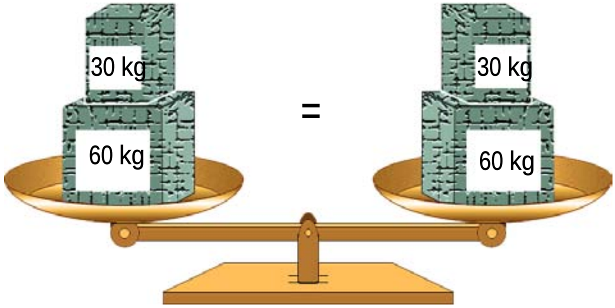


Figure 2.1 Balance

Rule 2: For all rational numbers a, b and c where $c \neq 0$, and

a. If an equation a=b is true, then ac = bc is true for any rational number c.

b. If an equation a=b is true, then $\frac{a}{c} = \frac{b}{c}$ is true for any rational number c.

To understand the multiplication property of equality, consider the following example. Suppose you start with a true equation such as 20=20. If both sides of an equations are multiplied by a constant such as 2 the result is also a true statement, see Figure 2.2 below.

a)

20 = 20
2×20 = 2×20
40 = 40

b)

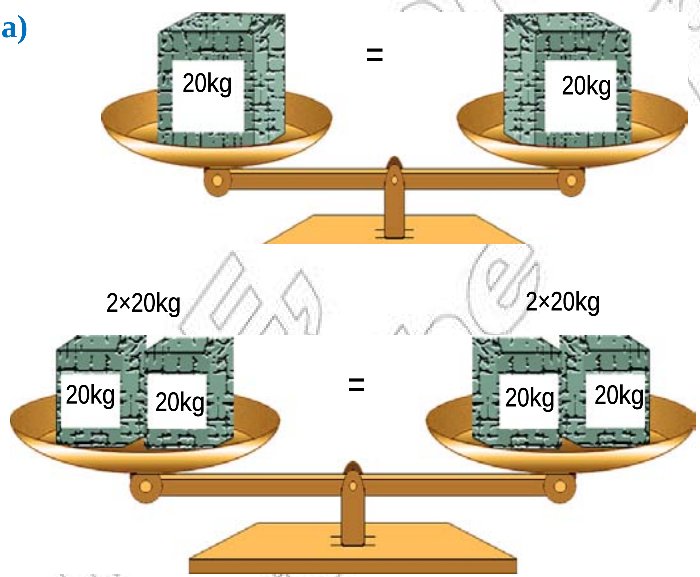


Figure 2.2 balance

Similarly, if an equation is divided by a non zero real numbers such as 2, the result is also a true statement, see Figure 2.3 below.

$$20 = 20$$

$$\frac{20}{2} = \frac{20}{2}$$

$$10 = 10$$

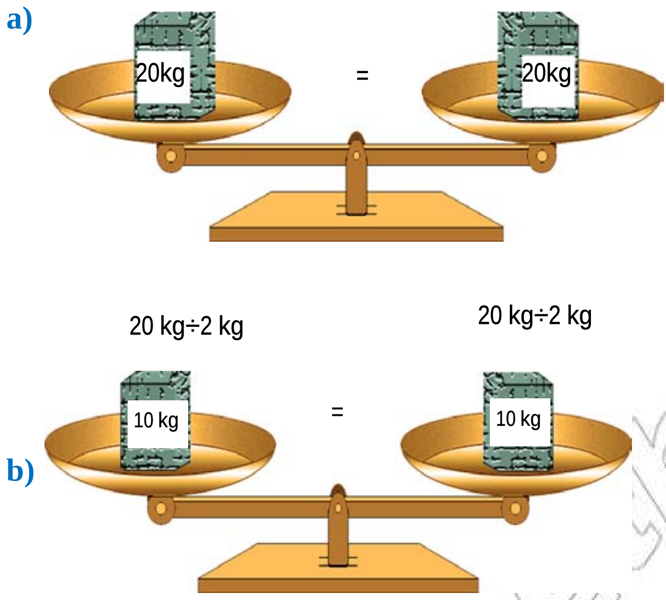


Figure 2.3 Balance

Example 7: To find X from X + 60 = 90

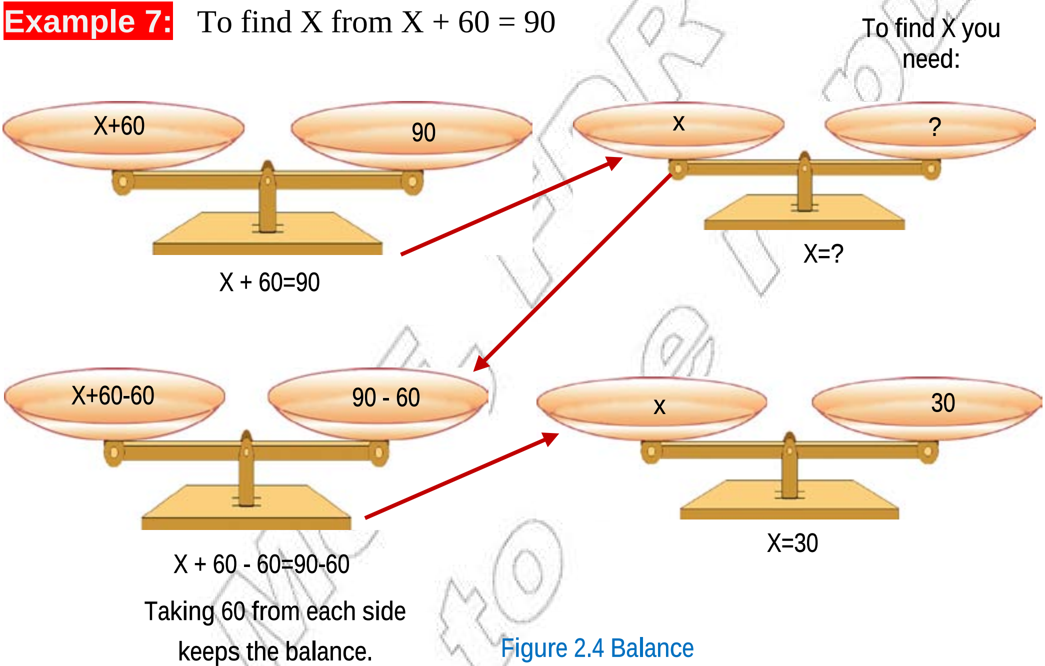


Figure 2.4 Balance

Example 8: Solve each of the following equations by using addition rules.

a. $x+\frac{3}{5}=\frac{8}{5}$ b. x -6 = -20

Solution:

a. $x+\frac{3}{5}=\frac{8}{5}$ . . . . . Given equation

$x+\frac{3}{5}+\left(\frac{-3}{5}\right)=\frac{8}{5}+\left(\frac{-3}{5}\right)$.......... Adding $\frac{-3}{5}$ on both sides.

x+0 = 1........Simplifying

x = 1 .... x is solved

✓ **Check:** When x = 1

$x+\frac{3}{5}=\frac{8}{5}$

$1+\frac{3}{5}\overset{?}{=}\frac{8}{5}$

$\frac{8}{5}=\frac{8}{5}$ .... True

Since $\frac{8}{5}=\frac{8}{5}$ is a true statement, x=1.

b. x - 6 =-20.......Given equation

x - 6 + 6=-20+6.......Adding 6 on both sides.

x + 0= -14......Simplifying

x = -14 .... x is solved

✓ **Check:** when x = -14

x-6 = -20

-14 – 6 $\overset{?}{=}$ -20

- 20 = -20 ....True

Since -20 = -20 is a true statement, x= -14

Example 9: Solve each of the following equations by using multiplication rules.

a. 8x = 72 b. $\frac{-4}{5}x=10$

Solution:

a. 8x = 72 …..Given equation

$\frac{1}{8}\times 8x = \frac{1}{8}\times 72$ .... Multiplying by $\frac{1}{8}$ on both sides

1×x=9………Simplifying

x= 9 ……..x is solved

✓ **Check**: When x= 9

8x= 72

$8\times 9 \overset{?}{=} 72$

72=72…….True

Since 72 = 72 is a true statement, x=9

b. $\frac{-4}{5}x = 40$ …….Given equation

$\left(\frac{-5}{4}\right)\times\left(\frac{-4}{5}x\right)=\left(\frac{-5}{4}\right)\times 40$ …..Multiplying by $\frac{-5}{4}$ on both sides.

1 × x=-50……..Simplifying

x = -50 ……..x is solved

✓ **Check:** When x = -50

$\frac{-4}{5}x = 40$

$\left(\frac{-4}{5}\right)\times -50 \overset{?}{=} 40$

40 = 40 ……True

Since 40 =40 is a true statement, x= -50.

2.1.2 Linear Equations in One Variable

? Consider the equation 3x+5 = 0, $\frac{-3}{5}\mathbf{x} - \mathbf{10} = \frac{3}{4}$, $\frac{1}{2}\mathbf{x} + \mathbf{10} = \mathbf{0}$ etc are examples of linear equations. Why? Discuss the reason with your teacher in the class.

Definition 2.5: A linear equation in one variable x is an equation which can be written in standard form ax + b= 0, where a and b are constant numbers with a ≠ 0.

From this definition, you can deduce that an equation of a single variable in which the highest exponent of the variable involved is one is called **a linear equation**.

Example 10: Which of the following equations are linear and which are not linear.

a. $5x+3\frac{5}{6}=10$

b. $\frac{-3}{2}x+20=10-\frac{1}{2}x$

c. $3x^2-\frac{8x^2}{2}+10=0$

d. $2x^2+2x=10$

Solution: a and b are linear equations. Because the highest exponent of the variable is one, but c and d are not linear equations. Why?

Briefly, all equations have two sides; with respect to the equality sign called left hand sides (L.H.S) and right hand sides (R.H.S) of the equality sign. These two sides are equal to each other like that of a simple balance. Thus equation is just a simple balance as shown in Figure 2.5 below.

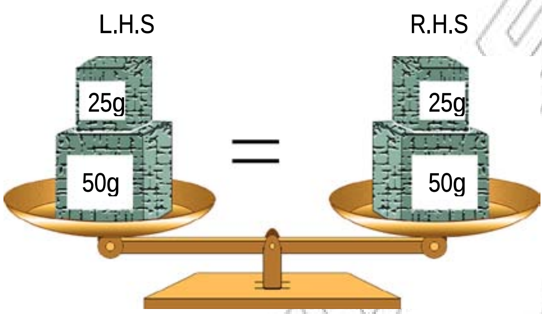


Figure 2.5 Simple balance

Note that the symbol " = " is read as "equals" or is "equal to".

Example 11: Identify the L.H.S and R.H.S of the following linear equations.

a. $\frac{5}{6}x+10=5$

b. $3y-16=19-6y$

Solution:

L.H.S	R.H.S
$\frac{5}{6}x+10$	5
3y-16	19-6y

Solving an equation means, applying the appropriate transformation rules to get a simplified equivalent equation in which the variable alone appears at one side and a constant (number) on the other side of the equality sign "=".

This constant number is called the **solution of the given equation.**

Note: Linear equations have exactly one solution. To see this, consider the following steps.

$ax + b = 0$ ......Given equation

$ax + b+(-b) = 0+(-b)$......Adding –b on both sides.

$ax = -b$ ....Simplifying

$\frac{ax}{a} = \frac{-b}{a}$...... Dividing both sides by a (since $a \neq 0$).

$x = \frac{-b}{a}$...... Simplifying

Thus, the equation $ax+b = 0$ has exactly one solution, that is $x = \frac{-b}{a}$.

Activity 2.1

1. Solve each of the following equations and mention the rules of transformation together.

   a. $0.8 + 2x = 3.5 – 0.5x$

   b. $8x – (3x – 5)=40$

   c. $(2x + 8) – 20 = -(3x – 18)$

   d. $5x – 17 – 2x = 6x – 1 – x$

2. $ax^2+bx+c=0$ is not a linear equation. Discuss the reason with your teacher in the class.

Example 12: Solve each of the following equations, in doing so indicate the rules you used.

a. $12x-14= 4x+10$

b. $3(7-2x)= 14 -8 (x-1)$

c. $8x+6-2x= -12-4x+5$

d. $7x-3(2x-5)= 6(2+3x)-31$

Solution:

a. 12x-14 =4x+10…..Given equation

12x-14+14= 4x+10+14…….Adding 14 to both sides.

12x=4x+24………Simplifying.

12x-4x= 4x-4x+24………Subtracting 4x from both sides.

8x= 24 …….Simplifying.

$\frac{1}{8}\times 8x = \frac{1}{8}\times 24$………..Multiplying both sides by $\frac{1}{8}$.

1×x =3 ……Simplifying

x= 3 …….x is solved.

✓ Check: When x= 3

12x -14= 4x + 10

12 ×3-14 $\stackrel{?}{=}$ 4×3 + 10

36-14 $\stackrel{?}{=}$ 12+10

22 = 22 …..True

Since 22 = 22 is a true statement, x=3

b. 3(7-2x) = 14-8(x-1)…….Given equation

21- 6x=14 - 8x + 8……..Removing parentheses by distributive property.

21- 6x= 22 - 8x…….Simplifying.

21 - 6x + 6x = 22 - 8x + 6x…….Adding 6x to both sides.

21 = 22 - 2x………Simplifying

21 – 22 = 22 – 22 - 2x…..Subtracting 22 from both sides

-1 = -2x ……Simplifying

$-1\times\left(\frac{-1}{2}\right) = -2x\left(\frac{-1}{2}\right)$ ……….. Multiplying both sides by $\left(\frac{-1}{2}\right)$.

$x = \frac{1}{2}$

✓ Check: When $x = \frac{1}{2}$

3 (7 - 2x) = 14 – 8 (x - 1)

$3\left(7 - 2\times\frac{1}{2}\right) \stackrel{?}{=} 14 - 8\left(\frac{1}{2} - 1\right)$

$3(7 - 1) = 14 - 8\left(\frac{-1}{2}\right)$

3 (6) = 14 + 4

18 = 18 ……True

Since 18 = 18 is a true statement, $x = \frac{1}{2}$ is the solution of the given equation.

c. $8x + 6 - 2x = -12 - 4x + 5$……..Given equation

$8x - 2x + 6 + (-6) = -12 + 5 + (-6) - 4x$……. Adding -6 on both sides.

$6x = -13 - 4x$…..Simplifying

$6x + 4x = -13 - 4x + 4x$……Adding 4x to both sides.

$10x = -13$…….Simplifying

$\frac{1}{10} \times 10x = \frac{1}{10} \times (-13)$........Multiplying by $\frac{1}{10}$ both sides.

$1 \times x = \frac{-13}{10}$ …...Simplifying

$x = \frac{-13}{10}$ .... x is solved

✓ Check: When $x = \frac{-13}{10}$

$8x + 6 - 2x = -12 - 4x + 5$

$$8\left(\frac{-13}{10}\right) + 6 - 2\left(\frac{-13}{10}\right) \overset{?}{=} -12 - 4\left(\frac{-13}{10}\right) + 5$$

$$\frac{-52}{5} + 6 + \frac{13}{5} \overset{?}{=} -12 + \frac{26}{5} + 5$$

$$\frac{-39}{5} + 6 \overset{?}{=} -7 + \frac{26}{5}$$

$$\frac{-9}{5} = \frac{-9}{5} \text{............ True}$$

Since $\frac{-9}{5} = \frac{-9}{5}$ is true statement, $x = \frac{-13}{10}$. or simply $\frac{-13}{10}$ is the solution of the given equation.

d. $7x - 3(2x - 5) = 6(2 + 3x) - 31$…..Given equation

$7x - 6x + 15 = 12 + 18x - 31$…..Distributive property.

$x + 15 = 18x - 19$ ……..Simplifying

$x + (-x) + 15 = 18x + (-x) - 19$…….Subtracting x from both sides.

$15 = 17x - 19$…….Simplifying

$15 + 19 = 17x - 19 + 19$…….Adding 19 from both sides.

$34 = 17x$……..Simplifying

$\frac{34}{17} = \frac{17x}{17}$ …….Dividing both sides by 17

$x = 2$…….. x is solved

✓ Check: When x =2

$7x - 3(2x - 5) = 6(2 + 3x) - 31$

$7\times2-3(2\times 2 - 5)\stackrel{?}{=}6(2 + 3\times 2) - 31$

$14 - 3(-1) \stackrel{?}{=} 6(8) - 31$

$14 + 3 \stackrel{?}{=} 48 - 31$

$17 = 17$ …..True

Since 17 = 17 is true statement, x = 2

The set that contains the solution of a given equation is called the **solution set of the equation.**

> **Definition 2.6:** Two equations are said to be equivalent if and only if they have exactly the same solution set.

Example 13: show that $2[9 - (x - 3) + 4x] = 4x - 5(x + 2) - 8$ and $\frac{x}{2} = -3$ are equivalent equations.

Solution: $2[9 - (x-3) + 4x] = 4x - 5(x+2) - 8$ .... Given equation

$2[9 - x + 3 + 4x] = 4x - 5x - 10 - 8$...................... Remove parentheses.

$2[12 + 3x] = -x - 18$……..Combine like terms

$24 + 6x = -x - 18$………Remove parentheses

$24 + 6x + x = -x + x - 18$….. Add x to both sides.

$24 + 7x = -18$…….Simplifying

$24 - 24 + 7x = -18 - 24$…..Subtract 24 from both sides.

$7x = -42$…….Simplifying

$\frac{7x}{7} = \frac{-42}{7}$ .... Dividing both sides by 7.

$x = -6$ ……. X is solved

And $\frac{x}{2} = -3$ .... Given equation

$x = -6$ ….. Multiplying both sides by 2

Therefore: $2[9 - (x-3] + 4x] = 4x - 5(x+2) - 8$ and

$\frac{x}{2} = -3$ are equivalent equations.

Exercise 2B

1. Which of the following pairs of equations are equivalent?

   a. $2x+8= 18$ and $2x=18-12$

   b. $9x-\frac{9}{8}=\frac{9}{4}$ and $9x=\frac{9}{8}$

   c. $21x = 38$ and $3x=36$

   d. $2x+(-6)=14$ and $2x=14+6$

   e. $3x=182$ and $x=\frac{182}{6}$

   f. $\frac{3}{5}x-\frac{3}{7}=10$ and $21x-365=0$

2. Show that $4(2x-1)=3(x+1)-2$ and $8x=3x+5$ are equivalent equations.

3. Solve the following linear equations and finally check your answers.

   a. $3x-9=4x+5$

   b. $2(3x+4)=6-(2x-5)$

   c. $2\left(\frac{x-3}{5}\right)=x-\frac{3}{5}$

   d. $2(2x+1)=3(x+3)+x-6$

   e. $270 \div x=540;\ x\neq 0$

   f. $4(2x-1)+6=7x-3(x+2)$

4. Show that $\frac{2}{3}(x+4)+\frac{3}{5}(2x+1)=0$ and $4(x+4)-3(2-x)=17$ are not equivalent equation.

Challenge Problems

5. Solve for x

   a. $ax+b=cx+d;\ a\neq c$

   b. $m(x-n)=3(r-x);\ m\neq 3$

   c. $ax+b=c;\ a\neq 0$

   d. $x+y=b(y-x);\ b\neq -1$

   e. $a_1x+b_1y=a_2x+b_2y;\ a_1\neq a_2$

2.1.3 Some Word problems

Group Work 2.2

1. Translate the algebraic expression $x+12$ in five different word phrases.
2. Translate the algebraic expression $x-7$ in six different word phrases.
3. Translate the algebraic expression $4x$ in four different word phrases.
4. Translate the algebraic expression $\frac{x}{6}$ in four different word phrases.

In this topic, you will apply the knowledge acquired on equations. The connection between an unknown number and other numbers which are known (constant) often arise out of practical life. To solve such problems verbal sentences needed to be changed into mathematical sentences. Relationship between such numbers or quantities given in word problem need to be expressed in the form of equation. You will now demonstrate how to solve a word problem by changing into mathematical equation.

Example14: Translate the algebraic expression x+10 in different word phrases.

Solution:

Word phrases	Algebraic expression (or symbols)
• A number plus ten. • The sum of a number and ten. • Ten added to a number. • A number increased by ten. • Ten more than a number.	x + 10

Example15: Translate the algebraic expression $\frac{x}{7}$ in different word phrases.

Solution:

Word phrases	Algebraic expression (or symbols)
• A number divide by seven. • The quotient of a number and seven. • The ratio of a number to seven. • one-seventh of a number.	$\frac{x}{7}$

Example 16: **(Relationship between temperature scales)**

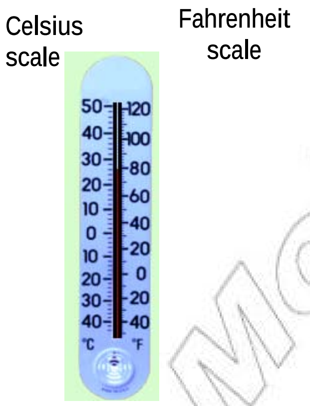


Figure 2.6 Temperature scales

The Celsius and Fahrenheit temperature scales are shown on thermometer in Figure2.6. The relationship between the temperature readings C and F is given by $C=\frac{5}{9}(F-32)$. (Express F in terms of C).

Solution: To solve for F you must obtain a formula that has F by itself on one side of the equals sign. You may do this as follows:

$$C = \frac{5}{9}(F - 32) \quad \text{.........Given equation}$$

$$\frac{9}{5}C = F - 32 \text{ ....Multiply both sides by } \frac{9}{5}$$

$$\frac{9}{5}C + 32 = F \text{........... Adding 32 from both sides.}$$

$$F = \frac{9}{5}C + 32$$

Example 17: **(Test average)**

A student take a mathematics test scores of 64 and 78. What score on a third test will give the student an average of 80?

Solution:

The unknown quantity is the score on the third test,
so you let x = score on the third test.
The average scores will be calculated on 64, 78 and x.

Thus average score = $\frac{64 + 78 + x}{3}$

$$\frac{64 + 78 + x}{3} = 80$$

$64 + 78 + x = 80 \times 3$ ....Multiplying both sides by 3

$142 + x = 240$.........Simplify

$x = 98$......... x is solved

✓ **Check:** If the three test scores are 64, 78 and 98, then the average is

$$\frac{64 + 78 + 98}{3} = \frac{240}{3} = 80.$$

Example 18: **(Age problem)**

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

Solution: let m = man age and w = wife age, then

m+w = 96 …….Translated equation(1)

m = 6+w ……..Translated equation (2)

6+w+w = 96……..Substituting equation (2) into(1)

6+2w = 96-6……Combine like terms

2w+6-6 = 96-6………Subtracting 6 both sides.

2w = 90 ………Simplifying

$\frac{2w}{2}=\frac{90}{2}$ ……………Divides both sides by 2

w = 45 …….w is solved

Therefore, the age of his wife is 45 years old.

Exercise 2C

Solve each of the following word problems.

1. If three fourth of a number is one-tenths, what is the number?
2. The sum of two consecutive integers is three times their difference. What is the larger number?
3. Can you find a number that satisfy the following property?
   a. If you multiply the number by 2 and add 4, the result you get will be the same as three times the number decreased by 7.
   b. If you increase the number by 4 and double this sum, the result you get will be the same as four times the number decreases by 6.
4. In a class there are 48 students. The number of girls is 3 times the number of boys. How many boys and how many girls are there in the class?
5. A farmer has sheep and hen. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?
6. 8 times a certain number is added to 5 times a second number to give 184. The first number minus the second number is -3. Find these numbers.
7. The perimeter of a rectangular field is 628m. The length of the field exceeds its width by 6m. Find the dimensions.

2.2. Solving Linear Inequalities

Activity 2.2

Discuss with your friends.

Solve each of the following linear Inequalities.

a. $2(5-x) \leq 3(1-2x) + 4$

b. $10(2x-4) \geq 12x-(2x+2)$

c. $8(2x-4)+6 \leq 14(2x+2)-12$

d. $0.5x + 0.5 > 0.2x + 2$

e. $0.7(x+3) < 0.4(x+3)$

From grade six mathematics lesson you have learnt about linear inequalities. Now in this sub topic you learn more about linear inequalities. The rules for transforming linear inequalities will be discussed in detail so as to find their solutions.

Definition 2.7: Mathematical sentence which contains one of the relation signs(symbols) $<, \leq, >, \geq$ or $\neq$ are called inequalities.

Example 19: Some examples of inequalities are:

a. $10x < 23$ b. $-2x > 5$ c. $\frac{1}{2}x \geq 4$ d. $\frac{3}{2}x \leq 10$

Definition 2.8: A linear inequality in one variable "**x**" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$ or $ax + b > 0$, $ax + b \geq 0$ where a and b are rational numbers and $a \neq 0$.

Example 20: Some examples of linear inequalities are:

a. $2x+10 > 0$

b. $5x+20 < 0$

c. $2x+12 \leq 0$

d. $6x+17 \geq 0$

2.2.1. Rules of Transformation for Inequalities

Group work 2.3

1. **Solve each of the following inequalities by using the addition rule.**

   a. $x + 8 > 3$

   b. $9x + 2.7 > 8x - 9.7$

   c. $x - 0.35 \leq 0.25$

   d. $x - 0.25 \geq -0.66$

2. Solve each of the following inequalities by using the multiplication rule.

a . $1 - 3x \geq 6$ **c. $3x < 18$**

b. $81x \leq 3$ **d. $5 - x \leq 2x - 1$**

The following rules are used to transform a given inequality to an equivalent inequality.

Rule 1: If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged. That is for any rational numbers a,b and c.

i. If $a < b$, then $a + c < b+c$.

ii. If $a < b$, then $a - c < b - c$.

Rule 2: If both sides of an inequality are multiplied or divide by the same positive number, the direction of the inequality is unchanged. That is for any rational numbers a,b and c.

i. If $a < b$ and $c > 0$, then $ac < bc$.

ii. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ (provided $c \neq 0$).

Rule 3: If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed. That is for any rational numbers a, b and c.

i. If $a < b$ and $c < 0$, then $ac > bc$.

ii. If $a < b$ and $c < 0$, then, $\frac{a}{c} > \frac{b}{c}$ (provided $c \neq 0$).

Example 21: Solve each of the following inequalities by using the transformation rules.

a. $10x - 4 \leq 8x - 2$ c. $4(x + 2) + 4 \leq 6(x + 1)-5$

b. $x + \frac{3}{4} > \frac{3}{8}$ d. $\frac{x+1}{3} \geq \frac{2x}{5} - 1$

Solution: a. $10x-4 \leq 8x-2$……….Given inequality

$10x-4+4 \leq 8x-2+4$…….Adding 4 both sides

$10x \leq 8x+2$……….Simplifying

$10x-8x\leq 8x-8x+2$…….Subtracting 8x from both sides

$2x\leq 2$……………..Simplifying

$\frac{2x}{2}\leq\frac{2}{2}$……………Dividing both sides by 2

$x\leq 1$…………….Simplifying

b. $x+\frac{3}{4}>\frac{3}{8}$ ……….Given inequality.

$x+\frac{3}{4}-\frac{3}{4}>\frac{3}{8}-\frac{3}{4}$………. Subtracting $\frac{3}{4}$ from both sides

$x>\frac{3}{8}-\frac{3}{4}$ ……… Simplifying

$x>\frac{3}{8}-\frac{3}{4}\times\frac{2}{2}$ ……………… Multiplying by $1=\frac{2}{2}$

$x>\frac{3}{8}-\frac{6}{8}$ …………….. Simplifying

$x>\frac{-3}{8}$ ……………….. Solved

c. $4(x+2)+4\leq 6(x+1)-5$……….Given inequality

$4x+8+4\leq 6x+6-5$……………Remove parenthesis by distributive property of "×" over "+"

$4x+12\leq 6x+1$…………….Combine like terms.

$4x-6x+12\leq 6x-6x+1$…….Subtracting 6x from both sides

$-2x+12\leq 1$…………..Simplifying

$-2x+12-12\leq 1-12$………Subtracting 12 from both sides

$-2x\leq -11$……… Simplifying

$\frac{-2x}{-2}\geq-\frac{11}{-2}$ …….Dividing both sides by -2

$x\geq\frac{11}{2}$

d. $\frac{x+1}{3}\geq\frac{2x}{5}-1$…….Given inequality

$15\left(\frac{x+1}{3}\right)\geq 15(\frac{2x}{5}-1)$ …….Multiplying by 15 which is the LCM of the denominators 3 and 5

$5x+5\geq 6x-15$……………..Remove parenthesis

$5x-6x+5\geq 6x-6x-15$………….Subtracting 6x from both sides

$-x+5\geq -15$………..Simplifying

$-x+5-5\geq -15-5$………..Subtracting 5 from both sides

$-x\geq -20$…………..Simplifying

$x\leq 20$ …………. Solved

Definition 2.9: Two inequalities are said to be equivalent if and only if they have exactly the same solution set.

Example 22: Some examples of equivalent linear inequalities are:

a. $5x < 20$ and $x < 4$

b. $x > 3$ and $x+8 > 3+8$

c. $\frac{x}{2} < \frac{10}{6}$ and $6x < 20$

Exercise 2D

1. Which of the following pairs of inequalities are equivalent?

   a. $2x - 6 > 4$ and $2x - 8 > 2$

   b. $6x + 22 < 4$ and $6x < -14$

   c. $3x + \frac{8}{12} < \frac{5}{12}$ and $36x + 8 < 5$

   d. $3x + 8x + 21 \geq 0$ and $x \geq \frac{-21}{11}$

   e. $\frac{4x}{3} < 12$ and $x < 9$

2. Identify whether each of the following inequalities is a linear inequality or not.

   a. $6x + 6 > 3x + 8$

   b. $2x + 6 \geq 0$

   c. $\frac{-x}{2} + \frac{3}{5} \leq 0$

   d. $4(x - 2) + 4(x + 1) - 6 \leq 0$

   e. $3x^2 + 6x \geq \frac{6x^2}{3} + 10$

   f. $13x^2 + 16 \leq 0$

3. Solve each of the following inequalities by using the transformation rules.

   a. $32 - 14x \geq 20x - 8$

   b. $5x + 5x + 2x \leq -24$

   c. $7(x - 2) < 4x - 8$

   d. $5(x - 3) < 7(x + 6)$

   e. $\frac{3x + 4}{2} \geq 10$

   f. $\frac{2 - 3x}{4} > x + 4$

   g. $\frac{3}{4}x + \frac{2}{3} < \frac{5}{6}x + \frac{4}{5}$

   h. $-5x + 7 \leq 1.4x - 17$

   i. $\frac{2}{3}x + \frac{3}{4} < \frac{4}{5}x + \frac{5}{6}$

Challenge problems

Solve for x

4. $x + 0.000894 \leq -0.009764$
5. $8x - 0.00962 \leq 7x + 0.00843$
6. $x + 0.001096 \geq -0.005792$
7. $6x - 0.000834 < 5x - 0.000948$

2.2.2 Solution Set of Linear Inequalities

Activity 2.3

Find the solution set of the following inequalities under the given domain.

a. $10x+14 < 25$; (domain is $\mathbb{N}$)

b. $5(2+x) > 18+6x$; (domain is $\mathbb{W}$)

c. $2-3x \geq 10$; (domain is $\mathbb{Q}$)

d. $10-2x \leq 4x-2$: (domain is $\mathbb{Z}$.)

In this topic you will solve linear inequalities by applying the necessary rules of transformation.

- To find the solutions of a given inequality, you will use the rules of transformation for inequalities to get successive equivalent inequalities so that the least simplified form is either $x > a$ or $x < a$ or $x \leq a$ or $x \geq a$.

In solving a linear inequality of the form $ax + b > 0$, $a \neq 0$, you have to consider two cases. These are:

When $a > 0$ and when $a < 0$

Case 1: when a > 0

$ax + b > 0$.........Given inequality

$ax + b - b > 0 - b$ ......Subtracting b from both sides.

$ax > -b$............Simplifying

$\frac{ax}{a} > \frac{-b}{a}$.......... Dividing both sides by a since $a > 0$

$x > \frac{-b}{a}$ ....Simplifying

Therefore, the solution set is $\left\{x : x > \frac{-b}{a}\right\}$.

Case 2: When a < 0

$ax + b > 0$..........Given inequality

$ax + b - b > 0 - b$............Subtracting b from both sides

$ax > -b$......Simplifying

$\frac{ax}{a} < \frac{-b}{a}$ .... Dividing both sides by a since $a < 0$

$x < \frac{-b}{a}$ .... Simplifying

Therefore, the solution set is $\left\{x : x < \frac{-b}{a}\right\}$.

Definition 2.10: The set of numbers from which value of the variable may be chosen should be meaningful and it is called **the domain of the variable.**

Example 23: Given the domain = {2, 4, 6, 8, 10, 12, 14}. Find the solution set of the inequality x - 5 > 6.

Solution:

$x - 5 + 5 > 6 + 5$

$x > 11$

Since 12 and 14 are the solution of the given inequality x - 5 > 6, these numbers the set containing is called the solution set of x – 5 > 6

You can now define the term solution set or truth set.

Definition 2.11: The set containing all the solutions of an inequality is **called the solution set or truth set of the inequality** and denoted by S.S or T.S.

Example 24: Find the solution set of the following inequalities under the given domain.

a. $2x + 10 < 10$: $x \in \mathbb{N}$

b. $-10x - (5 + 3x) \geq 0$; $x \in \mathbb{W}$

c. $2(x + 1) \leq 8x - (4x - 10)$; $x \in \mathbb{Q}$

d. $14(x - 4) < 8x - 16$; $x \in \mathbb{Q}^+$

Solution:

a. $2x + 10 < 10$; $x \in \mathbb{N}$……Original inequality

$2x + 10 + (-10) < 10 + (-10)$………..Subtracting 10 from both sides

$2x < 0$ ……………..Simplifying

$\frac{2x}{2} < \frac{0}{2}$ ………….. Dividing both sides by 2

$x < 0$

Solution set = { }. Because there is no natural number less than zero.

b. $-10x – (5 + 3x) \geq 0$; $x \in \mathbb{W}$ ……..Original inequality.

$-10x - 5 - 3x \geq 0$…………Remove parenthesis

$-10x - 3x - 5 \geq 0$ ....Collect like terms

$-13x-5 \geq 0$ ....Simplifying

$-13x - 5 + 5 \geq 0 + 5$ ....Adding 5 from both sides

$-13x \geq 5$ ....Simplifying

$x \leq \frac{-5}{13}$ ....Why?

Solution set= { }. Because there is no whole number less than or equal to $\frac{-5}{13}$.

c. $2(x + 1) \leq 8x- (4x - 10)$; $x \in \mathbb{Q}$ ....Original inequality

$2x + 2 \leq 8x - 4x + 10$ ....Remove parenthesis

$2x + 4x + 2 \leq 8x - 4x + 4x + 10$ ....Adding 4x from both sides

$6x + 2 \leq 8x + 10$ ....Simplifying

$6x - 8x + 2 \leq 8x - 8x + 10$ .... Subtracting 8x from both sides

$-2x + 2 \leq 10$ ...... Simplifying

$-2x + 2 - 2 \leq 10 - 2$ ....Subtracting 2 from both sides

$-2x \leq 8$ ....Simplifying

$\frac{-2x}{2} \geq \frac{8}{2}$ .... Dividing both sides by 2

$x \geq -4$ ....Remember to reverse the sign of the inequality.

Solution set = $\{x \in \mathbb{Q}: x \geq -4\}$

d. $14 (x - 4) < 8x-16$; $x \in \mathbb{Q}^+$ .... Original inequality

$14x - 56 < 8x - 16$ ....Remove parenthesis

$14x - 8x - 56 < 8x - 8x - 16$ ....Subtracting 8x from both sides

$6x - 56 < -16$ ....Simplifying

$6x - 56 + 56 < -16 + 56$ ....Adding 56 from both sides

$6x < 40$ ....Simplifying

$\frac{6x}{6} < \frac{40}{6}$ ....Dividing both sides by 6

$x < \frac{20}{3}$ .... Simplifying

$S.S = \left\{x \in \mathbb{Q}^+ : x < \frac{20}{3}\right\}$

2.2.3. Applications of Linear Inequalities

Provides several commonly used statements to express inequalities.

Table 2.1.

English phrase	Mathematical Inequality
✓**a is less than b**	**a < b**
✓**a is greater than b** ✓**a exceeds b**	**a > b**
✓**a is less than or equal to b** ✓**a is at most b** ✓**a is no more than b**	**a ≤ b**
✓**a is greater than or equal to b** ✓**a is at least b** ✓**a is no less than b**	**a ≥ b**

Example 25: Translating Expressions Involving Inequalities.

a. The speed of a car, S, was at least 220 km/hr.

b. Aster's average test score ,t, exceeded 80.

c. The height of a cave, h, was no more than 20m.

d. The temperature on the tennis court, t, was no less than 200°F.

e. The depth, d, of a certain pool was at most 10m.

Solution:

a. $s \geq 220$ c. $h \leq 20$ e. $d \leq 10$

b. $t > 80$ d. $t \geq 200°F$

Example 26: To earn grade A in a maths class, Aisha must have average score at least 90 on all of her tests. Suppose Aisha has scored 80, 86, 90, 94 and 96 on her first five maths tests. Determine the minimum score she needs on her sixth test to get an A in the class.

Solution:

Let x represent the score on the sixth test……….Lable the variable

(Average of all tests) ≥ 90………Create a verbal model.

$\frac{80+86+90+94+96+x}{6} \geq 90$……..The average score is found by taking the sum of the test scores and dividing by the numbers of scores.

$$\frac{446 + x}{6} \geq 90 \ldots\ldots\ldots\ldots\ldots\text{Simplify}$$

$$6\left(\frac{446 + x}{6}\right) \geq 90 \times (6) \ldots\ldots\ldots\text{..Multiply both sides by 6 to eliminate the}$$

denominator fractions.

$$446 + x \geq 540\ldots\ldots\ldots\ldots\ldots\ldots\text{.Solve the inequality}$$

$$446 - 446 + x \geq 540 - 446\ldots\ldots\text{.Subtracting 446 from both sides}$$

$$x \geq 94 \ldots\ldots\ldots\ldots\text{..Simplifying}$$

Aisha must score at least 94 on her sixth test to receive an A in the course.

Example 27: Eight times a natural number is increased by 4 times the number is less than 36. What are the possible value of this number?

Solution: let n be the number

$$8n + 4n < 36\ldots\ldots\ldots\ldots\text{Translated inequality.}$$

$$12n < 36\ldots\ldots\ldots\ldots\ldots\text{..Collect like terms.}$$

$$\frac{12n}{12} < \frac{36}{12} \ldots\ldots\ldots\text{Dividing both sides by 12.}$$

$$n < 3 \ldots\ldots\ldots\text{..Simplifying.}$$

Therefore, the required natural number is less than 3. Thus the number is either 1 or 2.

Example 28: For the region on the right figure. Find all values of x for which the perimeter is less than 37cm (see Figure 2.7 to the right).

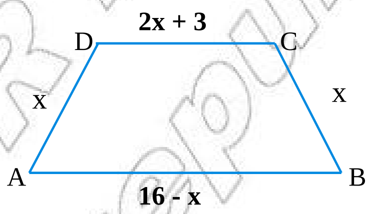


Figure 2.7

Solution: Consider the following Figure 2.8

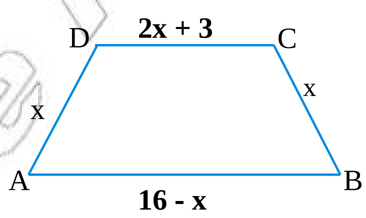


Figure 2.8

Let the perimeter = p

$AB + BC + CD + DA = P$

$16 - x + x + 2x + 3 + x = P$…….Substitution

Thus $P = 19 + 3x$, but we need $P = 19 + 3x < 37$

$$19 + 3x < 37$$

$$3x < 37 - 19$$

$$3x < 18$$

$x < 6$ and since x represents length, $x > 0$

Therefore, the values of x is $x < 6$ or $\{x \in \mathbb{N}: x < 6\} = \{1, 2, 3, 4, 5\}$…(Why)?

Exercise 2E

Solve each of the following word problems.

1. Twice a number x exceed 5 by at least 4. Find all possible values of x.
2. A natural number is less than the sum of its opposite and 8. Find all such numbers.
3. Find the two smallest consecutive even integers whose sum is at least 51.
4. The perimeter of a rectangle field is 118m. If the length of the rectangle is 7m less than twice the width, what is the length of the field?

Summary For Unit 2

1. **A constant (a number), a variable or product of a number and variable is called a term.**
2. **Like terms or similar terms are terms whose variables and exponents of the variables are exactly the same but only differ in the numerical coefficients.**
3. **In the product of a number and variable, the factor which is a numerical constant of a term is called** a numerical coefficient.
4. **An equation is a mathematical statement in which two quantities or two algebraic expressions are connected by the equality sign "=".**
5. **A linear equation in one variable x is an equation which can be written in standard form $ax + b = 0$, where a and b are constant numbers with $a \neq 0$.**
6. **Two equations are said to be equivalent, if and only if their solution sets are equal.**
7. **An inequality is a mathematical statements which contains the inequality symbols <, >, ≤ or ≥ to express that one quantity is greater than (or less than) another quantity.**
8. A linear inequality in one variable "x" is an inequality that can be written in the form of $ax + b < 0$, $ax + b \leq 0$ or $ax + b > 0$, $ax + b \geq 0$ **where a and b are rational numbers and** $a \neq 0$.

9. Rules of transformation for equation:

Let a, b and c be any rational numbers

a) If a =b, then a + c = b + c…………Addition property of equality.

b) If a = b, then a - c = b - c ……….Subtraction property of equality.

c) If a = b, then a × c = b × c………..Multiplication property of equality.

d) If a = b, then $\frac{a}{c} - \frac{b}{c}$ (provided c≠0). …..Division property of equality.

10. Rules of transformation for inequality:

Let a, b and c be any rational numbers

a) If a < b, then a + c < b + c………..Addition property of inequality.

b) If a < b, then a – c < b - c………….Subtraction property of inequality.

c) If c is positive and a < b, then ac < bc……Multiplication property of inequality.

d) If c is positive and a < b, then $\frac{a}{c} < \frac{b}{c}$ …….Division property of inequality.

e) If c is negative and a < b, then ac > bc ………Multiplication property of inequality.

f) If c is negative and a < b, then $\frac{a}{c} > \frac{b}{c}$ ………Division property of inequality.

Miscellaneous Exercise 2

1. Solve each of the following equations by using the rules of transformation.
   a) $-(-7x + 9) + (3x - 1) = 0$
   b) $5(3y) + 5(3 + y) = 5$
   c) $2x - \frac{1}{4} = 5$
   d) $-1.8 + 2.4x = -6.6$
   e) $\frac{6}{7} = \frac{1}{7} + \frac{5}{3}y$
   f) $5x - 3 - 4x = 13$
   g) $16y - 8 - 9y = -16$
   h) $6x-5-16x = -7$
   i) $\frac{3}{7}x - \frac{1}{4} = \frac{-4}{7}x - \frac{5}{4}$
2. Solve the equations using the steps as out lined in the text and finally check the result.
   a) $4(x + 15) = 20$
   b) $4(2y + 1) - 1 = 5$
   c) $5(4 + x) = 3(3x - 1) - 9$
   d) $6(3x - 4) + 10 = 5(x - 2) - (3x + 4)$
   e) $-5y + 2(2y + 1) = 2(5y - 1) - 7$
   f) $-2(4p + 1) - (3p - 1)) = 5(3 - p) - 9$
   g) $5 - (6y + 1) = 2((5y - 3) - (y - 2))$
   h) $7(0.4y-0.1) = 5.2y+0.86$
3. Explain the difference between simplifying an expression and solving an equation.
4. Which properties of equality would you apply to solve the equation $4x + 12 = 20$?
5. Which properties of equality would you apply to solve the equation $4x - 12 = 20$?
6. The sum of two consecutive integers is -67. Find the integers?
7. The sum of the page numbers on two integers facing pages in a book is 941. What are the page numbers?

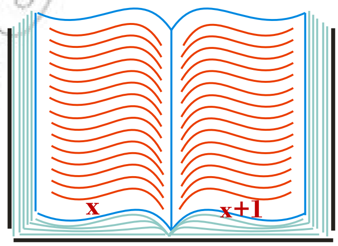


Figure 2.9

8. If y represents the smallest of three consecutive odd integer, write an expression to represent each of the next two consecutive odd integers.

9. Three consecutive odd integers are such that 3 times the smallest is 9 more than twice the largest. Find the three numbers.

10. a) Simplify the expression: 6(x + 2) – (4x - 14)

b) Simplify the expression: -(10x – 1) – 4(x + 8)

11. Solve each of the following inequalities by using the rules of transformation.

a) $-4x - 8 \leq 22$

b) $-14y - 6 \leq 6y$

c) $4x + 2 < 6x + 8$

d) $8 - 6(x - 3) > -4x + 12$

e) $3 - 4(y - 2) > -5y + 6$

f) $\frac{7}{6}x + \frac{4}{3} \geq \frac{11}{6}x - \frac{7}{6}$

12. Find the solution set of each of the following inequalities under the given domain.

a) $4x - \frac{1}{3} < 6x + 4\frac{2}{3}, x \in \mathbb{W}$

b) $9x - 4 < 13x - 7,\ x \in \mathbb{Z}$

c) $0.7(x + 3) < 0.4(x + 3),\ x \in \mathbb{Q}$

d) $3(x + 2) - (2x - 7) \leq (5x - 1) - 2(x + 6),\ x \in \mathbb{N}$

e) $6 - 8(y + 3) + 5y > 5y - (2y - 5) + 13;\ x \in \mathbb{Q}^{+}$

13. Find all values of X for which the perimeter is at most 32.

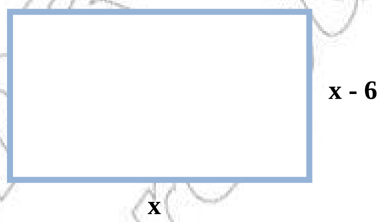


Figure 2.10

14. A board with 86 cm in length must be cut so that one piece is 20 cm longer than the other piece. Find the length of each piece.

Figure 2.11

15. Solve for X: $\frac{1}{2}x + \frac{1}{3}x - 2\left[\frac{x-2}{4}\right] \geq 2(x-1) + 3x.$