UNIT 4

INTEGERS

Unit outcomes: **After completing this unit you should be able to:**

- understand the concept of integers
- represent integers on a number line
- perform the operations addition and subtraction on integers

Introduction

In earlier grades, you have learnt about whole numbers, their properties and basic mathematical operations upon them. In the present unit you will continue studying a new number system, the set of integers. Here you will learn about integers, their properties and the operations addition and subtraction on integers.

4.1. Introduction to Integers

Activity 4.1

Place 4,5,6,7,8 and 9 in the empty circles so that each side has the same sum.

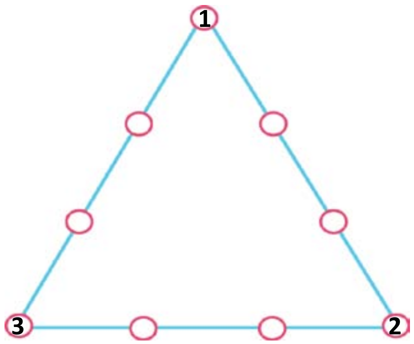

Figure 4.1

Figure 4.2

Do you like cold weather? If so, you might want to move to Debre Birhan. In some days of the year 2002 E.C the temperature in Debre Birhan was 5 degrees centigrade below zero. Have you ever heard of temperature below zero degree centigrade? Although the whole numbers have many uses, they are not adequate for describing such situations. But you can express this temperature using the **negative number written as -5**. This number is a member of the **set of integers**. In this sub-unit you will study about the set of integers. Integers are used to represent real-world quantities such as temperatures below zero.

Definition 4.1: The set of integers = $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

- The set of integers is usually denoted by Z.
- Integers greater than 0 are positive integers. Integers less than zero are negative integers. Zero is neither positive nor negative. Positive integers are written without the +sign, so +6 and 6 are the same.

Activity 4.2

Use the Venn diagram below to express relations between the sets N, W and Z. Which one of the following relations is correct?

a) $Z \subset W \subset N$ or

b) $N \subset W \subset Z$?

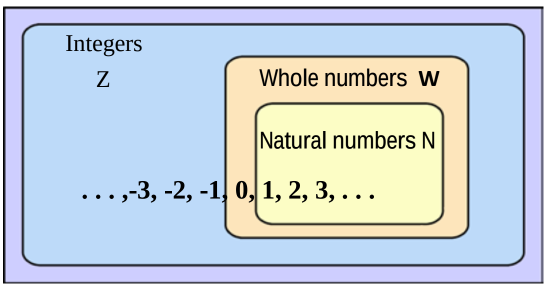

Figure 4.3

- You can represent integers as points on a number line. On a horizontal number line, positive integers are represented as points to the right of 0, and negative integers as points to the left of 0.

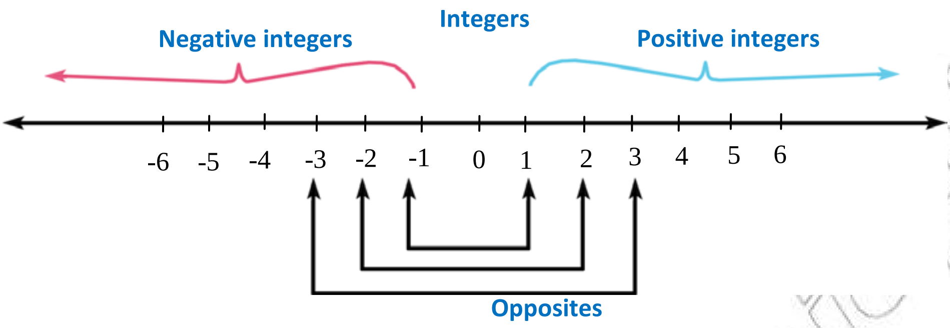

Figure 4.4

Definition 4.2: Two numbers are **opposites** of one another if they are represented by points that are at the same distance from zero, but on opposite sides of zero.

Example 1

a) -3 is the opposite of 3.

b) 7 is the opposite of -7.

Note: For any integer x, -(-x)=x. That is opposite of an opposite is itself.

Example 2

a) – (-4) = 4

b) – (-10)= 10

Exercise 4.A

1. Represent each of the following integers on a number line.

   a) -5   b) -7   c) -10

2. What integers are represented by the variables on the number line given below?

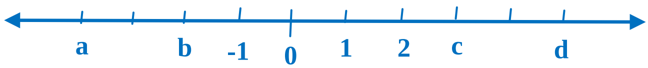

Figure 4.5

3. Write the opposites of the following integers.

   a) 10
   b) -8
   c) 120
   d) -1000
   e) 0

4. Write an integer for each situation.

   a) A profit of birr 4.
   b) A withdrawal of birr 5.
   c) 12 days behind.
   d) 9 steps forward.
   e) A loss of 20 points.
   f) 10°c below zero
   g) The opposite of 16
   h) The opposite of -20
   i) -(-30)

5. What is the greatest negative integer? And what is the least positive integer?

6. Write a number the opposite of which is

   a) Positive.
   b) Negative.
   c) Neither positive nor negative.

4.2. Comparing and Ordering Integers

Here you will study how you compare integers and the way of ordering integers.

Activity 4.3

1. What is the predecessor of 4? What is its successor?
2. Write down the three integers that come before the integers listed below.

   a) ______, ______, ______, -2, -1, 0, 1,2,3,4.

   b) ______, ______, ______, -10,-9,-8,-7,-6,-5,-4,-3.

   c) ______, ______, ______, -23, -22,-21,-20,-19,-18,-17.

Assegid and Bahiru were playing a game resulting in loss and gain. In such a game a player can end up with negative scores. Assegid's final score was -1000, and Bahiru's final score was -400. Whose score was greater?

You can use a number line to answer this question. On a number line, values increase as you go right and decrease as you go left.

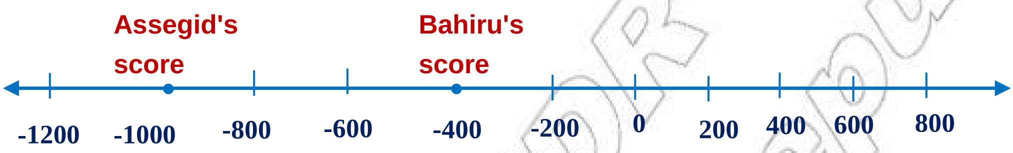

Figure 4.6

Since Bahiru's score is to the right of Assegid's score on the number line, Bahiru's score is greater than Assegid's. You can write -400 > -1000 or -1000 < -400.

Note: On a number line, any positive integer is to the right of a negative integer. So when comparing a positive and negative integer, the positive integer will always be greater. For example, 1> -100 or -100 <1.

Example 3

Replace ☐ with <, >, or = in -5 ☐ -1 to make a true sentence.

Solution: plot a point for each integer on a number line.

Figure 4.7

Since -5 is to the left of -1 on the number line, -5 <-1. Can you compare -6 and -2? Which is greater? -6 or -2?

Example 4

Order the integers -5,6,3,-2 and 0 from least to greatest.

Solution: plot the point for each integer on a number line.

Figure 4.8

Order the integers by reading from left to right.

-5, -2, 0, 3, 6

Can you order the integer -6, 4, 2, -1, and 0 from least to greatest?

Group work 4.1

The table shows the average temperatures of a certain city, for several months. In which month is the average temperature lowest?

Monthly	Temperatures
January	-12° C
March	-13° C
May	20° C
July	23° C

Example 5

Since -9<-4 and -4<1, you see that -9<1.

Example 6

7>4 but -4>-7 or -7 < -4

- **Properties of order of integers:**
  1. The greater the integer, the smaller is its opposite, i.e. if a and b are integers such that a>b, then –a<-b.
  2. In general, for any two integers a and b, a<b means the point corresponding to a is on the left of the point corresponding to b on the number line.
  3. For any two integers a and b, either a=b or a<b or a>b.
  4. For three integers a,b and c, if a<b and b<c, then a<c.

Exercise 4.B

1. Identify whether each of the following statements is true or false.
   a. For two integers on a number line, the integer which is to the right of the other is larger.
   b. Every positive integer is greater than every negative integer.
   c. Zero is less than every negative integer.
   d. -8 is the predecessor of -7.

2. Replace each $\boxed{}$ with >, <, or = to make a true statement.
   a) 2 $\boxed{}$ -10 c) -5 $\boxed{}$ 5 e) -20 $\boxed{}$ -90
   b) 3 $\boxed{}$ - 87 d) -101 $\boxed{}$ -1,001 f) -81 $\boxed{}$ -51

3. Order the integers from least to greatest.
   a) 7, -4, 0, -11, 29, -78 c) 17, 5, -9, 0, -1, -44,44
   b) 2, -13, -67, -91, 35, 18, -6

4. Order the integers from greatest to least.
   a) 10, 8, -3, 18, -70, -5 c) 17, -40, 31, -28, 26, -52
   b) 6, -77, -55, -3, 15, -19

5. Complete the table.

Predecessor	Integer	Successor
	29	
-16		
		-73
	-152	

6. List a) All negative integers greater than -9.
   b) Integers between -20 and -15.

4.3. Addition and Subtraction of Integers

Here you will be introduced how to find the sum of integers and also the difference of integers.

Addition of Integers

Activity 4.4

1. Find an integer which is 3 more than -4.
2. What is the sum of an integer and its opposite?
   e.g. -3+3=?

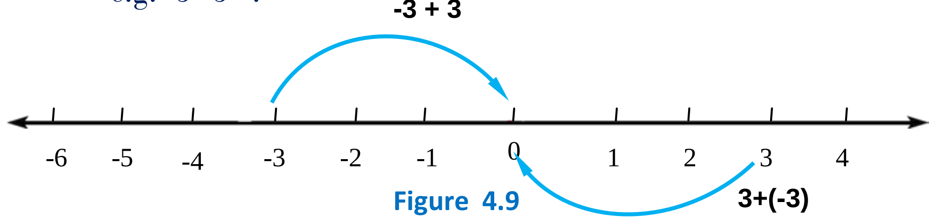

Figure 4.9

You know how to add two whole numbers on the number line. Addition of integers can be done in the same way. The only difference is that when you add a negative integer, you move to the left of the number.

Example 7

Add 5 and -7.

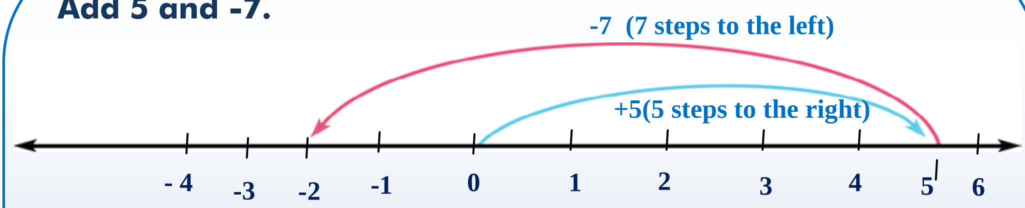

Figure 4.10

Solution: Start from 0, move 5 units to the right and turn in opposite direction. Move 7 units to the left and reach -2 which is the answer. Thus, we obtain 5+(-7)= -2. We may think of, for better understanding, adding 5 and -7 as follows: suppose a man made a gain of Birr 5 followed by a loss of Birr 7. Does he have profit or loss in the whole? The answer to this question will lead us to the result of adding 5 and -7.

Note: In the addition, 5+(-7), 5 and -7 are called addends and the result -2 is called the sum. We say 5 plus -7 is equal to -2.

Example 8

Add -4 and 6.

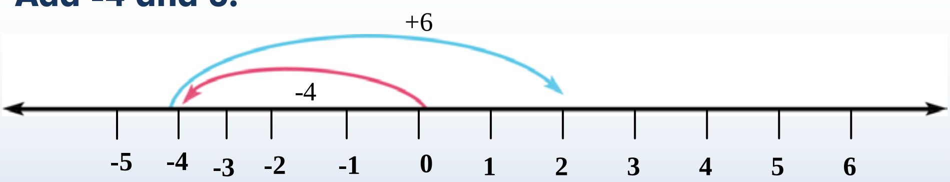

Figure 4.11

Solution: We draw a number line, we start from 0, move 4 units to the left of 0 and reach at -4. Now we move 6 units to the right and reach at 2. Therefore,

-4+6= 2. Can you add -3 and 7? What is the sum?

Example 9

Add -2 and -3.

Solution: On the number line, we start from 0. Move 2 units to the left of 0 and reach at-2. Now we move further 3 units to the left and reach at -5. Therefore -2+(-3)=-5. Can you add -6 and -9? What is the sum?

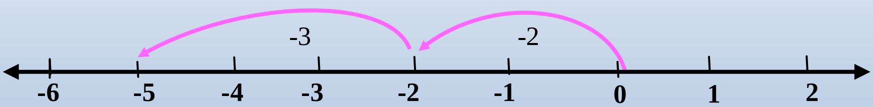

Figure 4.12

Subtraction of Integers

Before we can find difference, let's see how addition and subtraction of integers are related. Let's compare the subtraction 5-2 to the addition 5+(-2).

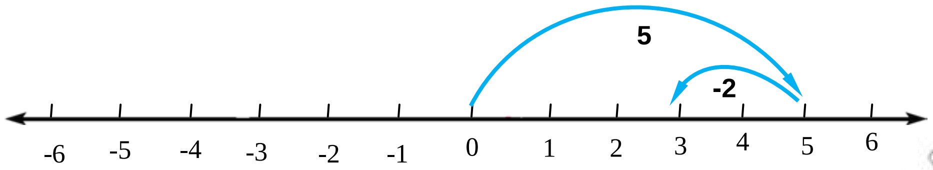

Figure 4.13

The figure shows that 5-2=5+(-2). This example suggests that adding the opposite of an integer produces the same result as subtracting it.

Note: To subtract an integer, add its opposite. That is, if a and b are two integers, then a - b= a+(-b).

Example 10

Subtract

a) 27-35 b) 43-(-37) c) -60-45

Solution

Subtracting 35 from 27 is the same as adding -35 to 27.

That is, 27-35=27+(-35)= -8

Remember that for any integer x, -(-x)=x.

Thus, 43-(-37) = 43+(-(-37))= 43+37=80

-60-45= -60+(-45)= -105

Group work 4.2

Discuss

(i) **Will 8 + (-3) and (-3) +8 give the answer? Why or why not?**

(ii) **Is the sum of any two integers an integer?**

(iii) **What will happen to the sum when you add 0 to an integer?**

The answers to this group work may lead you conclude the following important properties of addition of integers.

Properties of addition of integers: we have learnt the properties of addition of whole numbers. All those properties hold true for integers also.

Property I : The sum of two integers is always an integer. For example,

3+4=7, 7 is an integer.

3+(-4)=-1 ; -1 is an integer.

-3+(-4)= -7; -7 is an integer.

-3+4=1 ; 1 is an integer.

PropertyII (Commutative Property of Addition): For any two integers a and b, a+b= b+a.

For example, -3+5=5+(-3)=2

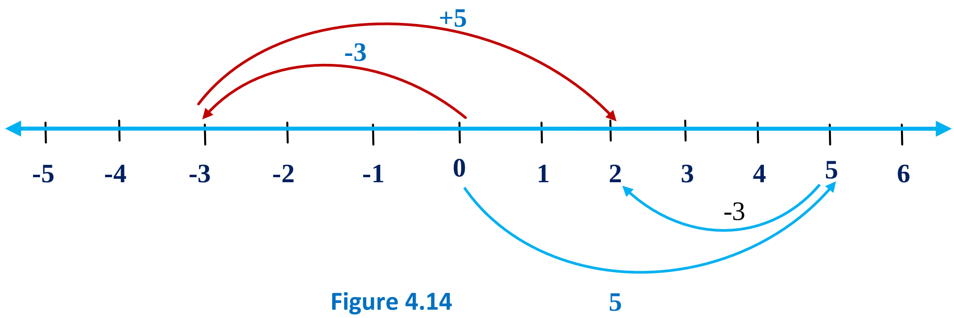

Figure 4.14

Property III(Associative Property of Addition)

If a, b and c are three integers, then (a+b)+c= a+(b+c).

For example, $(2+3)+(-5)= 5+(-5)=0$ and

$2+(3+(-5))= 2+(-2)=0$

Therefore, $(2+3)+(-5) = 2+(3+(-5))$

Property IV :When 0 is added to any integer, the sum remains unchanged.

For example, $3+0=0+3=3$; $0+(-2)=-2+0=-2$. In general, for any integer a, $a+0=0+a=a$.

Property V: The sum of an integer and its opposite is zero. That is, for any integer a, $a+(-a)=0$

For example, $3+(-3)= (-3)+3=0$

Exercise 4.C

1. Identify whether each of the following statements is true or false.
   a) If a is an integer, a-1 is its predecessor.
   b) $a-a= -a+a$ for any integer a.
   c) $3-(4-6)=5$
   d) $-3-5 = 5-3$
   e) The opposite of a negative integer is positive.
2. Subtract
   a) 10-23
   b) -18-(-34)
   c) 35-(-61)
   d) -69-(-31)
   e) (10-(-41))-28
   f) 40-(12-49)
3. If a and b are two integers such that
   i) a is the successor of b, then what is the value of a-b?
   ii) a is the predecessor of b, then what is the value of a-b?
4. Use >, < or = to compare the following.
   a) $-8-(-12)$ $\boxed{}$ 0
   b) $-27+30$ $\boxed{}$ 0
   c) $-79+36$ $\boxed{}$ $-28-(-28)$
   d) $-45-(-45)$ $\boxed{}$ $54-54$
   e) $-38-57$ $\boxed{}$ $57-38$
5. Evaluate $x + y$ for $x = 42$ and $y = -71$
6. The Maths club's income from a cake sale was Birr 286. Expenses were Birr 198. Use integer addition to find the club's total profit or loss.

UNIT SUMMARY

Important facts you should know:

- **The set of integers =Z**
  Z = $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.
- 1, 2, 3,...are **positive integers**.
- ...-4, -3, -2, -1 are **negative integers**.
- 0 is neither positive nor negative.
- Every positive integer is greater than every negative integer.
- Two numbers are **opposites** of one another if they are represented by points that are the same distance from zero but on opposite sides of zero.
- For any integer x, -(-x)=x.
- For any two integers a and b.

i) a<b means the point corresponding to a is on the left of the point corresponding to b on the number line.

ii) a=b or a<b or a>b.

- For three integers a, b and c, if a<b and b<c, then a<c.
- The greater the integer, the smaller is its opposite, i.e. if a and b are integers such that a>b, then–a<-b.
- If a,b and c are integers, then
  - i) a+b is an integer
  - ii) a+b= b+a
  - iii) (a+b)+c= a+(b+c)
  - v) a+(-a)=0
  - iv) a+0=0+a=a
- If a, b, and c are integers, then
  - i) a-b is an integer
  - ii) a-b ≠ b-a
  - iii) (a-b)-c≠ a-(b-c)
  - iv) a-0≠0-a

Review Exercise

1. Identify whether each of the following statements is true or false.
   a) If a and b are integers, then a-b is always an integer.
   b) There does not exist any smallest integer.
   c) If a is an integer, then a+1 is its successor.
   d) 4 – 2 = 2– 4.
   e) If a and b are opposites, then a + b = 0.
   f) If x is an integer, then x is always greater than –x.
2. Draw a number line and write all integers between -8 and 3.
3. Draw a number line and mark the following points on it:
   -6, 0, 5, -10, 6 and 11.
4. Complete the sequence given below.
   a) -18, -14, -10, _______, _______, ______, ________
   b) ______, ______, ______, ______, -17, -12, -7
   c) -100, -82, -64,_______, ______, ______, _______
5. Order the integer from least to greatest.
   -123, -541, -120, -97, 0, -58, 2
6. Subtract
   a) 30 from the sum of 12 and 13.
   b) - 420 from the sum of 354 and 147.
7. Perform the indicated operations.
   a) 3+ (-24)+13-8
   b) 1-10+63-(-21)+(-54)-27
   c) 10- [10-[10-(10-1)]]
   d) -10- [-10-[-10-(-10-10)]]
8. Use a number line to find the sum or difference.
   a) -7 + (-5) b) -13 + 15 c) 4 – 8 d) -2 – 7
9. Evaluate
   a) x + y for x = 8 and y = -12.
   b) x – y for x = -9 and y = - 13.
   c) x + y – z for x = -2, y = -5 and z = 7.
10. At 3 P.M the temperature was 9°C. By 11p.m, it had dropped 21°C. What was the temperature at 11 p.m?