Physics

Student Textbook

Grade 11

ISBN 978-99990-0-034-5

FDRE Ministry of Education

2023

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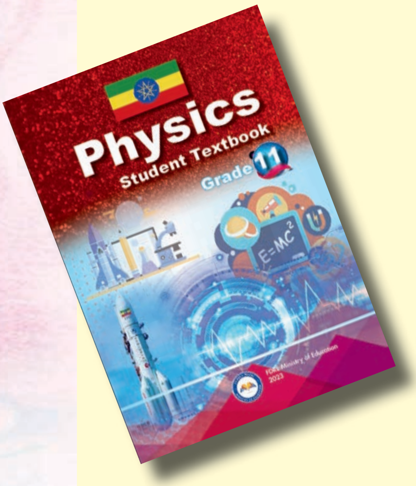

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FDRE, MINISTRY OF EDUCATION

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First Published in 2023 by the Federal Democratic Republic of Ethiopia, Ministry of Education, under the General Education Quality Improvement Program for Equity (GEQIP-E) supported by the World Bank, UK's Department for International Development/DFID-now merged with the Foreign, Common wealth and Development Office/FCDO, Finland Ministry for Foreign Affairs, the Royal Norwegian Embassy, United Nations Children's Fund/ UNICEF), the Global Partnership for Education (GPE), and Danish Ministry of Foreign Affairs, through a Multi Donor Trust Fund.

The Ministry of Education wishes to thank many individuals, groups, and other bodies involved – directly or indirectly – in publishing this Textbook. Special thanks are due to Hawassa University for their huge contribution to the development of this textbook in collaboration with Addis Ababa University, Bahir Dar University, and Jimma University.

Printed by:

GRAVITY GROUP IND LLC

13th Industrial Area,

Sharjah, UNITED ARAB EMIRATES

Under Ministry of Education Contract no. MOE/GEQIP-E/LICB/G-01/23

ISBN: 978-99990-0-034-5

CONTENTS

UNIT 1

PHYSICS AND HUMAN SOCIETY

Introduction

Physics is a major driver of innovation and economic growth in world-wide in sectors ranging from energy, security, advanced manufacturing, data extraction and analysis to healt care and diagnostic, finance and communication for the benefit of society.

In this chapter you will understand the career opportunity of physics, the contribution of physics for every individuals and economic growth of a country. In addition to see you will practice the method of gaining knowledge, It is important in providing fundamental knowledge for understanding nature, in providing applied knowledge for the development of technology.

After completing this unit, student will be able to:

- *get acquainted with the impact of physics on society.*
- *familiarize themselves with physics communities and their roles.*
- *develop basic understanding of the making of physics knowledge.*
- *familiarize themselves with basic principles and applications of physics in various disciplines.*
- *acquire basic knowledge and understandings of nature and appreciate it.*
- *update themselves with the current status of physics.*

1.1 Importance of Physics to Society

At the end of this section you will be able to:

- *explain the importance of physics to society*

? Brainstorming

1. What is the benefit of physics for society?
2. What are the technologies directly related to physics that benefit society?

Physics is one of the sciences that generate fundamental knowledge needed for the future technological advances that will continue to drive the economic engines of the world. Physics contributes to the technological infrastructure and provides trained personnel needed to take advantage of scientific advances and discoveries. Physics is an important element in the education of chemists, engineers and computer scientists, as well as practitioners of the other physical and biomedical sciences. The future of any country and its contribution to the global economy is linked to its ability to develop its human capital in the field of science and engineering. Through its scientists and engineers, it is able to develop infrastructure for trade and industry.

Physics extends and enhances our understanding of other disciplines, such as the earth, agricultural, chemical, biological, and environmental sciences, plus astrophysics and cosmology subjects of substantial importance to all peoples of the world.

Physics can play an important role in developing strategies to combat climate change, in the development of cleaner energies, and in the development of technological advancements.

Physics improves our quality of life by providing the basic understanding necessary for developing new instrumentation and techniques for medical applications, such as computer tomography, magnetic resonance imaging, positron emission tomography, ultrasonic imaging, and laser surgery.

Physics and technology must work together to resolve the need for new technologies that will decrease the damage to our planet, for strategies to ensure that the people of developing countries have the tools to progress, the need for solutions to deadly diseases that remain a threat, and the need for solutions to the increasing demands we place on our resources before they depleted. The role of physics in our modern world is more important than in any other time in history.

Society's reliance on technology represents the importance of physics in daily life. Many aspects of modern society would not have been possible without the important scientific discoveries made in the past. These discoveries became the foundation on which current technologies were developed. Discoveries such as magnetism, electricity, conductors and others made modern conveniences, such as television, computers, phones and other business and home technologies possible. Modern means of transportation, such as aircraft and telecommunications, have drawn people across the world closer together all relying on concepts in physics.

The Influence of Physics on Society

Explain the important of physics in transport, electronics and health.

Simple mechanical devices such as pulleys and levers, as well as complex modern machines like quantum computers and nuclear reactors, would be impossible to create without the use of physics.

Physics lies at the root of many inventions that have had an enormous impact on the everyday life of the average person. There are numerous ordinary objects that people use regularly that rely on the science of physics to function, including semiconductors, lasers, X-rays, GPS devices, radio transmitters and bar code scanners.

Transportation vehicles such as automobiles, airplanes and space shuttles could not be constructed without the help of physics experts. Physics is also useful for military purposes, informing the design of weapons. Many of the scientists responsible for inventing the atomic bomb were physicists, and today physicists are involved in the creation of nuclear weapons.

Activity 1.1

1. Explain the influence of physics on society in transport, health, economy, technology, etc

1.2 Physics Communities and Their Roles

At the end of this section you will be able to:

- *discuss about physics communities and their roles.*

Brainstorming

1. What is the purpose of establishment of physics communities? What are the technologies directly related to physics that benefit society?

Physics communities are an organized group of persons associated together for scientific purposes. This organization is established at the level of national, continent as well as worldwide based on their aims. For example, Ethiopian Physical Society (EPS) was established by Ethiopian physics professionals to enhance physics research and education, thereby contributing its share to the overall development of the country.

Objectives of EPS

- To promote physics education and research in the country;
- To organize and coordinate various conferences on physics education and exchange of scientific information;
- To popularize physics in order to make students develop interest in physics;
- To promote active participation of Ethiopian physicists and the general public in the design and implementation of the physics curriculum;
- To create a means for disseminating scientific information; etc.

Ethiopian Physics Society in North America (EPSNA) was established to promoting physics education and research among Ethiopians in Ethiopia and North America. The mission of EPSNA was to promote and support the education and training of Ethiopian Physics professionals by donating educational materials, field equipment, computing facilities, provide scholarship…etc.

African association of physics students (aaps) is Student-run educational association. It comprises students and recent graduates who are interested in physics. The purpose of the organization is to encourage physics students in their scientific and professional work in an African and an international context as well as to promote relations between physics students from all over the world. The 'aaps' is committed to supplement physics education with an international scientific experience and it is affiliated with Physics societies and departments. 'aaps' is open to all persons committed to physics.

Group Work

Make a group and do a project work on searching for detailed information about Ethiopian space science society (ESSS) mission, strategies, goals etc and present in the class.

Activity 1.2

What are the benefits of physics communities?

1.3 Making of Physics Knowledge

At the end of this section you will be able to:

- *discuss how scientific knowledge is constructed.*
- *discuss the roles that the learning of physics plays to the individual intellectual satisfaction.*

? Brainstorming

1. What is the process of gaining knowledge in physics?

One way of understanding the concept of knowledge is to look at the different ways in which we acquire knowledge. There are two types of knowledge from two entirely different sources. First, there is knowledge through experience: seeing something, hearing about something, feeling something called *a posteriori* knowledge. Second, there is knowledge that does not come from experience, but perhaps instead is intuitively supplied from reason itself, such as logical and mathematical truths called *a priori* knowledge.

Experimental knowledge

Experiential (*a posteriori*) knowledge is of many types. Four of them are sensory perception, introspection, memory, and testimony: these are the four main ways of acquiring knowledge through experience.

I. Sensory perception: sensory perception is perhaps the dominant source of experiential knowledge, it immediately raises a critical question. We gather knowledge by seeing, touching hearing, etc.

II. Introspection: Introspection is like a sixth sense that looks into the most intimate parts of our minds, which allows us to inspect how we are feeling and how our thoughts are operating. If I go to a doctor complaining of an aching back, she'll ask me to describe my pain. Through introspection I then might report, "Well, it's a sharp pain that starts right here and stops right here." The doctor herself cannot directly experience what I do and must rely on my introspective description.

III. Memory: a memory is like a recording device that captures events that one can experience more or less in the order that they occur.

IV. Testimony: Testimonies from written sources are usually more reliable than oral sources, but much depends on the integrity of the author, publisher, and the methods of fact-gathering.

Non-Experiential Knowledge

Turning next to non-experiential (*a priori*) knowledge, this source of information is more difficult to describe. We presumably gain access to this knowledge through rational insight. Usual examples of non-experiential knowledge are mathematics and logic. Take, for example, 2+2=4. Indeed, I might learn from experience that two apples plus two more apples will give me four apples.

Scientific method

The **scientific method** is an ordered series of steps to acquire knowledge based on experimental evidence.

Examples of Scientific Method in Physics

Observation: the water boiled at a lower temperature when I was visiting the mountains than when I was in other cities with low altitudes.

Question: Why does my water boil at different temperatures?

Research: In a chemistry book, you read that the boiling temperature of a substance depends on the strength of the molecular bonds of a substance and the pressure.

Hypothesis: Since the atmospheric pressure changes with altitude, the boiling temperature of water is different at different altitudes.

Experiment: You decide to heat water at different altitudes and record the boiling temperature.

Analysis:

Altitude (m)	Boiling point of water (C°)
0	100
150	99.5
305	99
610	98
1524	95

Your measurements indicate that as the height increases, the boiling temperature of water decreases!

Conclusion: The original hypothesis was correct. The boiling temperature of water decreases approximately by one degree Celsius.

Activity 1.3

1. Give some examples in the process of scientific method of knowledge gain.

1.4 The Mission of Physics and Career Awareness

At the end of this section you will be able to:

- *explain the job opportunities concerning to physics.*

Brainstorming

1. Mansion lists of career related to physics.

The diversity of physics careers is an appealing aspect for prospective students. Physics graduates have skills that are in high demand in diverse sectors. These include skills relating to numeracy, problem-solving, data analysis and the communication of complex ideas, as well as a wider understanding of how the world works on a scientific and human level.

Typical careers in physics

Whether you want to explore space, time, matter or the many other intriguing elements of the physical world, a physics degree can do wonders for your career path. While many physics graduates go on to work within research roles, these are spread across many different industries – including education, automotive and aerospace industries, defense, public sector, healthcare, energy, materials, technology, computing and IT.

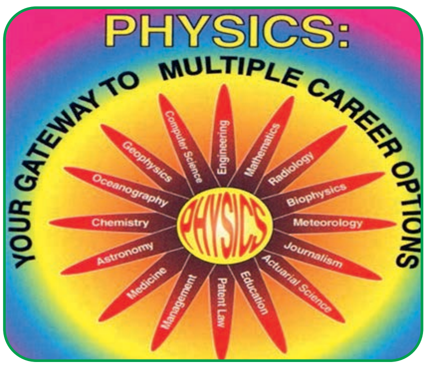


Figure 1: Career opportunities

Physics careers in space and astronomy

As an astronomer, your job would be to study the universe, collecting data from global satellites and spacecraft and operating radio and optical telescopes. Other tasks within this sector include investigation and research of new materials and technologies, measuring performance of existing materials and technologies, and problem-solving at the design stage.

Figure 2: A person observing stars through telescope

Physics careers in healthcare

Figure 3: observing CT scan result

Physics careers in the healthcare sector are numerous. Medical physics overlaps significantly with biomedical engineering, and physicists work alongside biomedical engineers to create, review and maintain medical technologies and equipment. Although cardiology and neurology are areas reserved for those with an additional medical degree, physicists are regularly employed within areas such as radiology, radiation oncology and nuclear medicine, in order to test and approve the latest technologies and equipment.

Physics careers in engineering

The engineering sector provides many careers in physics, particularly within manufacturing and technology-based roles. Physics graduates are often tasked with improving and developing products and manufacturing processes, and benefit from a large range of potential employers spanning multiple industries such as medicine, energy, transport, defense, space exploration and telecommunications.

Figure 4: engineers working place

Physics careers in energy

Figure 5: wind energy production site

Whether we're talking about renewable or non-renewable energy, there are plenty of careers in physics within the energy sector. Alongside the rise of renewable energy, oil and gas companies remain big players in the energy market and are major employers for physics graduates. Oneb area of focus is on extracting fossil fuel reserves in the most efficient way possible, using knowledge of the Earth's characteristics and the newest technologies.

With the prospect of fossil fuels running out, energy companies are also branching out into renewable alternatives such as wind and solar energy and are investing heavily in research and development in this area, offering much career potential. Your role here could be to collaborate with other scientists and engineers to develop efficient and functional energy systems which harness the Earth's energy sustainably and cost-effectively.

Physics careers in technology

A broad arena of continual growth and innovation, the technology sector is a constant source of new opportunities, challenges and career paths. For physics graduates, there is scope to work alongside other specialists in order to develop new ideas and products. Fields with particularly high demand for research and development workers from various backgrounds include relatively young fields such as robotics, nanoscience and nanotechnology.

Figure 6: application of machines

Technology careers in physics may be based in public or private-sector research centers. Many opportunities for graduates are available within large technology companies, as these businesses are keen to attract innovative and talented researchers from around the world.

Geophysics and meteorology careers

Those who study physics are also prime candidates for environmental careers, thanks to their scientific understanding of the ways in which the Earth functions. While geophysicists are more concerned with the prediction of natural disasters, meteorologists focus on areas such as daily weather forecasting, as well as researching the long-term effects of climate change.

Figure 7: Metrological report

Research scientist careers

The main reason to study physics is to help you gain more in-depth, specialized knowledge to prepare you to work effectively in a specific field. Potential areas of specialization include astrophysics, particle physics, biotechnology, nanotechnology, meteorology, aerospace dynamics, atomic and laser physics, atmospheric, oceanic and planetary physics and climate science.

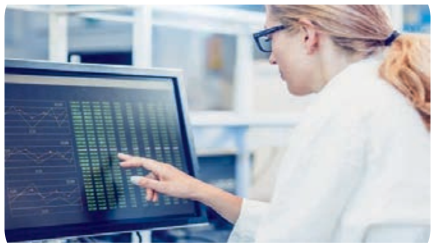


Figure 8: Data scientist

The mission of physics is to advance science, engineering, and innovation throughout the world for the benefit of all a*nd serving Society.*

1.5 Current Status of Physics

At the end of this section you will be able to:

- *list at least five recent new developments or discoveries in the fields of physics.*

? Brainstorming

Go to library and websites search out what are the newest discoveries in physics

Nowadays, there is a lot of discoveries and inventions in different parts of the world. Among this some are listed below.

Major recent discoveries in Physics

Physics is the study of physical world from the smallest particles to the gigantic structures known to human; galaxies, clusters of galaxies and black holes. Some of them are,

Discovery of Exoplanets:

Exoplanet or extrasolar planet is a planet outside our solar system. As of 1 October 2022, there are 5,197 confirmed exoplanets in 3,833 planetary systems.

Michel Mayor and Didier Queloz won the Nobel Prize in Physics in 2019.

Figure 9: Artist's impressio of exo planet orbiting two stars

https://www.nobelprize.org/interactive-visualisations-the-discovery-of-exoplanets/
https://en.wikipedia.org/wiki/Michel Mayor
https://en.wikipedia.org/wiki/Didier Queloz

Black hole:

Black hole is a region of space-time the gravity of it so strong that nothing can escape. Not even, light (electromagnetic wave).

Roger Penrose, Reinhard Genze. And Andrea M. Ghez have won the Nobel Prize in 2020 for discovery black hole formation and discovery of supermassive compact object (Black hole) at the center of our Milky-way Galaxy.

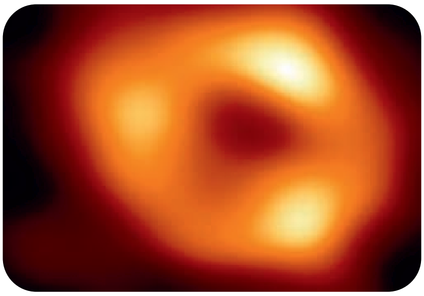


Figure 10: Image of Black Hole

https://en.wikipedia.org/wiki/Roger_Penrose,
https://en.wikipedia.org/wiki/Reinhard_Genzel
https://en.wikipedia.org/wiki/Andrea_M._Ghez

Quantum cryptography:

Cryptography is a way of sending information and messages in secret. Think of your bank account number, password etc. without the secret encryption all these are at risk. Quantum cryptography is an application of quantum physics that enhances the security of the information exchange at higher level.

High energy physics (Particle Accelerators)

July 4, 2012 EC, CERN scientists (thousands of them doing collaboratively from almost every corner of the world) discovered a particle called Higgs Boson. It is a subatomic particle like photons, protons and electrons but at the very basic level. The experiment took 60 years to prove the existence of this particle.

Gravitational wave (Large scale structures)

Even more years have taken to prove Albert Einstein's theory of gravity that says gravity is a wave that travels with the speed of light $3.00 \text{ X } 10^8$ m /s. The hypothesis was formulated in 1916, it was proved to be right in 2016 by LIGO, it took again thousands of scientists' collaborative work. They use Laser Interferometer as shown in the figure.

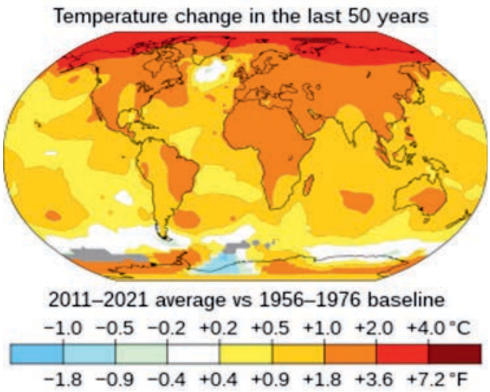
Global warming:

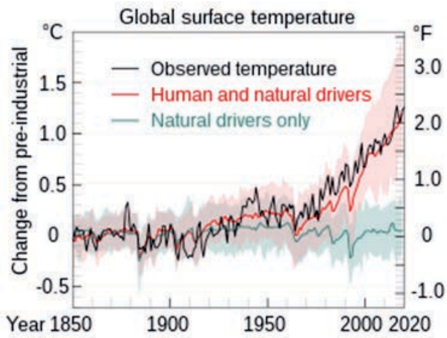
Global warming is an increase in earth's temperature and its impact on earth's climate.

Climate is a long-term weather variability averaged over many years and decades. Some of the variables are temperature, humidity, atmospheric pressure and wind. According to World Meteorological Organization (WMO) a typical variability average year is 30.

Figure 11: gravitational wave experimenting areas

Syukuro Manabe and Klaus Hasselmann have won the Nobel Prize in Physics in 2021 for modeling earth's climate and reliability predicting global warming. The following images from Wikipedia illustrate the effect of global warming.





https://en.wikipedia.org/wiki/Syukuro_Manabe
https://en.wikipedia.org/wiki/Klaus Hasselmann

Figure 12: illustrating the effect of global warming.

James Webb Space Telescope (JWST)

It is an infrared telescope located between earth and sun at a distance of 1,500,000 km beyond Earth's orbit around the Sun. It is operating at the temperature of (−223 °C) important for infrared detection of the galaxies and other large-scale structures.

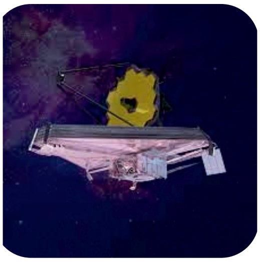


Figure 13: image of JWST

It will tell us about assembly of galaxies, birth of stars and atmospheres of exoplanets and objects in our solar system.

Figure 14: Star formation in Carina Nebula.

Future perspectives.

Research at the forefront is very expensive (Billions of dollars), it is also needing a collaboration of scientists from many fields. It also involves many countries.

Rich countries have huge laboratories of their own with many physicists involved. Ethiopia as developing country must start doing physics at the highest level for its national security and sustainable growth. the Ethiopian Science and Art Museum inaugurated by PM Abiy Ahmed 4 October 2022 would stimulate and attract young students towards science. It is situated in Addis Ababa.

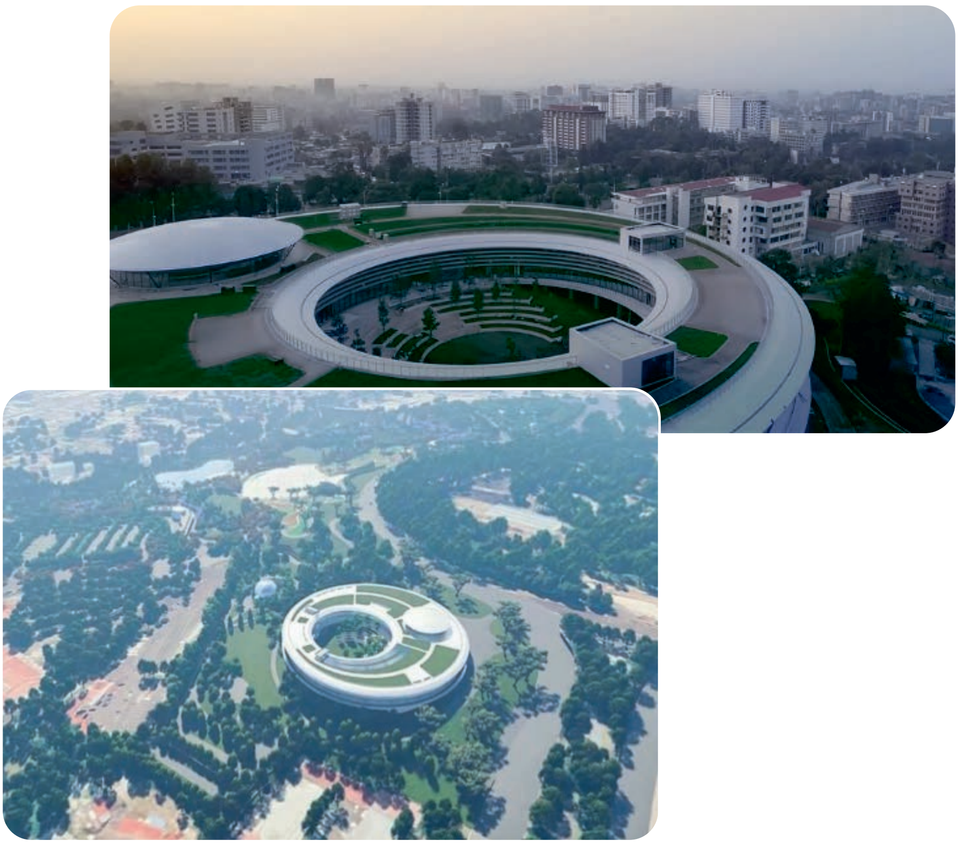


Figure 15: Some photographs of the newly inaugurated Ethiopian Science and Art Museum.

The museum holds exhibition hall dedicated to scientific and developmental research. It also includes several building complexes dedicated to interactive display screens, cyber security, geographical information system (GIS), data analysis, manufacturing, and robotics to name a few.

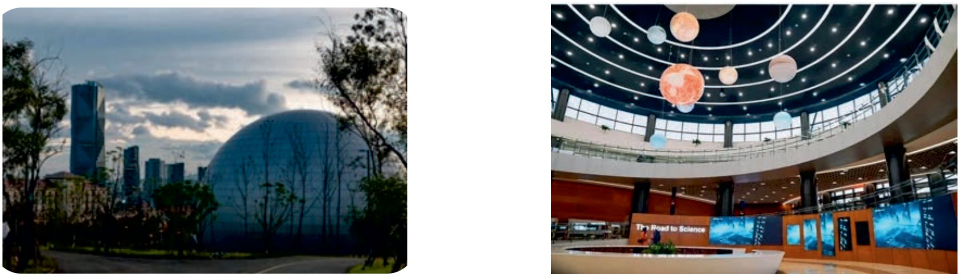


Figure 16: Interior of the Dome-shaped Science and Art Museum

Planetarium: Inside this dome you can seat and watch from our solar system to stars and galaxies. It is an amazing and inspiring place for young and old alike.

Activity 1.4

1. Google the web or go to the library to find some information to find out what is going on https://en.wikipedia.org/wiki/Ethiopia_Museum_of_Art_and_Science.

Questions 1.1

Explain current status of physics.

1. What do you think the future of physics looks like? Your opinions in one paragraph or two with justification.
2. What is the significances of the two discoveries- gravitational wave, and Higgs boson?

UNIT SUMMARY

- Physics improves our quality of life by providing the basic understanding necessary for developing new instrumentation and techniques for medical applications, such as computer tomography, magnetic resonance imaging, positron emission tomography, ultrasonic imaging, and laser surgery.
- Transportation vehicles such as automobiles, airplanes and space shuttles could not be constructed without the help of physics experts.
- Physics is also useful for military purposes, informing the design of weapons. Many of the scientists responsible for inventing the atomic bomb were physicists, and today physicists are involved in the creation of nuclear weapons.
- Physics communities are an organized group of persons associated together for scientific purposes. This organization is established at the level of national, continent as well as worldwide based on their aims.
- The scientific method is an ordered series of steps to acquire knowledge based on experimental evidence.
- The mission of physics is to advance science, engineering, and innovation throughout the world for the benefit of all and serving Society.
- Physics have a lot of job opportunities in the fields of Astronomy, healthcare, engineering, energy, technology, meteorology, etc
- New discoveries of Exoplanets, Black hole, Quantum cryptography, High energy physics, Gravitational wave, Global warming, James Webb Space Telescope, etc have been seen in recent years.

END OF UNITQUESTIONS

1. What is the importance of learning physics in your opinion?
2. What did you benefit from knowing physics at this level?
3. List at least ten technological benefits that physics has contributed for society.
4. What is the benefit of establishing physics communities?
5. Is there a physics community in your high school? If the answer is yes, what is it helpful for?
6. What process skills should you use investigating a phenomenon?
7. What are the process skills you are using in your in investigating a phenomenon?
8. What are the career opportunities physics have?
9. At the forefront in physics research, there are many countries collaborating. Why is that?

UNIT 2

VECTORS

Introduction

Many physical quantities in nature have both magnitude (size) and direction. To describe the motion of an airplane for instance, we must state its speed and the direction of its motion. To describe a force, we use a number to tell how big the force is and indicate its direction of application. The principal aim of this unit is to discuss ways of describing two dimensional 2-D vector physical quantities and the mathematical operations involving vectors.

After completing this unit you should be able to:

- *familiarize themselves with basic principles of vector operations*
- *interpret physical phenomena using the concept of vector*
- *develop skills of using the concept of vector in solving various problems.*

2.1 Vectors and Types of Vectors

After completing this section, you should be able to:

- *describe the difference between vector and scalar quantities.*
- *list down the common vector quantities in our everyday life.*
- *discuss geometric representation of vectors.*
- *give the definitions of the different types of vectors.*

? Brainstorming

1. Why do we need to know the direction of some quantities in order to have a complete sense of them? If a girl told you that the velocity of a delivery truck was 50 km/h, has she fully expressed the motion of the truck?

Some physical quantities such as mass, speed, distance, density, etc., can be expressed completely by their magnitudes and units only. But many others that are common in our day to day activities such as force, velocity, acceleration, displacement need both their magnitude as well as direction for their specification.

To refresh your memory of category of vectors, the table below has some of the most common physical quantities.

Activity 2.1

Categorize the quantities listed in Table 2.1 as scalars or vectors and prepare your own list of vector and scalar quantities that are common in our daily practices.

Table 2.1 Table of physical quantities

Quantity	Scalar/vector	Quantity	Scalar/Vector
Area		Density	
Distance			
Weight			
Pressure			

Representation of vectors

In Grade 10 you have learnt that a vector can be represented graphically by an arrow. Recall that the length of the arrow drawn to scale represents the magnitude and the tip of the arrow represents the direction of the vector. When drawing diagrams with vectors, we use a scale similar to those used for maps. For example, a displacement of 10 km might be represented in a diagram by a vector 1 cm long and a displacement of 20 km by a vector 2 cm long. In a diagram for velocity vectors, a vector that is 1 cm long might represent a velocity of magnitude 5 m/s. A velocity of 40 m/s would then be represented by a vector 8 cm long.

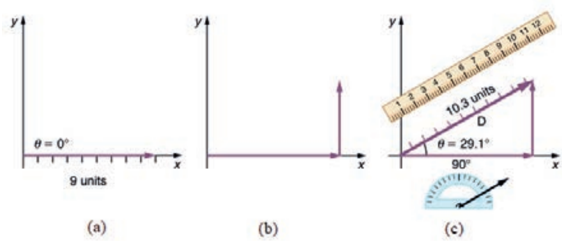
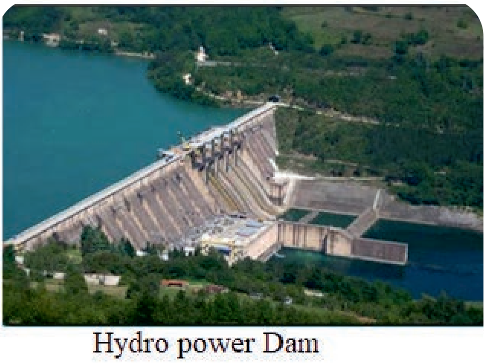


Figure 2.1 Representing a vector by an arrow drawn to scale. Ruler, Protractor and Graph paper used

Illustration of the concept of vectors in practice



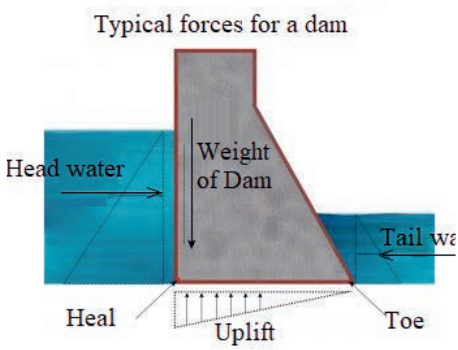
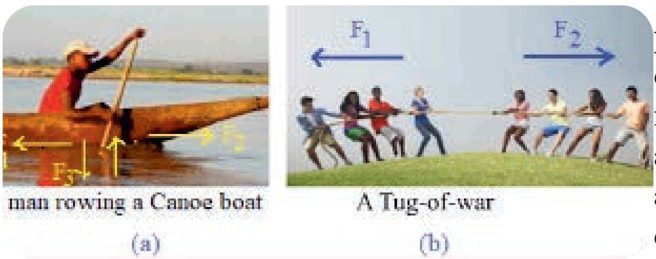


Figure 2.2 Hydro power dam and force vectors

By researching the size and direction of all forces on a dam, engineers design a structure that is able to resist all these forces, Figure 2.2



While rowing a boat, the person pushes the water backward using oars and the water exerts a forward force on the boat. There are also the weight of the boat and an upward force (upthrust) exerted by the water on the boat, Figure 2.3a.

Figure 2.3 Force vectors in action

In a tag-of-war the rope is the means to transfer forces, Figure 2.3b. The concepts of vectors plays important role in analyzing all of the above situations

Activity 2.2

Ask your friend to stand at a mark in the middle of the volleyball field. Tell her to walk through a certain distance in any direction she chooses. Use a measuring tape to measure the distance between her starting point and the ending point. Take the starting point as the origin of coordinates with the x axis pointing toward East and the y axis toward North. Show the displacement of the girl graphically on a graph paper.

Types of vectors

- Parallel vectors: Vectors that are in the same direction are said to be Parallel, Figure 2.4a.

Antiparallel vectors: When vectors have opposite directions, whether their magnitudes are the same or not, they are Antiparallel, Figure 2.4b.

Equal vectors: If two vectors representing the same quantity have the same magnitude and the same direction, they are Equal vectors, Figure 2.4c, no matter where they are located in space.

- Collinear vectors: These are vectors that lie along the same line or parallel lines. Two vectors are collinear if they are parallel to the same line irrespective of their magnitudes and direction.
- Co-planar vectors: Three or more vectors lying in the same plane or parallel to the same plane are known as co-planar vectors.
- Zero vector: A zero vector is a vector when the magnitude of the vector is zero and the starting point of the vector coincides with the terminal point.
- Orthogonal vectors : Vectors that are perpendicular to one another.
- A unit vector: A vector which has a magnitude of unit length is called a unit vector.

The vector $-\vec{A}$ in Figure 2.6d is not equal to vector $\vec{A}$ because its direction is opposite to that of $\vec{A}$. We define the **Negative of a vector** as a vector having the same magnitude as the original vector but the opposite direction. The negative of vector $\vec{A}$ is denoted as- $\vec{A}$.

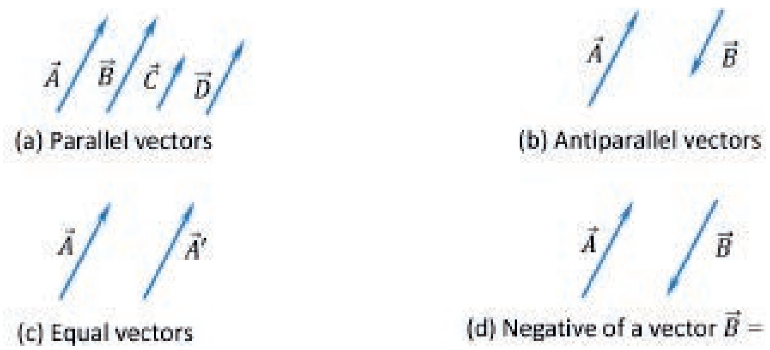


Figure 2.4 (a) Parallel vectors, (b) Antiparallel vectors, (c) Equal vectors, (d) Negative of a vector

Example 2.1

Find the negative of each of the following vectors.

a. dosplacement vector= 60km toward North

b. Velocity vector = 100km/h towards 200 southeast.

Solution

The negative of a vector has the same magnitude as the original vector in opposite direction.

(a) If $\vec{S}$ = 60 km, toward North, then - $\vec{S}$ = 60 km, toward South

(b) $\vec{S}$ = 100 km/h, toward 20o South of East, then - $\vec{S}$ = 100 km/h, toward 20o North of West

Review questions 2.1

1. How do you represent a vector graphically?
2. Why do we use of scale diagram in vector drawing?
3. How does vector $\vec{A}$ differ from vector - $\vec{A}$?

2.2 Graphical Method of Addition of Vectors in Two Dimensions (2-D)

After completing this section, you should be able to:

- *define the term resultant vector.*
- *explain the geometric method for addition and subtraction of vectors in a plane.*
- *apply geometric method of addition of vectors to find resultant of vectors in two dimensions.*

? Brainstorming

1. In grade 10 physics you have studied the properties of vectors in one dimension. Is it possible to add two vector quantities such as displacement vectors the way we do with scalar quantities like mass? Share your answer with your classmates.

There are two commonly used methods employed to determine the vector sum of two or more vectors. The two methods that we will discuss in this unit are Graphical method and Analytic (algebraic) method of vector addition.

Figure 2.5 A vector diagram with scale of a trip in a park

In your physics lesson in grade 10 you have learnt about vectors in one dimension (1-D). Here we are going to develop further the concept of vectors in two dimensions (2-D) which in physics has physical representation and importance in our daily lives.

Graphical method of vector addition gives visual understanding of vectors and it is commonly used in navigation. Figure 2.5 represents a trip that starts at point A then to point B and ending at point C. You see that total distance traveled for the whole trip is 1.2 km + 2 km = 3.2 km, but what is the displacement from A to C? How do you get the resultant displacement given displacements from A to B and from B to C?

In this section we will discuss Triangle law, Parallelogram law and Polygon law of addition of vectors.

When two or more vectors are added they must have the same units. For example, it doesn't make sense to add a force vector carrying units of newtons, to a velocity vector carrying units of meters per second. Scalars obey the same rule: It would be similarly meaningless to add mass to volume or density to time intervals. Graphical method of addition of vectors involves the use of equal vectors. As long as its length is not changed, a vector is not altered if it is displaced parallel to itself. This process is known as Translating a vector.

Triangle Law of Vector Addition

Suppose a particle undergoes displacement $\vec{s}_1$ followed by a second displacement $\vec{s}_2$. The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{S}$.

We call displacement $\vec{S}$ the vector sum, or resultant, of displacements $\vec{s}_1$ and $\vec{s}_2$. We express this relationship symbolically as

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

This can be shown graphically by placing the vectors head to tail and then the arrow drawn from the tail of the first vector to the head of the second vector represents the resultant, Figure 2.6. This is known as Triangle Law of vector addition.

Figure 2.6 Triangle law of addition of vectors: $\vec{\mathbf{S}}_1$ *and* $\vec{\mathbf{S}}_2$ *are placed head to tail.*

Activity 2.3

1. Use graph paper, a protractor, and a ruler and apply triangle law of vector addition to show that $\vec{A}+\vec{B}=\vec{B}+\vec{A}$. Use vector $\vec{A}$ of magnitude 5 units and vector $\vec{B}$ of 9 units, and the angle between the vectors to be 60^o.

Parallelogram law of vector addition

Two vectors can also be added graphically by constructing a parallelogram using the vectors as the sides of the parallelogram. The vectors are now placed tail to tail and the diagonal drawn from the tails of the vectors to the opposite corner of the parallelogram represents the resultant, Figure 2.7.

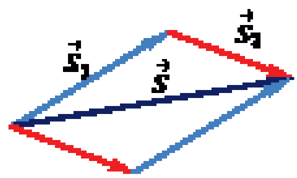


Figure 2.7. Paralelogram Law of addition of vectors: : $\vec{S}_1$ and $\vec{S}_2$ are placed tail to tail.

Activity 2.4

Discussion with your classmates

It's a common error to conclude that if $\vec{C}=\vec{A}+\vec{B}$, then the magnitude C should equal the magnitude. A plus the magnitude B. What is wrong with this conclusion?

When we need to add more than two vectors, we may start with one of the vectors and then place the tail of the next vector to the head of the first and do the same until all the vectors are included. The vector drawn from the tail of the first vector to the head of the last vector represents the resultant. This is known as polygon law of addition of vectors, Figure 2.8.

$$\vec{R}=\vec{A}+\vec{B}+\vec{C}+\vec{D}$$

Figure 2.8 polygon law of vector addition

Exercise 2.1

What is the result of adding a vector to the negative of itself?

Activity 2.5

How can you add individual displacement vectors to determine total displacement?

Materials

- A 20 m tape measure, Field marker (or flag), Graph paper ruler and Protractor

Procedure

- Mark a point at the center of the field with a flag.
- Indicate the North-South direction taking the marked point as the origin
- Ask your friend to walk the following distances starting from the flag: 12 m toward North, and 16 m toward East, 10 m toward South-East.
- Mark the final point and use the measuring tape to measure magnitude of his/her displacement from the starting point.
- Show your vector diagram on a graph paper
- Use a scale of 1 cm for 1 m of displacement.
- Use a protractor to determine the direction if displacement.

What is the magnitude and direction of the displacement?

Subtraction of Vectors

We can subtract vectors as well as add them. Recall that vector $-\vec{\mathbf{B}}$ has the same magnitude as $\vec{\mathbf{B}}$ but the opposite direction. Subtracting a vector is the same as adding a vector of the same magnitude but opposite in direction.

We define the difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ of two vectors and $\vec{\mathbf{B}}$ to be the vector sum of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$, Figure 2.9.

$$\vec{C} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Figure 2.9 Subtraction of vectors

Review questions 2.2

1. Which one of the vector diagrams (Figure 2.10) satisfies the vector equation $\vec{C} = \vec{A} + \vec{B}$?

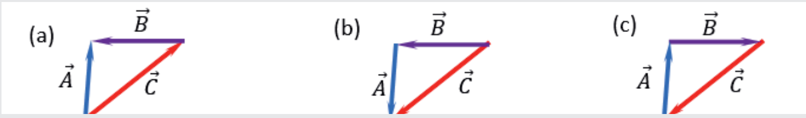


Figure 2.10

2. Three displacement vectors $\vec{A}, \vec{B}, \vec{C}$ are specified by their magnitudes A = 10 cm, B = 7 cm, C = 8 cm, respectively, and by their respective direction angles (with the +x direction) of $\alpha = 53^o$, $\beta = 120^o$, $\gamma = 210^o$. Choose a convenient scale and use a ruler and a protractor to find the magnitude and direction of the vector sum:

   (a) $\vec{R} = \vec{A} + \vec{B}$ (b) $\vec{P} = \vec{A} - \vec{B}$ (c) $\vec{Q} = \vec{A} - 2\vec{B} + \vec{C}$

   a) the magnitude C equal to the sum of the magnitude A and magnitude B?
   b) the magnitude C equal to the difference of magnitude A and magnitude B?

3. Suppose an insect lands on a sheet of graph paper at a point located 12 cm to the right of the left edge and 8 cm above the bottom edge and walks slowly to a point located 7 cm from the left edge and 4 cm from the bottom edge. Choose the rectangular coordinate system with the origin at the lower left-side corner of the paper and find the displacement vector of the insect. Use a ruler, a graph paper and make use of graphical representation of vectors
4. What is the minimum number of vectors of equal magnitudes required to produce a zero resultant?

2.3 Algebraic Method of addition of vectors in Two Dimensions (2-D)

After completing this section, you should be able to:

- *apply algebraic method to find the resultant of two collinear vectors, and perpendicular vectors.*
- *apply trigonometry to determine the direction of a resultant vector in terms of the angle it forms with a reference direction.*
- *compare the resultant of two given vectors as determined by way of geometric construction and by analytic method.*
- *resolve a vector into its components.*
- *find the resultant of two or more vectors applying component method.*
- *define unit vector and determine a unit vector in the direction of a given vector.*
- *apply the unit representation of vectors to determine the resultant of two or more vectors.*

? Brainstorming

Do you recall the different ways in which vectors that represent similar quantities can be added to/ subtracted from one another? Mention two ways of vector addition based on your physics lesson in grade 10.

Adding two collinear vectors

As scalars are fully described by their magnitude, addition or subtraction of scalars can be done by applying simple arithmetic. Vector addition, on the other hand involves both the magnitude and direction of the vectors.

Before beginning to add vectors in two dimensions, let us review important points about addition of vectors in one dimension.

Consider two force vectors $\vec{F}_1$ and $\vec{F}_2$ acting at a point along the same direction, Figure 2.11a. The resultant of these vectors $\vec{R}$ has a magnitude equal to the sum of the magnitude of each of the vectors and it is directed along the direction of any one of the vectors, Figure 2.11b.

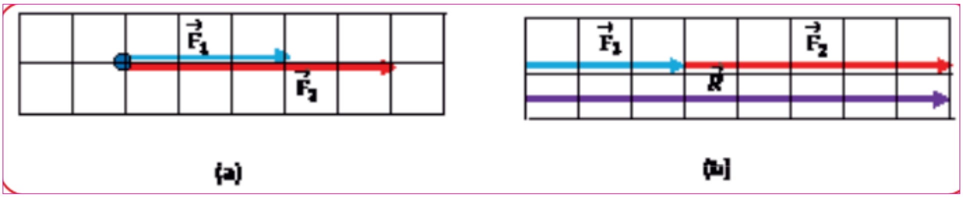


Figure 2.11 Vectors in the same direction

We write the resultant vector $\vec{\mathbf{R}}$ symbolically as

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

When force vectors $\vec{F}_1$ and $\vec{F}_2$ are acting in opposite directions, Figure 2.12a, their resultant $\vec{R}$ will have magnitude equal to the absolute value of the difference in magnitudes of the vectors and it is directed along the direction of the larger vector, Figure 2.12b.

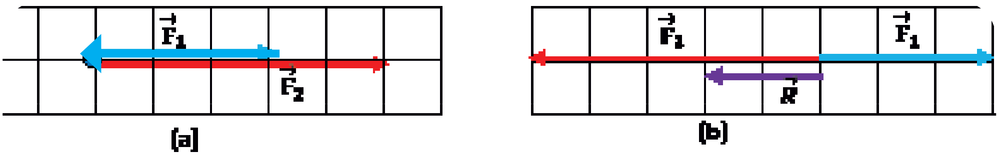


Figure 2.12 Vectors in opposite directions

Exercise 2.2

The magnitudes of two vectors $\vec{A}$ and $\vec{B}$ are 13 units and 9 units, respectively. What are the largest and smallest possible values for the magnitude of the resultant vector $\vec{R} = \vec{A} + \vec{B}$?

Example 2.2

A man rows his boat along the direction of flow of a river (downstream). If the boat can sail in still water at 0.50 m/s, and the river flows at 0.30 m/s, what is the resultant velocity of the boat?

Let the magnitude of velocity of the river be A and that of the boat be B.

A = 0.30 m/s

B = 0.50m/s

The resultant velocity is $\vec{R} = \vec{A} + \vec{B}$

Since both vectors are along the same direction, the magnitude of the resultant velocity is given by:

$R = A + B = 0.30\ m/s + 0.50 m/s.$

$R = 0.80\ m/s.$

The direction of the resultant is along the direction of the river flow.

Therefore, the resultant velocity is:

$\vec{R} = 0.8 m/s$, downstream

Exercise 2.3

If the man in Example 2.2 rows his boat upstream (in opposite direction to the river flow), what would his resultant velocity be?

Example 2.3

Two men are pushing a block along a horizontal surface by exerting oppositely directed forces of magnitudes 200 N and 100 N respectively, as shown in Figure 2.13. What is the resultant force applied on the block by the two men?

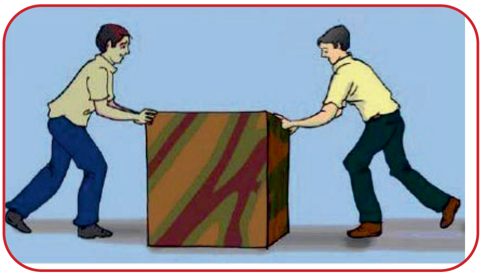


Figure 2.13 Two men pushing the block in opposite directions

Solution:

The two men applied two different forces on the same block.

Let the magnitude of the force exerted by the man on the right be $F_1 = 200$ N and that of the man on the left be $F_2 = 100$ N.

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

The magnitude of the resultant force is

$R = F_1 - F_2 = 200N - 100N = 100\ N$

The direction of the resultant is along the direction of the 200 N (larger) force.

$\vec{R} = 100N$, toward the right.

Adding two perpendicular vectors

The rules for addition of vectors that lie in a plane – that is, when they are in two dimensions become more complicated than the rules for addition of vector in one dimension. We have to use graphical method of vector addition to construct resultant vectors, followed by trigonometry to find magnitudes and directions. To get the resultant vector of two perpendicular vectors, $\vec{A}$ and $\vec{B}$, Figure 2.14, we use Pythagorean theorem for the magnitude of the resultant and apply trigonometry for the direction of the resultant.

Recall

According to Pythagoras theorem applied on a right angled triangle, Figure 2.14, the square of the hypotenuse, c, is equal to the sum of squares of the other two sides, a and b.

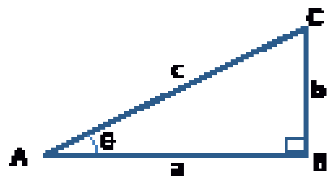


Figure 2.14 A Right-angled triangle

$$c^2 = a^2 + b^2, c = \sqrt{a^2 + b^2}$$

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{b}{a}$$

Example 2.4

An ant, starting at a point, walked through a distance of 40 cm due South and then 50 cm due West. What is the resultant displacement of the ant from its starting point?

Solution

Placing the tail of the vectors at the origin of coordinates, the vector diagram of the motion is shown in Figure 2.15.

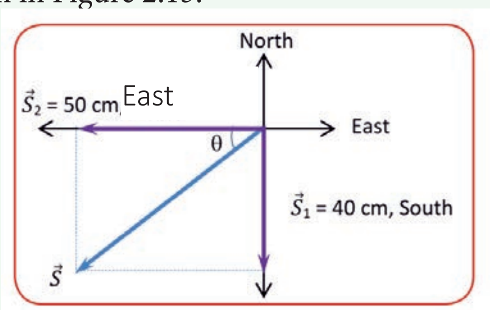


Figure 2.15 Vector diagram of motion of the ant

using pythagorean theorem, the magnitude of the resultant displacement is

$$S=\sqrt{S_1^2+S_2^2}=\sqrt{(40cm)^2+(50cm)^2}=\sqrt{4100cm^2}=64cm$$

Direction of the reultant displacement is shown by the angle θ, where

$$\tan\theta=\frac{opposite}{adjacent}=\frac{40cm}{30cm}=0.8, \theta=38.6^o$$

the resultant is $\vec{S}=64cm, 38.6^o\ South\ of\ West$

Components of a vector

In the previous sections we discussed addition of vectors by applying graphical method (a scale diagram) and by using properties of right angled triangles. Graphical method offers only very limited accuracy and calculations with right triangles work only when the two vectors are perpendicular. This section introduces a simple but general method for adding vectors known as Component Method.

In your mathematics lesson you have studied about rectangular (Cartesian) coordinate system of axes. Consider a vector placed on the xy-plane with its tail at o, the origin of the coordinate system, Figure 2.16.

The projection of the vector on the coordinate axes is known as component of the vector. The projection of vector $\vec{A}$ on the x axis, A_x, is known as the x component of vector $\vec{A}$ and that on the y axis, A_y, is known as the y component of vector $\vec{A}$. Components A_x and A_y are numbers but they are not vectors. $\vec{A}_x$ and $\vec{A}_y$ are the vector components of $\vec{A}$ and in symbols

$$\vec{A}=\vec{A}_x+\vec{A}_y$$

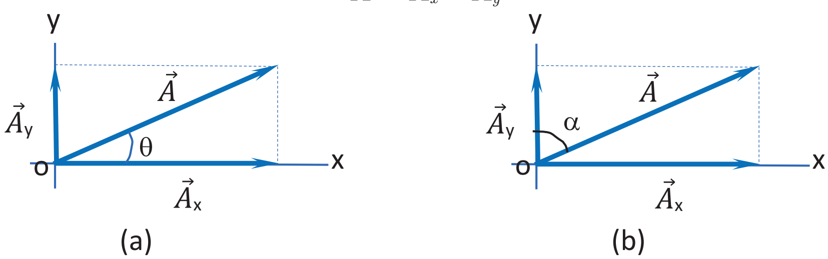


Figure 2.16 Vector \vec{A} in terms of its vector components $\vec{A}_x$ *and* $\vec{A}_y$

The x and y components of vector $\vec{A}$ (Figure 2.16a) have magnitudes of $A_x=A\cos\theta$ and $A_y=A\sin\theta$, respectively. In the arrangement shown in Figure 2.16a, both components are positive numbers. But in a more general situations, components may be positive or negative umbers depending on how the vector is oriented, i.e., the angle it makes with the x axis. The process of breaking up a vector into its components is known as resolution of a vector.

Note that: In finding the x- and y- components of a vector, we associate cosine with the x-component and sine with the y-component, as in Figure 2.16a. This association is due to the fact that we chose to measure the angle α with respect to the positive x-axis. If the angle were measured with respect to the y-axis, as in Figure 2.16b, the components would be given by

$$A_x = A\sin\alpha \quad and \quad A_y = A\cos\alpha$$

Key terms

- *Component of a vector: the projection of a vector on the coordinate axes.*
- *Resolution of a vector: Breaking a vector into its components*

Example 2.5

A force vector $\vec{F}$ of magnitude of 35 N is pushing on a box placed on the horizontal ground as shown in Figure 2.17a. If the force makes an angle of 40° above the negative x axis, what are the x and y components of the force?

Figure 2.17 A pushing force

Solution

Force F can be A placed on the xy plane as shown in Figure 2.17b. Remember that translation of a vector does not alter the magnitude and direction of the vector. The x component of the force is F_x = F cos 40° = 35 N x (0.766) = 26.8 N, and the y component is F_y = -F sin 40° = -35 N x (0.643) = -22.5 N

Finding the resultant of vectors by component method

In order to determine the resultant of two or more vectors using component method, we resolve each of the vectors into their x and y components. The x component of the resultant, Rx, will be the sum of the x components of the individual vectors and the y component of the resultant, Ry, will be the sum of the y components of the individual vectors, Figure 2.18.

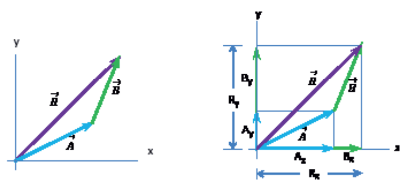


Figure 2.18 Vectors \vec{A} and \vec{B} added by Component Method

In terms of the vector components we write

$$\vec{R} = \vec{R}_x + \vec{R}_y, \text{ where } \vec{R}_x = \vec{A}_x + \vec{B}_x, \text{ and } \vec{R}_y = \vec{A}_y + \vec{B}_y$$

The magnitude of the resultant vector will be;

$$R = \sqrt{R_x^2 + R_y^2}$$

$R_x = A_x + B_x$, and $R_y = A_y + B_y$ (taking the signs of the components into consideration)

The direction of the resultant is given in terms of the angle it makes with the x axis. Place $\vec{R}_x$ and $\vec{R}_y$ on the xy-plane and find the angle as

$$\tan\theta = \frac{R_y}{R_x} \qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Attention

- if both Rx and Ry are positive numbers, angle θ is above the +x axis
- if R_x is negative number and Ry is positive number, angle θ is above the -x axis
- if both Rx and Ry are negative numbers, angle θ is below the -x axis
- if Rx is positive number and Ry is negative number, angle θ is below the +x axis

Exercise 2.4

Under what circumstances would a vector have components that are equal in magnitude?

Exercise 2.5

A vector has an x-component of - 35 units and a y-component of 40 units. Find the magnitude and direction of the vector considering that the positive x direction indicates East and that the positive y direction indicates North.

Example 2.6

Consider two displacement vectors $\vec{A}$ and $\vec{B}$ of magnitudes 100 m and 200 m respectively, Figure 2.21 . Find the magnitude and direction of the resultant displacement vector.

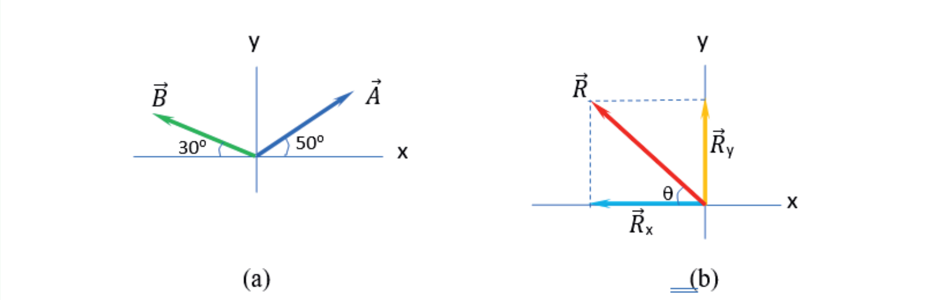


Figure 2.19

Solution

We first resolve the vectors into their x and y components.

Using trigonometry, we get A_x = A cos 50° = 100(0.643) m = 64.3 m, and

A_y = A sin 50° = 100(0.766) m = 76.6 m

B_x = -B cos 30° = -200 (0.866) m = -173.2 m, and

B_y = B sin 30° = 200(0.50)m = 100 m

The vector components of the resultant are $\vec{R}_x = \vec{A}_x + \vec{B}_x$, and $\vec{R}_y = \vec{A}_y + \vec{B}_y$, where Rx = Ax + Bx = 64.3 m + (-173.2 m) = -108.9 m and Ry =Ay + By = 76.6 m + 100 m = 176.6 m.

Then we place the components of the resultant on the xy-plane as in Figure 2.19 (b). The magnitude of the resultant is

$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(-108.9m)^2 + (176.6m)^2} = 207.5m$$
$$and\ \theta = \tan^{-1}\left(\frac{176.6m}{108.9m}\right) = \tan^{-1}(1.62) = 58.3°$$
$$Therefore,\ \vec{R} = 207.5m,\ \theta = 58.3°\ above\ the\ negative\ x\ axis$$

Exercise 2.6

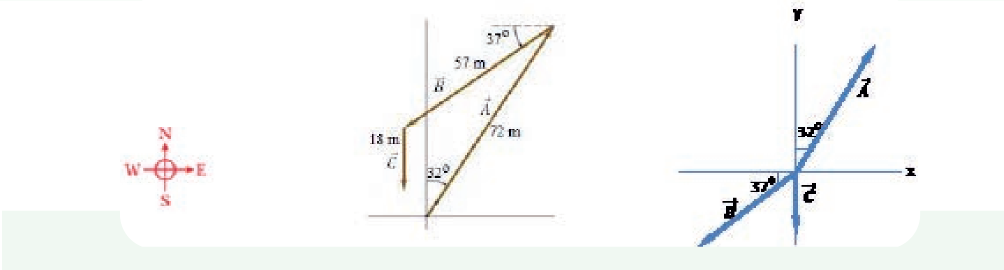
Vector $\vec{A}$ has a magnitude of 30 units and points in the positive y-direction. When vector $\vec{B}$ is added to $\vec{A}$, the resultant vector $\vec{A}$ + $\vec{B}$ points in the negative y-direction with a magnitude of 10 units. Find the magnitude and direction of $\vec{B}$.

Example 2.7

A girl walked across a large field through the following distance in the given order:72 m toward 32° East of North, 57 m toward 37°South of West, 18 m toward South. Find the magnitude and direction of the resultant displacement of the girl?

Solution

The displacement vectors are placed on the xy-plane as shown in Figure 2.20.



(a) (b)

Figure 2.20 (a) The actual path the girl followed, (b) Vector diagram

Solution

The components of the displacement vectors are calculated as in Table 2.2 below.

Table 2.2

	x component	y component
S_1	72 (sin 32°) m = 38.2 m	72 (cos 32°) m = 61.1 m
S_2	-57(cos 37°) m = - 45.6 m	-57 (sin37°) m = -34.2 m
S_3	0 m	-18 m
	$R_x = S_{1x} + S_{2x} + S_{3x}$ = -7.4 m	$R_x = S_{1y} + S_{2y} + S_{3y}$ = 8.9 m

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-7.4m)^2 + (8.9m)^2)} = 11.6m$$

$$\text{and } \theta = tan^{-1}\frac{(8.9m)}{(7.4m)} = tan^{-1}(1.2) = 50.2^\circ \text{ above the nagative x axis}$$

$$\text{Therefore, } \vec{R} = 11.6m, \theta = 50.2^\circ \text{ N of W}$$

Did you know?

Analytic methods of vector algebra are used routinely in mechanics, electricity, and magnetism. They are important mathematical tools of physics.

Exercise 2.7

A car travels 20 km due north and then 25 km in a direction 60° West of North. Use both graphical and algebraic methods to find the magnitude and direction of a single vector that gives the net effect of the car's trip.

Unit Vectors

As we have seen in the previous discussion we defined a unit vector as a vector of magnitude one, with no unit. Unit vectors are used to represent the direction of vectors in space. The symbol of unit vectors is denoted by a 'hat' (^) to distinguish it from ordinary vectors.

In an xy-coordinate system we define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector \hat{j} that points in the direction of the positive y-axis, see Figure 2.21 .

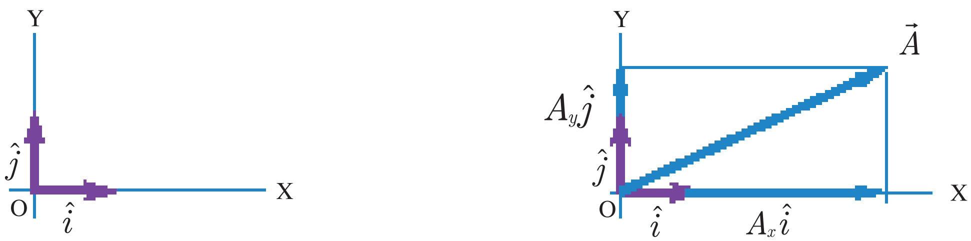


Figure 2.21 Unit vectors on the xy-plane

Vector components of vector \vec{A} and its components are related as

$$\vec{A}_x = A_x\hat{\imath}$$
$$\vec{A}_y = A_y\hat{\jmath}$$

Vector A can be written in vector equation as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

Using the unit vectors the resultant of two vectors A and B can be expressed as

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} \\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} \\
\vec{R} &= \vec{A} + \vec{B} \\
\vec{R} &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) \\
&= (A_x + B_x)\,\hat{\imath} + (A_y + B_y j)\,\hat{\jmath} \\
&= R_x\hat{\imath} + R_y\hat{\jmath}
\end{aligned}$$

The unit vector along the direction of a given non-zero vector is defined as the vector divided by its magnitude.

If vector A $\vec{A} = A_x\hat{i} + A_y\hat{j}$, then the unit vector along $\vec{A}$ is

$$\vec{U}_A = \frac{\vec{A}}{A}$$

Example 2.8

Given force vector $\vec{F} = (12\hat{i} - 16\hat{j})N$, what is a unit vector along $\vec{F}$?

solution

$$\vec{U}_F = \frac{\vec{F}}{F} = \frac{12\hat{i} - 16\hat{j}}{12^2 + (-16)^2} = \frac{12\hat{i} - 16\hat{i}}{20} = (0.6\hat{i} - 0.8\hat{j})N$$

See that the magnitude of $\vec{U}_F is\ U_F = \sqrt{(0.6)^2 + (0.8)^2} = 1$

Example 2.9

Vector $\vec{A}$ has x and y components of 4 units and 2 units, respectively and vector $\vec{B}$ has the corresponding components of -9 units and 3 units respectively. Find

(a) the vector components of their resultant,

(b) the magnitude and direction of their resultant.

solution

Given $\vec{A} = A_x\hat{i} + A_y\hat{j} = (4\hat{i} + 2\hat{j})\ units$

$\vec{B} = B_x\hat{i} + B_y\hat{j} = (-9\hat{i} + 3\hat{j})\ units$

The resultant

$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y j)\hat{j} = (4 - 9)\hat{i} + (2 + 3)\hat{j} = (-5\hat{j} + 5\hat{j})\ units$

(a) The x and y components of

$$\vec{R}\ are = R_x = -5\ units,\ \vec{R}_x = R_x\hat{i} = -5\hat{i}$$

$$R_y = -5\ units,\ \vec{R}_y = R_y i\hat{j} = -5\hat{j}$$

(b) the magniture of

$$\vec{R}\ is\ R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-5)^2 + 5^2} = \sqrt{50} = 5\sqrt{2}\ units$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{5}{5}\right) = 45^\circ$$

As $\vec{R}$ is along the negative x direction and $\vec{R}_y$ is along the positive y direction, we see that $\vec{R}$ is in the second quadrant and it makes angle of 45° above the negative x axis.

Example 2.10

An unknown vector $\vec{\mathbf{D}}$ is added to vector $\vec{\mathbf{C}} = (-4\hat{i} + 5\hat{j})$ units and the resultant $\vec{\mathbf{R}} = \vec{\mathbf{C}} + \vec{\mathbf{D}}$ has x and y components of each -1 and 1 units, respectively Find the magnitude of the unknown vector.

Solution

$$Given\ \vec{C} = (-4\hat{i} + 5\hat{j}),\ \vec{D} = (D_x\hat{i} + D_y\hat{j})\ and\ \vec{R} = (-\hat{i} + \hat{j})$$

$$For\ \vec{R} = \vec{C} + \vec{D},\ we\ have\ R_x = C_x + D_x,\ -1 = -4 + D_x,\ D_x = 3\ units$$

$$R_y = C_y + D_y,\ 1 = 5 + D_y,\ D_y = -4\ units$$

$$Vector\ \vec{D} = (3\hat{i} - 4\hat{j})\ units$$

Exercise 2.8

The vector sum $\vec{P}$ + $\vec{Q}$ is a unit vector along the positive x axis. If $\vec{P} = \hat{i} - \hat{j}$, find $\vec{Q}$.

Exercise 2.9

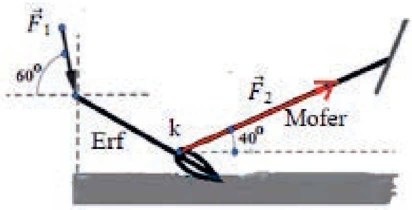
If vector 0.4 $\hat{i}$ + b $\hat{j}$ is a unit vector, then what is the value of b?

Example 2.11

Traditional Maresha

A farmer is ploughing the field using traditional Maresha (plow) pulled by two oxen Figure 2.24a. The two animals are pulling the beam (Mofer) with a force $\vec{F}_2$ of 1200 N at angle of 40° from the horizontal and the farmer's force $\vec{F}_1$ on the handle (Erf) is 150 N at 60° above the horizontal as shown in Figure 2.24b. Find the resultant horizontal pulling force exerted by the farmer and the oxen on the Maresha? At this point assume all the forces are acting at point K.
For the whole system to work the resultant downward vertical force must be slightly greater than the upward. Explain why.

Figure 2.22 (a) A farmer plowing his farm



(b) Schematic diagram of the Maresha

Solution

The forces that are acting on the Maresha are the force exerted by the farmer on the Erf, $\vec{F}_1$, and the pulling force on the Mofer, $\vec{F}_2$.

Both $\vec{F}_1$ and $\vec{F}_2$ have x components and the sum of these components gives the resultant horizontal pulling force. Both F_{1x} and F_{2x} are along the positive x direction.

$$F_x = F_{1x} + F_{2x} = F_1 \cos\theta_1 + F_2 \cos\theta_2$$

$$= 150N \cos 60^\circ + 1200N \cos 40^\circ$$

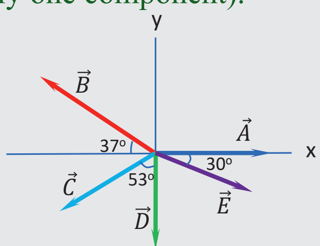
$$= 75N + 919N = 994.3\ N.$$

If the upward forces on the marseha exceed the downward focrces(which is the sum of weight of the plow and the downward component of the force exerted on the Erf), the maresha will be lifted above the ground and plaughing will not be possible.

Review Questions 2.3

1. Two vectors give a resultant of magnitude 8 units when are parallel and 2 units when they are antiparallel. Determine the possible values of each of the vectors.
2. State the condition where the vector sum of two vectors is a null vector.
3. Suppose you are in a large hall and that you mark a point on the floor and start walking the following distances in the given order: 50 m [forward], 10 m [backward], 10 m [forward], 10 m [backward], 20 m [forward], 10 m [backward], 40 m [forward], and 10 m [backward].
   A. What is the total distance you traveled?
   B. What is the magnitude and direction of your displacement from the starting point?
4. What vector must be added to vector $\vec{C}$ of 10km, East in order to give a resultant vector of
   A. 15 km, East? B. 15 km, West

5. A sailor boards a paddle boat and heads the boat Westward directly across a river. The river flows South at 50 cm/s and the woman paddles the boat with a speed of 100 cm/s.
   A. Determine the resultant velocity of the boat – both magnitude and direction.
   B. How far down stream relative to the straight-across direction will woman be when she reaches the opposite shore?
6. Given three displacement vectors $\vec{B}, \vec{C}, and\ \vec{D}$ such that $\vec{B} = (-\hat{i} + 2\hat{j})\,m, \vec{C} = 3\hat{i} - 2\hat{j})\,m$ $\vec{D}$ of unknown components, determine the magnitude and direction of $\vec{D}\ for\ 3\vec{B} - \vec{C} + \vec{D} = 0$
7. Given displacement vectors of $\vec{S}_1\ and\ \vec{S}_2$ as $\vec{S}_1 = (3\hat{i} + 6\hat{j})\,m$, $\vec{S}_2 = (-2\hat{i} - 4\hat{j})\,m$, find a unit vector along the direction of $\vec{S}_1 + \vec{S}_2$.
8. Five displacement vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}\ and\ \vec{E}$ are placed on the coordinate axes as shown in Figure 2.23. Fill Table 2.3 with the correct values of the components of the vectors. (Hint: A vector on either of the coordinate axis has only one component).



	x component	y component
$\vec{A}$		
$\vec{B}$		
$\vec{C}$		
$\vec{D}$		
$\vec{E}$		

Figure 2.23 The five vectors on the xy plane

2.4 Product of Vectors

After completing this section, you should be able to:

- *define dot product of a vector*
- *apply the technique of dot product to solve practical problems*

In the previous sections you have learnt about different techniques of finding a resultant vector. We can also express many physical relationships by using products of vectors. Vectors are not ordinary numbers, so we can't directly apply ordinary multiplication to vectors.

There are two different kinds of products of vectors: Scalar (Dot) product and Vector (Cross) product. The scalar product of two vectors yields a result that is a scalar quantity while the vector product of two vectors yields another vector. In this section we will discuss the scalar product of two vectors.

Brainstorming

Do you know any physical quantity that can be expressed as a product of two vector quantities?

Multiplying or dividing a vector by a number or scalar

A vector quantity such as a force can be multiplied by a scalar quantity (an ordinary number). Multiplying a vector by a number gives a vector. When we multiply a vector $\vec{A}$ by a scalar c, the result c$\vec{A}$ has a magnitude equal to |c|$\vec{A}$ (the absolute value of c multiplied by the magnitude of vector $\vec{A}$). If c is positive, $\vec{A}$ and c$\vec{A}$ are in the same direction (they are parallel) but if c is negative, $\vec{A}$ and c$\vec{A}$ are in opposite directions (they are antiparallel).

For example, if vector $\vec{A}$ is multiplied by a scalar number 2, the result, represented by 2$\vec{A}$, is a vector with magnitude two times that of $\vec{A}$ pointing along the direction of $\vec{A}$. On the other hand, if vector $\vec{A}$ is multiplied by a scalar number -2, the result, represented by -2$\vec{A}$, is a vector with magnitude two times that of $\vec{A}$ pointing in opposite direction to $\vec{A}$, see Figure 2.24.

Figure 2.26 (a) $\vec{A}$ *and 2* $\vec{A}$ *are parallel (b)* $\vec{A}$ *and -2* $\vec{A}$ *are antiparallel*

Example 2.12

Displacement vector given as $\vec{S}$=35km, towards 60 60° North of east. Find the magnitude direction of a) 1.2$\vec{S}$ and b) -1.2$\vec{S}$?

Solution

Given $\vec{S}$=35km, towards 60 60° North of east

a) Vector 1.2$\vec{S}$ is a displacement with magnitude 1.2 times that of S along the direction of $\vec{S}$.

1.2S= 1.2 X 35=42km and 1.2$\vec{S}$=42km, 60^0 North of East.

b) Vector -1.2$\vec{S}$ a displacement vector with magnitude 1.2 times that of $\vec{S}$ pointing opptisite to the direction $\vec{S}$. Therefore -1.2$\vec{S}$=42km, 60^0 South of West.

Dot product

The scalar product of two vectors is denoted by $\vec{A} \cdot \vec{B}$. Because of this notion, the scalar product is also called the dot product. Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A}.\vec{B}$ is a scalar.

The dot product of vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where θ is the angle between the vectors, Figure 2.25 notes that the righthand side of the equation is a number (without direction).

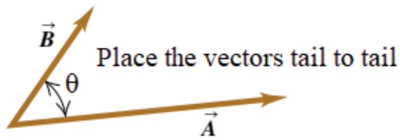


Figure 2.25 Vectors are placed tail to tail to form angle θ

The angle between the vectors ranges from 0º to 180º and it is the angle formed when vectors are joined tail to tail.

Depending on the value of the cosine of the angle between the vectors, the scalar product $\vec{A} \cdot \vec{B}$ may be positive, negative, or zero.

Key terms

- *Scalar (dot) product: product of vectors that yields a scalar.*
- *Angle between vectors: the angle two vectors form when placed tail to tail*

As shown in Figure 2.26a, or the magnitude of the component of $\vec{\mathbf{B}}$ in the direction of $\vec{\mathbf{A}}$, as shown in Figure 2.26b.

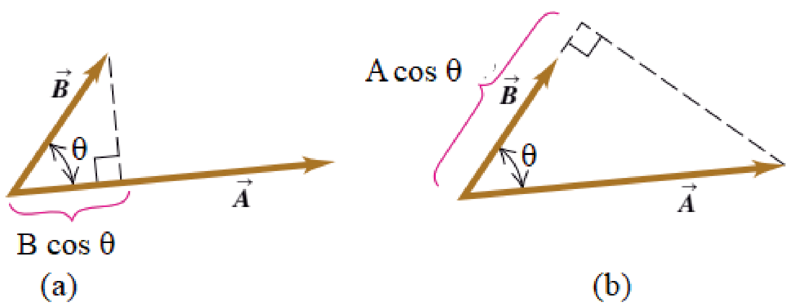


Figure 2.26: Alternative ways of finding the dot product of $\vec{A}$ and $\vec{B}$

Exercise 2.10

The dot product of $\vec{A}$ and $\vec{B}$ can be defined alternatively as the magnitude of multiplied by the component $\vec{B}$ of in the direction of $\vec{A}$. Find the angle between each of the following pairs of vectors.

a) $3\hat{i}-\hat{j}$ *and* $\hat{i}-2\hat{j}$ b) $3\hat{i}-2\hat{j}$ *and* $\hat{i}-2\hat{j}$ c) $3\hat{i}-2\hat{j}$ *and* $4\hat{i}+6\hat{j}$

Activity 2.6

Group discussion

Apply the definition of dot product to show that the dot product of a unit vector with itself is one (unity) while the dot product of a unit vector with another unit vector is zero, and use the result to prove that

$$\vec{P}\bullet\vec{Q}=P_xQ_x+P_yQ_y, \text{ where } \vec{P}=P_x\vec{i}+P_y\vec{j} \text{ and } \vec{Q}=Q_x\vec{i}+Q_y\vec{j}$$

Note that

There are two ways of writing the dot product of two vectors:

$$\vec{A}\bullet\vec{B}=AB\cos\theta$$

$$\vec{A}\bullet\vec{B}=A_xB_x+A_yB_y$$

Exercise 2.11

Given a position vector $\vec{P}=-4\hat{i}+6\hat{j}$, What is the cosine of the angle between $\vec{P}$ and (a) the x-axis? (b) the y-axis? (The cosine of the angle between a vector and the positive coordibate axis is called direction cosine of the vector.

Example 2.13

Vector $\vec{A}=\hat{i}+\hat{j}$ and vector $\vec{B}=-2\hat{i}+3\hat{j}$.

What is (a) the scalar product of $\vec{A}$ *and* $\vec{B}$?

(b) the nagle between $\vec{A}$ *and* $\vec{B}$?

Solution

(a) $\vec{A}.\vec{B} = A_x.B_x + Ay.B_y = (1)(-2) + (1)(3)$

(b) From $\vec{A}.\vec{B} = AB\cos\theta, \cos\theta = \dfrac{\vec{A}.\vec{B}}{AB}$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

$$\cos\frac{\overrightarrow{AB}}{AB} = \frac{1}{\sqrt{2}\sqrt{12}} = 0.196, \theta = \cos^{-1}(0.196) = 78.7^{a}$$

Example.2.14:

Force and displacement

Consider a block placed on a horizontal surface and that force $\vec{F}$ is applied on the block to move the block through displacement $\vec{S}$. The work done by the force is defined as the dot product of the force and the displacement. If $\vec{F} = (5\hat{i} + 3\hat{j})$ N and $\vec{S} = (-2\hat{i} + 4\hat{j})$ m, what is the work done by the applied force?

solution

$$Work\ W = \vec{F}.\vec{S} = (5\hat{i} + 3\hat{j})N.(-2\hat{i} + 4\hat{j})m = (5(-2) + 3(4) = 2Nm = 2J$$

Example 2.15:

Force and velocity

Consider force $\vec{F}$ of a certain machine is applied on a body to move the body with an average velocity $\vec{V}$. The power developed by the machine is defined as the dot product of the force and the average velocity. If $\vec{F} = (50\hat{i} + 30\hat{j})$ and $\vec{V} = (3\hat{i} + 4\hat{j})$ m/s, what is the power developed by the machine?

Solution

The power $P = \vec{F}.\vec{V} = (50\hat{i} + 30\hat{j})N.(3\hat{i} + 4\hat{j})m/s$

$$= 50(3) + 30(4) = 270Nm/s = 270W$$

Review Questions 2.4

1. What is the dot product of vector $\vec{A}$ with itself?
2. Show that the dot product of two vectors obeys:
   (a) commutative property, $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$?
   (b) distributive property, $\vec{A} . (\vec{B} + \vec{C}) = (\vec{A} . \vec{B}) + (\vec{A} . \vec{C})$
3. Is $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$?
4. For what angle between $\vec{A}\ and\ \vec{B}$ will $\vec{A}.\vec{B}$ be equal to $\vec{B}.\vec{A}$
5. The force applied on a body and the displacement it underwent are given respectively as $\vec{F} = (25\hat{i} - 30\hat{j})$ N and $\vec{S} = (-2\hat{i} - 3\hat{j})$ m. What is the work done by the force?
6. For what values of the angle between $\vec{A}\ and\ \vec{B}$ will the dot product $\vec{A}.\vec{B}$ be positive, negative or zero?
7. If $\vec{D} = (5\hat{i} - 3\hat{j})$ N and $\vec{E} = (2\hat{i} + \hat{j})$, what is the angle between $\vec{D} + \vec{E}$ and the x axis?
8. If $\vec{P}.\vec{Q}$ = PQ, what is the angle between $\vec{P}\ and\ \vec{Q}$?

UNIT 2 SUMMARY

- When a physical quantity is described by a single number, we call it a scalar quantity. In contrast, a quantity described by both a magnitude (the "how much" or "how big" part) and direction in space is a vector.
- Depending on their direction, magnitude and orientation we can categorize vectors as collinear, coplanar, parallel, antiparallel, equal, null, unit vector etc.
- Graphical method of vector addition makes use of translation of vectors.
- Triangle law – Vectors are connected head to tail to form a triangle.
- Parallelogram law – vectors are connected tail to tail and a parallelogram is constructed using the vectors as its sides.
- Polygon law - used to find the resultant of more than two vectors. The vectors are joined head to tail without regard to the order they are taken. The vector drawn from the tail of the first vector to the head of the final vector represents the resultant.
- Analytic methods of vector addition/subtraction allow us to find resultant of sums or differences of vectors without having to draw them. They are exact, contrary to graphical methods which are approximate.
- The resultant of two vectors in the same direction has a magnitude equal to the sum of the magnitude of each of the vectors and it is directed along the direction of any one of the vectors.
- The resultant of two vectors at right angles has a magnitude equal to the square root of the sum of the squares of the magnitude of each of the vectors and the direction of the resultant is determined by the angle it makes with respect to a reference line (commonly the x axis).
- A unit vector vector along the x axis is denoted by $\hat{i}$ and that along the y axis is denoted by $\hat{j}$. A vector that is represented by its vector components as $\vec{A} = \vec{A}_x + \vec{A}_y$ can also be represented interms of its components as $\vec{A} = A_x\hat{i} + A_y\hat{j}$.
- If $\vec{R}$ is the resultant of $\vec{A}$ *and* $\vec{B}$ then $\vec{R} = R_x\hat{i} + R_y\hat{j}$, where $R_x = A_x + B_x$ and $R_y = A_y + B_y$. The angle that $\vec{R}$ makes with the x axis is $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$
- The scalar product of two vectors $\vec{A}$ *and* $\vec{B}$ is defined as $\vec{A}.\vec{B} = AB\cos\theta$ and also $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y$.
- The scalar product of two vectors can be used to determine the angle between the vectors and it can also be used to determine the component of one of the vectors in the direction of the other.

END OF UNIT 2 QUESTION

1. Identify the following quantities as vectors and scalars: Speed, velocity, Displacement, Volume, Acceleration, Power.
2. Which one of the following is not true about the three vectors shown in Figure 2.27?
   a. $-\vec{A}=\vec{B}+\vec{C}$
   b. $\vec{B}=-\vec{C}-\vec{A}$
   c. $\vec{C}=-\vec{A}-\vec{B}$
   d. $\vec{A}+\vec{B}-\vec{C}=0$

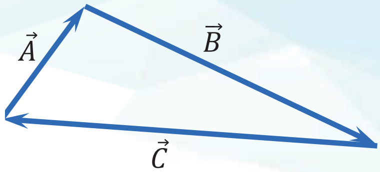


Figure 2.27

3. If you and your friend have to apply a force to move an object, what would be the best way to apply the forces to maximize your resultant force?
4. What is the least number of unequal vectors required to produce a zero resultant?
5. If you and your friend have to apply a force to move an object, what would be the best way to apply the forces to maximize your resultant force?
6. What is the least number of unequal vectors required to produce a zero resultant?
7. Suppose you are adding two vectors $\vec{P}$ *and* $\vec{Q}$, how would you add them to get the resultant with the least magnitude?
8. What will happen to the magnitude of the resultant of two vectors as the angle between them increases from 0° to 180°?
9. A vector $\vec{A}$ lies in the xy plane. For what orientations of $\vec{A}$ will both of its components be negative? For what orientations will its components have opposite signs?
10. Two vectors have unequal magnitudes. Can their sum be zero? Explain.
11. Given displacement vectors $\vec{S}_1=(-2\hat{i}+3\hat{j})\,m$, $\vec{S}_3=(3\hat{i}-2\hat{j})\,m$, $\vec{S}_1=(-2\hat{i}+3\hat{j})\,m$. Find the magnitude and direction of the resultant displacement $\vec{S}=\vec{S}_1+\vec{S}_2+\vec{S}_3$
12. Three force vectors $\vec{A}$, $\vec{B}$ *and* $\vec{C}$ are acting at a point. If $\vec{A}=4\hat{i}-3\hat{j}$, $\vec{B}=2\vec{i}+\vec{j}$, and $\vec{A}+\vec{B}-\vec{C}=0$. Find
    a) the magnitude and direction of vector $\vec{C}$ and
    b) a unit vector along $\vec{C}$.
13. Determine a velocity vector that has a magnitude of 5 m/s directed along the direction of vector $\vec{D}=1.5\hat{i}+2\hat{j}$.
14. The angle between vector $\vec{A}$ and vector $\vec{B}$ is 20°. If the magnitude of vector $\vec{A}$ is 6 units and that of $\vec{B}$ is 4 units, what is the component of vector along the direction of vector $\vec{B}$
15. The initial position vector of a particle on the xy plane is (5,2) and its final position vector is (−2,−3). If distance is measured in centimeter,
    a) what is the displacement of the particle?
    b) what is the magnitude and direction of the displacement?

16. A postal employee drives a delivery truck along the route shown in Figure 2.30. Determine the magnitude and direction of the resultant displacement by drawing a scale diagram.

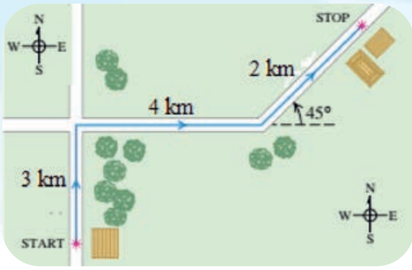


Figure 2.28 Path of the truck

17. A girl pushes a box placed on the floor with a force of 50 N at an angle of 37° below the horizontal. If the box moves toward the positive x direction, what are the vector components of the force?

18. The Tug-of-War Game

Four dogs nicknamed A, B, C, and D play a tug-of-war game with a toy, Figure 2.31. A pulls strongly with a force of magnitude A = 160 N. B pulls even stronger than A with a force of magnitude B = 200 N, and C pulls with a force of magnitude C = 140 N. When D pulls on the toy in such a way that his force balances out the resultant of the other three forces, the toy does not move in any direction. With how big a force and in what direction must D pull on the toy for this to happen? (Hint: If the toy does not move, the vector sum of the four forces, $\vec{F}_A + \vec{F}_B + \vec{F}_C + \vec{F}_D = 0$.

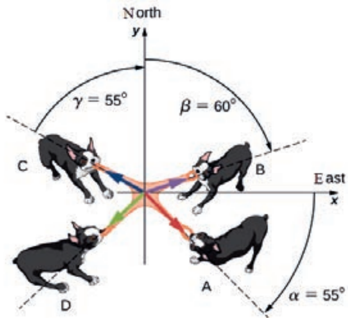


Figure 2.29 Dogs enjoying the tug-of-war game

19. Given two vectors $\vec{X}$ and $\vec{Y}$ such that $\vec{X} = 4\hat{i} + \hat{j}$ and $\vec{Y} = 6\hat{i} + 3\hat{j}$,

 (a) what is the angle between the two vectors?

 (b) Find the component of $\vec{X}$ along the direction of $\vec{Y}$?

20. What is the vector product $\vec{A}.\vec{B}$ of the vectors shown in Figure 2.30?

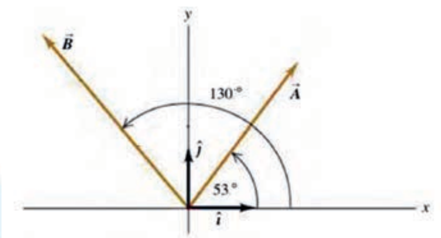


Figure 2.32 Vectors on the xy plane – x axis as a reference line for measuring angles

UNIT 3

MOTION IN ONE AND TWO DIMENSIONS

Introduction

Motion is one of the significant topics in physics. Everything in the universe moves. If you give a glimpse to your environment, you might see several things in motion: a car on a road, children playing on a playground, an airplane flying in the sky, even an ant crawling on the ground. An object is said to be in motion if its position changes with time.

The concept of motion is a relative. A body that may be in motion relative to one reference system, may be at rest relative to another. For example, if you are sitting in the classroom, you are at rest with respect to the earth; but actually, you are in motion with respect to the Sun.

In this unit we will learn about motion in one dimension relative to any fixed reference frame, and its applications in our day-to-day life. In addition to motion in 1D, we will also discuss uniform circular motion as an example of motion in 2D.

Learning outcomes: Students will be able to:

- *gain an understanding of the fundamental principles of kinematics in one and two dimensions.*
- *develop skills in applying equations of motions to solve practical problems.*
- *recognize the effect of air resistance and force of gravity on motion of a body.*
- *describe technological advances related to motion; and identify the effects of societal influences on transportation and safety issues.*

3.1 Uniformly Accelerated Motion in 1D

By the end of this section, you will be able to:

- *explain a uniformly accelerated motion in 1D;*
- *explain the difference between average velocity and instantaneous velocity; and*
- *solve problems involving average velocity, instantaneous velocity and acceleration.*

? Brainstorming

After revising what you have learnt in grades 9 and 10 about motion in a straight line and uniformly accelerated motion discuss about the following questions in group and present your group's answers to the class.

1. Define the terms position, distance, displacement, speed, and velocity in motion.
2. Explain the relationship between
   - position and displacement,
   - distance and displacement,
   - distance and speed,
   - displacement and velocity,
   - average velocity and instantaneous velocity.
3. When do we say an object is accelerating?
4. How will it be possible for a car to move but not accelerating?

Acceleration

An object is said to accelerate when its velocity changes in magnitude, in direction or both in magnitude and direction. Acceleration is a vector quantity, and is defined as the rate of change of velocity or change in velocity per unit time. The SI unit of acceleration is m/s^2.

Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. If the velocity and acceleration have the same sign whether positive or negative it is speeding up. If the velocity and acceleration have different signs, then the object is slowing down.

Activity 3.1:

Discuss in group and present your group's understanding of the following questions to the class.

1. Does negative acceleration necessarily mean slowing down?
2. Does positive acceleration necessarily mean speeding up?

Average acceleration

The average acceleration, $\vec{a}_{av}$ of a particle is defined as the ratio of the change in velocity $\triangle\vec{v} = \vec{v}_f - \vec{v}_i$, to the time interval $\triangle\vec{t} = \vec{t}_f - \vec{t}_i$.

$$a_{av} = \frac{\vec{v}_f - \vec{v}_t}{\Delta t}$$

where $\vec{v}_f$ is the final velocity, $\vec{v}_i$ is the initial velocity and t is the time interval. The SI unit of acceleration is $m/s^2 = ms^{-2}$

Key term

Acceleration is the rate of change of velocity. It is a vector quantity, with a unit of m/s^2.

Example: 3.1

If the speed of a certain car is increased from rest to 20m/s in 6 seconds, what is the average acceleration of the car?

Solution

We have $\mathbf{v}_0 = 0$, at and at $t_0 = 0$, and $\mathbf{v_f} = \mathbf{0}$ at $t_\mathbf{f} = \mathbf{6s}$

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0} = \frac{20m/s - 0}{6s - 0} = 3.67m/s^2$$

Instantaneous acceleration

Instantaneous acceleration, $\vec{a}_{ins}$ or acceleration at a specific instant in time, is obtained using the same process discussed for instantaneous velocity. That is, we calculate the average acceleration between two points in time separated by Δt and let Δt approach zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Example 3.2.

The velocity of a particle that moves along a straight line varies as a function of time with a velocity equation as: $v(t)=t^2-2t$. Find the acceleration of the particle at t = 2 s.

Solution:

To find the acceleration at t =2s, we first find the acceleration at any time t, that is

$$a(t)=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\lim_{\Delta t\to 0}\left(\frac{v(t+\Delta t)-v(t)}{\Delta t}\right)$$

$$\text{Sin}\,ce, v(t)=t^2-2t$$

$$\Rightarrow v(t+\Delta t)=(t+\Delta t)^2-2(t+\Delta t)$$

$$a(t)=\lim_{\Delta t\to 0}\left(\frac{v(t+\Delta t)-v(t)}{\Delta t}\right)$$

$$=\lim_{\Delta t\to 0}\left(\frac{(t+\Delta t)^2-2(t+\Delta t)-(t^2-2t)}{\Delta t}\right)$$

$$=\lim_{\Delta t\to 0}\left(\frac{t^2+2t\Delta t+(\Delta t)^2-2t-2\Delta t-t^2+2t)}{\Delta t}\right)$$

$$=\lim_{\Delta t\to 0}\left(\frac{2t\Delta t+(\Delta t)^2-2\Delta t)}{\Delta t}\right)$$

$$=\lim_{\Delta t\to 0}\left(2t+(\Delta t)-2)\right), Since\,\Delta t\approx 0$$

$$=\left(2t-2\right)$$

This expression is the acceleration of the particle at any time t. Substituting the given time t = 2s in this expression gives us the acceleration of the particle at t =2s. Therefore $a(t=2s)=2(2)-2=2m/s^2$

Motion with Constant Acceleration

Although there is a possibility to have varying acceleration (non-uniform acceleration) in reality, in many common types of one-dimensional motion, the acceleration is assumed to be constant or uniform. It means that, the velocity of the object is changing by equal amount in equal interval of time. Such a motion is known as a uniformly accelerated rectilinear motion. In this case, the average acceleration equals the instantaneous acceleration, and the average velocity is the average of the initial and final velocities.

$$v_{av}=\frac{v_i+v_f}{2}$$

Exercise 3.1:

A Cheetah, shown in the figure, is the fastest land animals. It could run up to 120 km/h. Starting from rest a Cheetah can reach to a speed of 96.6 km/h in only three seconds. What is its (a) average acceleration? and (b) average velocity in these 3 seconds?

Activity 3.2:

Experiment

In this experiment the acceleration of a ball/wooden block/trolley/tin can is determined by rolling or sliding it down a slight incline plane. To do so, the distances they covered in successive time intervals is measured.

Apparatus and Materials

- Ball/wooden block/trolley/tin can
- Ramp,
- meter stick.
- electronic stopwatch
- Ticker-timer with power supply unit
- Ticker-tape
- Sellotape

Ramp Construction

For the ramp you can use any flat, level object such as a board or a cardboard box. It should be at least 1.25 meters long. If you don't have anything like that then you will have to improvise. It doesn't matter what the ramp is made of as long as it is flat and strong enough to support your rolling/sliding object without bending.

To raise one end of the ramp, use a pile of books. You may not be able to get the top of the ramp at exactly the height given in the instructions. Get it as close as you can and record the actual height in data tables A and B.

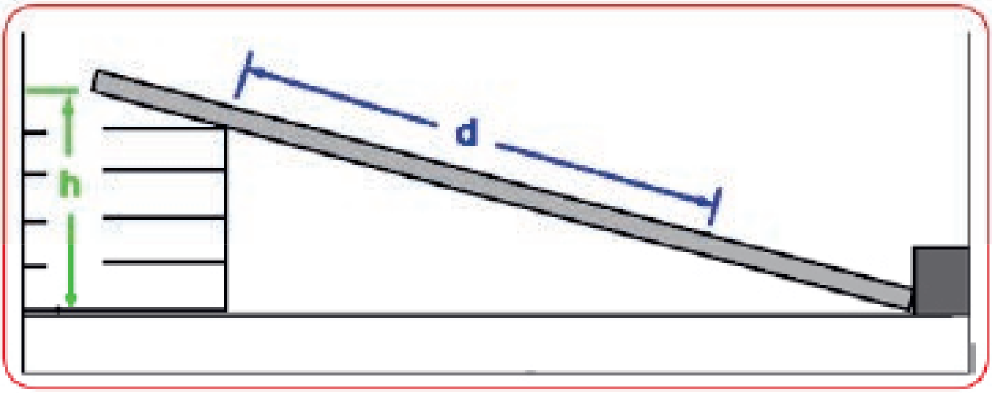


Figure 3.1

Objectives

1. Become familiar with the concept of hypothesis testing by experiment.
2. Observe and measure motion on an inclined plane.
3. Understand the relationships between distance, time, and acceleration.
4. Understand how to draw and interpret a "best fit" or regression line on a graph of experimental data.
5. Appreciate the difficulties in analyzing experimental data to make conclusions, even if the data is carefully and correctly collected.

Procedure

1. Measure distance, d along the ramp with arbitrary starting point.
2. Set up the ramp with h = 0.10 m above the table, (as shown in Figure 3.9)
3. Starting with the ball/wooden block/trolley/cylinder at rest, use the stopwatch to measure the time to roll distance d = 1.8 meter down the ramp.
4. Take 6, time measurements, record in data table. Cross out the highest and lowest times and determine the average of the remaining four times. (Sum four times and divide by four to find the average.)
5. Repeat steps 2 to 4 for distances of 1.6 m, 1.4 m, 1.2 m, and 1.0 m (see data table).
6. Using cell-phone record the video of the motion and use it for plotting the s vs. t, v vs. t and a vs. t graphs.

A countdown of "3, 2, 1" is a good way to begin the timing whether there is one person or two involved in the experiment.

To stop the timing, it is best to use a flat object such as a ruler or the cover of a book as a physical stop. This allows you to use your sense of hearing along with sight to coordinate the stopwatch with the stopping point.

Data Table

Make six trials and discard the highest and lowest in each trial so you will average the middle three values. Enter the average time in the data table. In each case use $t_{(avg)}$ for the times to calculate t squared.

d (m)	t_1	t_2	t_3	4	t_5	t_6	t_{av}	t_{av}^2	$a = \frac{2d}{t^2}$
1.8									
1.6									
1.4									
1.2									
1.0									

1. Draw a graph of distance versus average time squared for your data, (d vs. t2)
2. Determine the slope the best fit line.
3. The value of acceleration is twice the slope.
4. Analyze the data using spreadsheet (Microsoft excel).
5. Using distance 'd' and average time draw: distance vs. time, velocity vs. time and acceleration vs. time graph of the motion using spreadsheet (Microsoft excel).

N.B. Contact your IT teacher to get an assistance about how to use Microsoft excel in analyzing your data and draw graphs.

1. Compare the measured acceleration with the theoretical estimate ($a = g\sin\theta$), friction neglected by making the surface as smooth as possible.
2. Write a report on your findings.

Review Questions 3.1

1. Define the terms average and instantaneous acceleration.
2. State the conditions in which an object is said to speed up and slow down.
3. What is the relationship between acceleration and deceleration?
4. A car driver accelerates her car by 1.4 m/s^2, starting from rest. (a) How long does it take her to reach a speed of 2.0 m/s? (b) If she then brakes to a stop in 0.8 seconds, what is her deceleration?
5. A particle is in motion and is accelerating. The velocity of the motion as a function of time is given by: $\vec{v}(t) = 20t - 5t^2$ m/s. Find (a) the functional form of the acceleration, (b) the instantaneous velocity at t = 3s, and (c) the instantaneous acceleration at t = 3 s.

3.2 Equations of Uniformly Accelerated Motion in 1D

By the end of this section, you will be able to:

- *derive and apply the equations of uniformly accelerated motion in 1D.*
- *solve various problems of uniformly accelerated motion in 1d.*

Derivation of equations of motion for uniformly accelerated motion in a straight line.

The symbols used have the following meanings:

$$s = displacement\,,\ v_i = initial\ velocity\,,\ v_f = final\ velocity\,,$$

$$t = time\ for\ the\ acceleration\,,\ \text{and}\ a = acceleration$$

Directly from the definition of acceleration, we get

$$a = \frac{v_f - v_i}{t}\text{, which can be written as}$$

$$v_f = v_i + at \qquad \textbf{Equation 1}$$

From the definition of average velocity, we get

$$s = v_{av}t\,, \qquad \text{but for uniformly accelerated motion,}$$

$$v_{av} = \frac{v_i + v_f}{2}. \qquad \textbf{Equation 2}$$

Remember that average velocity and instantaneous velocities are different. Instantaneous velocity is the velocity at a particular instant of time, but average velocity is the velocity that represents a motion in a certain interval of time.

Substituting this equation of average velocity into the above equation of displacement gives

$$s = \left(\frac{v_i + v_f}{2}\right)t$$

Equation 3

By substituting for final velocity from equation 1 into equation 2 and rearranging gives

$$s = \left(\frac{v_i + v_f}{2}\right)t = \left(\frac{v_i + v_i + at}{2}\right)t = \left(\frac{2v_i + at}{2}\right)t$$

$$s = v_i t + \frac{1}{2}at^2$$

Equation 4

Instead of final velocity if we substitute for initial velocity into equation 2 and rearranging gives

$$s = v_f t - \frac{1}{2}at^2$$

Equation 5

Rearranging equation 1 for time (t) and substituting into equation 2 gives

$$s = \left(\frac{v_i + v_f}{2}\right)t = \left(\frac{v_i + v_f}{2}\right)\left(\frac{v_f - v_i}{a}\right) = \left(\frac{v_f^2 - v_i^2}{2a}\right)$$

$$v_f^2 = v_i^2 + 2as$$

Equation 6

Note that, the above 6 equations all refer to uniformly accelerated motion in a straight line. It means that, they do not apply if the acceleration is changing.

The above equations are important kinematics equations to solve any problem of motion with constant acceleration. They summarized in Table 3.1 below.

Table 3.1: Kinematics equations for uniformly accelerated motion

Equation	Missing quantity
$v_f = v_i + at$	Displacement (s)
$v_{av} = \frac{v_i + v_f}{2}$	Displacement (s), acceleration (a), and time (t)

$s = \left(\frac{v_i + v_f}{2}\right)t$	Acceleration (a)
$s = v_i t + \frac{1}{2}at^2$	Final velocity (v_f)
$s = v_f t - \frac{1}{2}at^2$	Initial velocity (v_i)
$v_f^2 = v_i^2 + 2as$	Time (t)

Note that the above equations are vector equations, it means that we need to take of the direction of the quantities to be added. If they are in the same direction we add, but if they point in opposite directions we subtract.

Example 3.3

A car accelerates uniformly from rest to a speed of 72 km/h in 8.0 s. (a) Find the acceleration of the car. (b) Find the distance the car travels. (c) If the car brakes to a stop over a distance of 50 m, then find its uniform deceleration.

Solution:

(a) In this problem we are given $v_i = \mathbf{0}, v_f = 72km/h = 20m/s$, and t = 8.0 s

Using the equation $a = \frac{v_f - v_i}{t} = \frac{20m/s - 0}{8s} = 2.5m/s^2$

(b) To find the distance that the car travelled, we can use the equation:

$$s = v_i t + \frac{1}{2}at^2$$

thus $s = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2.5)(8^2) = 80\,m$

The distance can also be found from the equation: $s = \left(\frac{v_i + v_f}{2}\right)t$

So that $s = \left(\frac{v_i + v_f}{2}\right)t = \left(\frac{0 + 20}{2}\right)8 = 80m$

(c) In this case the car breaks from 20m/s to a stop over a distance of 50 m, then its deceleration will be:

$$a = \frac{v_f^2 - v_i^2}{2s} = \frac{0 - (20)^2}{2(50)} = \frac{-400}{100} = -4m/s^2$$

This negative acceleration implies that the car's speed decreases from time to time.

Misconception Alert !!!

Note that all negative accelerations are not always decelerations. Negative acceleration can also occur due to the change in direction with respect to a certain fixed reference point.

Example: 3.4

The Ethiopian Airlines Boeing airplane lands on one of the Bole international airport runways with an initial velocity of 40.0 m/s and then decelerates at 1.50 m/s^2. (a) What length of runway will it need? (b) For how long will it move on the runway?

Solution:

Identify the known quantities: $v_i = 70m/s$, $a = -1.5m/s^2, v_f = 0$

$$s = \frac{v_f^2 - v_i^2}{2a} = \frac{0-(40)^2}{2(-1.5)} = \frac{-1600}{-3.0} = 533.3m$$

The time to stop is

$$t = \frac{v_f - v_i}{a} = \frac{0-40}{-1.5} = 26.67s$$

:

Determining the stopping distance of a vehicle

What is stopping distance?

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. Calculating stopping distance is important for avoiding potential hazardous situations. There are many factors that can affect the stopping distance of a moving vehicle. Here are some;

- Speed of the vehicle
- Weight of the vehicle
- Road conditions (slick, icy, snow, dry, wet)
- Vehicle brake conditions (old or worn pads and rotors)
- Braking technology in the vehicle
- Tire conditions

This leads us to the actual formula for working out braking distances. The formula is based on the velocity (speed) of the vehicle and the coefficient of friction between the wheels and the road (μ).

$$\text{Braking distance}\left(s_{break}\right) = \frac{v^2}{2a} = \frac{v^2}{2\mu g}$$

This is because, for a car coming to a stop due to friction: $a = \mu g$. Experiments have shown that for dry roads, the coefficient of friction is about 0.7, and for wet roads it is about 0.4. The stopping distance is then the sum of reaction distance and braking distance.

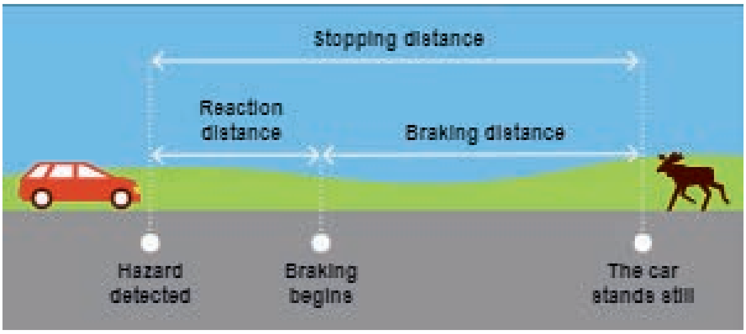


Figure 3.2: Stopping distance

The reaction distance is the distance you travel from the point of detecting a hazard until you begin braking or swerving. The time between being where we first start measuring, and where the car begins braking, is called the reaction time. As an equation, looking at the car's speed, v, the distance v it travels over a period of time v is:

The reaction distance = The car's speed $\times$ reaction time.

$$\text{Reaction distance}\left(s_r\right) = vt$$

Stopping distance = reaction distance + braking distance

$$\text{Stoppind distance} = vt + \frac{v^2}{2\mu g}$$

Example 3.2:

(a) Compare the stopping distance for the following speeds of an automobile. Assumes the road is dry and the co-efficient of friction between the tire and the road is 0.70. (b) Discuss the potential damage that could be caused by the automobile running with 80 km/h compared to that moving with 20 km/h.

Speedometer reading (km/h)	Speedometer reading (m/s)	**reaction distance** $s_r = vt$ $t = \text{your reaction time}$	**braking distance** $s_b = \frac{v^2}{2\mu g}$	**Stopping distance** $s_r + s_b$
80 km/h				
60 km/h				
30 km/h				
20 km/h				

Activity 3.3:

1. Discuss in group about the meanings of the traffic signs shown in the figure below and the consequences resulted if one is going to miss it.
2. Discuss about the main causes of car accidents in your area, and the ways to reduce the accidents. Your teacher can invite a guest from traffic offices.

Figure 3.3: Traffic sign

Review Questions 3.2

1. Derive the five kinematic equations.
2. It is found experimentally that, if a street surface is dry, a good driver can safely decelerate an automobile with reasonably good tires at the rate of about 4.57 m/s^2. So, for a car moving at 60 km/h what should be the minimum distance between two cars so that they won't crash during a sudden stop?
3. A driver of a vehicle traveling at a speed of 30 m/s on a motorway brake sharply to a standstill in a distance of 100m. Calculate the deceleration of the vehicle.
4. A motorist travelling at 18 m/s approaches traffic lights. When he is 30 m from the stop line, they turn red. It takes 0.7 s before he can react by applying the brakes. The car slows down at a rate of 4.6 m/s^2. How far from the stop line will he come to rest and on which side of it?
5. The driver of a train travelling at 40 m/s applies the brakes as the train enters a station. The train slows down at a rate of 2m/s. The platform is 400 m long. Will the train stop in time?
6. The distance covered in n seconds and the distance covered in the n^{th} second are different questions. Derive the equation for the distance covered in the n^{th} second.

3.3 Graphical Representation of Uniformly accelerated Motion in 1 D

By the end of this section, you will be able to:

- *graphically, represent position versus time graph, displacement time graph, velocity time graph, and acceleration time graph of a uniformly accelerated motion in 1d.*
- *interpret a graph of velocity vs. time, displacement vs. time and acceleration vs. time.*
- *draw graphs from the kinematic equations.*
- *determine the velocity of a motion from the slope of displacement-time graph, and acceleration from the slope of velocity – time graph.*
- *determine the displacement covered from the area under velocity – time graph.*

In this section, we will discuss about graphs that help us picture the concepts of displacement, velocity and acceleration.

3.3.1 Displacement-Time Graphs

? Brainstorming

What are dependent and independent variables in drawing graphs, and what is the relationship between them?

Displacement-Time Graph

A displacement-time graph shows how displacement changes with time. In drawing displacement-time graph, we record displacement and time for the motion, and the displacement (the dependent variable) is plotted on the Y-axis, and time (the independent variable) is plotted on the X-axis.

The slope of displacement-time graph

Slope is a measure of the steepness of a line. It is given by the equation:

$$Slope = \frac{rise}{run} = \frac{Vertical\ increase}{Horizontal\ increase} = \frac{x_2 - x_1}{t_2 - t_1}$$

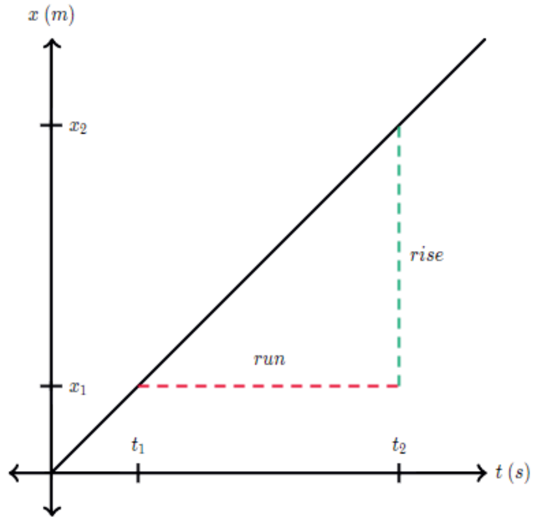


Figure 3.4: Slope of Position-time graph

This expression for slope is the same as the definition of velocity:

$$velocity = \frac{\Delta x}{\Delta t}$$

So, the slope of a position-time graph equals the velocity of the motion.

a. Displacement-time graph for uniform motion

For a body moving at constant velocity (uniform motion), its graph of displacement against time is a straight line with a constant slope/gradient, Figure 3.5 (b). This is because distance is directly proportional to time ($s = vt$).

b. Displacement-time graph for uniformly accelerated motion

For a body moving at constant acceleration, its graph of displacement against time is a parabola (curved upward), Figure 3.5 (c). This is because distance is directly proportional to the square of time ($s = \frac{1}{2}at^2$). In this case slope is increasing (sloping up or getting steeper), hence velocity is increasing resulting in a positive acceleration.

Displacement-time graph for uniformly decelerated motion

Figure 3.5 (d), shows the case where the velocity of an object is decreasing, slope is decreasing, resulting to a deceleration.

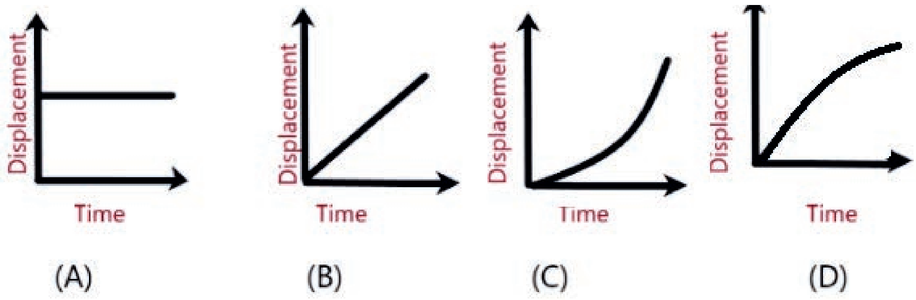


Figure 3.5: Some common displacement-time graphs

Interpreting a displacement-time graphs

In interpreting or describing the motion of an object, we decide whether the object is moving at a constant speed, accelerating or decelerating or is the object stationary (at rest). To understand how to interpret these graphs, we need to know something about the slope of a line. The slope of a line on a graph tells you how quickly the quantity being graphed is increasing or decreasing. For this purpose, we use the following properties of the slope of a dispacement time-graph.

- A flat line (slope equals zero), means that the object is at rest.
- An upwards straight line means that the object is moving forwards at a constant speed.
- For an upward curved line, for each second that ticks on the horizontal time axis, the distance travelled is greater than the second before. In other words, distance is increasing at an increasing rate. We say that the object is accelerating.
- If the line on the distance-time graph curves the other way (when the slope becomes flatter), then the object will be decelerating, as each second the distance travelled is less than the previous second.

So, the displacement vs. time graphs of Figure 3.5 are interpreted as follows.

a. shows the graph for an object stationary over a period of time. The slope is zero, so the object has zero velocity.

b. shows the graph for an object moving at a constant velocity. You can see that the displacement is increasing as time goes on. The slope, however, stays constant (remember: it's the slope of a straight line), so the velocity is constant. Here the slope is positive, so the object is moving in the direction we have defined as positive.

c. shows the graph for an object moving at a constant acceleration. You can see that the slope of the curve increases with time (becomes more vertical); thus, the velocity is increasing with time and the object is accelerating.

d. shows the graph for an object moving at a constant deceleration. You can see that the slope of the curve decreases with time (becomes more horizontal); thus, the velocity is decreases with time and the object is decelerating.

Example 3.5:

Draw the displacement-time graph of a ball thrown directly upwards and caught when it returns.

Solution:

- Immediately after leaving the throwers hand, the velocity is positive and large so the slope is positive and large.
- As the ball rises, its velocity decreases so the slope decreases.
- At the maximum height, its velocity is zero so the slope is zero.
- As the ball descends, its velocity becomes increasingly negative, corresponding to increasing speed in a downward direction. So, the slope becomes increasingly negative.

The slope of the line represents the object's acceleration. This is constant and negative, equal to the acceleration of free fall, g. The acceleration of the object is the same when it descends as when it ascends so the slope of the line is always equal to $-9.8m/s^2$.

The area under the line represents the displacement of the object from its starting position.

- The area between the positive section of the line and the time axis represents the displacement during the ascend.
- The area under the negative section of the line and the time axis represents the displacement during the descent.

Summing up theses two areas gives zero. This corresponds to zero total displacement, because it returns back to the thrown point.

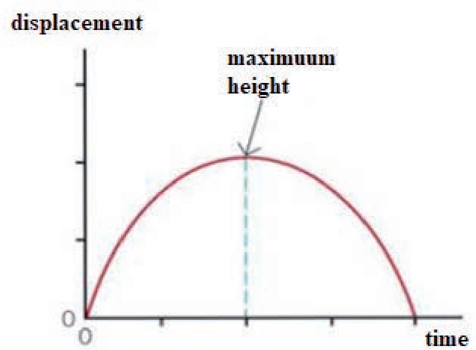


Figure 3.6: Displacement-time graph of an object thrown vertically upward

Exercise 3.2:

For the motion of a ball thrown vertically upward whose displacement-time graph is shown in Figure 3.13, draw its distance time graph.

Velocity-Time Graph

A velocity-time graph shows how velocity changes with time. In drawing velocity-time graph, we record velocity and time for the motion, and the velocity (the dependent variable) is plotted on the Y-axis, and time (the independent variable) is plotted on the X-axis. That is vertical axis represents the velocity of the object, and the horizontal axis represents the time taken. Figure 3.7, shows the velocity-time graph of a motion with a constant acceleration, i.e., the velocity increases by equal amount in equal interval of time.

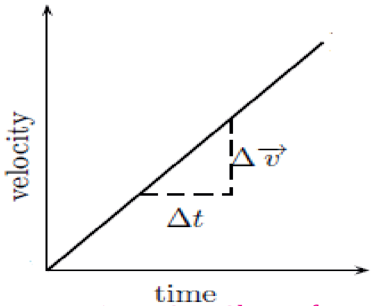


Figure 3.7: Slope of velocity-time graph

$$slope = \frac{Vertical\ increase}{Horizontal\ increase} = \frac{\Delta v}{\Delta t}$$

This is just the expression for acceleration. So, the slope of a velocity graph represents the acceleration of the motion. Because the slope is the same at all points on this graph, the acceleration is constant.

We can also get the displacement travelled from the velocity vs. time graph by finding the area of the region between the line and the time axis.

Note:

- The slope of a velocity-time graph gives the acceleration.
- The area between a velocity-time graph and the `time' axis gives the displacement of the object.

The velocity-time graph for (a) uniform motion, (b) uniformly accelerated motion, and (c) uniformly decelerated motion is shown in the Figure 3.8, below.

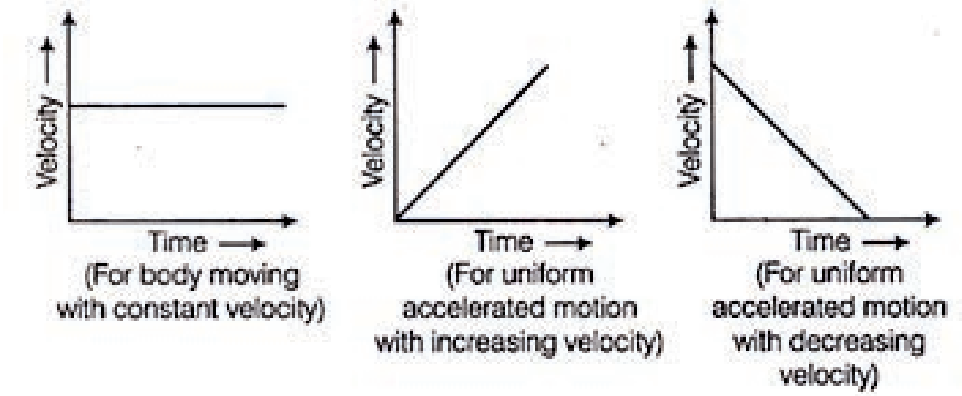


Figure 3.8: Some common velocity-time graphs:

Key terms

- *Slope of displacement-time graph give average velocity.*
- *Slope of velocity-time graph give the acceleration of the motion.*
- *The area under the velocity-time curve and the time axis gives the magnitude of the displacement covered.*

Exercise 3.4

By looking a velocity vs. time graph, you can tell if an object is accelerating, decelerating, or moving at a constant velocity. Forming a group with 5 members make a discussion and interpret the above three velocity vs. time graphs.

Example 3.6

Draw the velocity-time graph of a ball thrown directly upwards and caught when it returns.

Solution:

The velocity of the ball decreases from its initial positive (upward) value to zero at a maximum height then increases in the negative (downwards) direction as it falls. Figure 3.9 shows how the velocity of the ball changes with time.

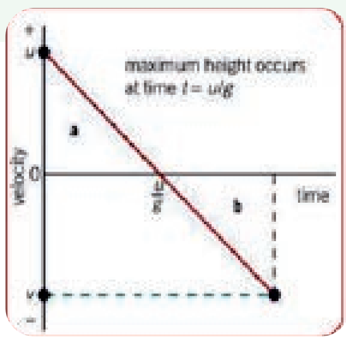


Figure 3.9 Velocity-time graph of a ball thrown vertically up and came back to the throwers hand

Exercise 3.5

There is a difference between displacement vs. time graph and distance vs. time graph of a motion, and also between velocity vs. time and speed vs. time graph of a motion. Draw (a) distance vs. time, and (b) speed vs. time graph of a ball thrown vertically upward and returns back to the throwers hand.

Acceleration-Time Graph

Acceleration-time graph shows how acceleration changes with time. In drawing acceleration-time graph, we record acceleration and time for the motion, and the acceleration (the dependent variable) is plotted on the Y-axis, and time (the independent variable) is plotted on the X-axis. In this section, we will only deal with objects moving at a constant acceleration, thus all acceleration-time graphs are parallel to the time axis. Figure 3.10 below shows acceleration vs. time graph of a uniform motion (a), a uniformly accelerated motion (b), and a uniformly decelerated motion (c).

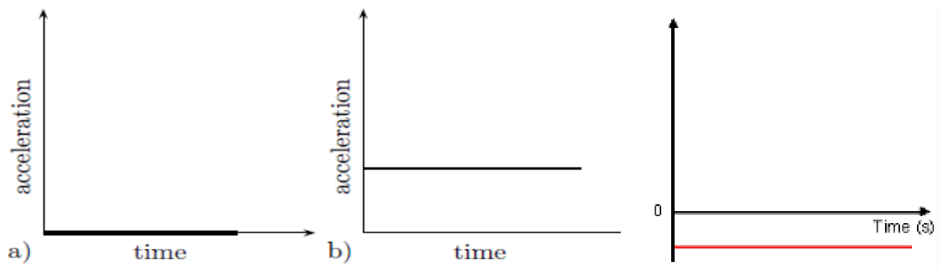


Figure 3.10 acceleration-time graph of (a) uniform motion, (b) uniformly accelerated motion, (c) uniformly decelerated motion.

Example 3.7:

A car starts from rest and accelerates at a rate of 2.0m/s^2 for 5seconds, then maintains its speed for 4 seconds. It then applied the brakes and stopped within 2 seconds. Then it moves at a constant speed of 15m/s in the opposite direction until it reaches the starting point.

Determine the position of the car for each interval of time.

a) Draw the displacement versus time, velocity versus time and acceleration b) versus time graph of the motion of the car.

c) From the velocity time graph, show that the displacement is zero.

Solution:

a) The car has different motions in the four intervals of time.

In the first interval of time between t = 0 and t = 5s it accelerates from rest at a rate of 2.0m/s2. The displacement s in this interval will then be: $s_1 = ut + \frac{1}{2}\mathbf{a}t^2 = 0 + \frac{1}{2}(2m/s^2)(5s)^2 = \mathbf{25m}$.

In the second interval of time, it maintains its speed for 4 seconds (i.e. between t = 5 s and t = 9s). The constant speed it attained is the final speed at t = 5s. That speed is

$$v_f = v_i + \mathbf{a}t = 0 + (2.0m/s^2)5s = \mathbf{10m/s}$$

The displacement in the second interval will then be

$$s_2 = \mathbf{v}t = (10m/s)(4s) = \mathbf{40m}$$

Since $s = \mathbf{x_2} - \mathbf{x_1}$ then the position of the car x^2 at the end of the second interval will be

$$x = S_2 + S_1 = 40m + 25m = 65m$$

In the third interval of time, the car applied its brakes and reduces its speed at a constant rate for 2 seconds until its velocity becomes zero from 10m/s. The acceleration during this time interval is

$$\mathbf{a} = \frac{\mathbf{v_f} - \mathbf{v_0}}{t} = \frac{0 - 10m/s}{2s} = \mathbf{-5.0m/s^2}.$$

The displacement during this time interval will therefore be

$$s_3 = \frac{\mathbf{v_f^2} - \mathbf{v_0^2}}{\mathbf{2a}} = \frac{0 - (10m/s)^2}{2(-5m/s^2)} = \frac{-100m^2/s^2}{-10m/s^2} = \mathbf{10m}$$

The position of the car at the end of the third interval will be

$$\mathbf{x_3} = \mathbf{x_2} + s_3 = 65m + 10m = \mathbf{75m}$$

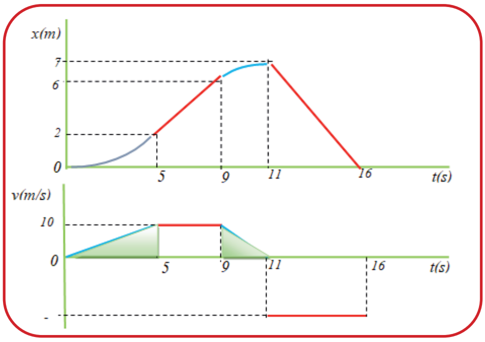
In the fourth interval of time, the car moves at a constant speed of 15m/s in opposite direction until it reaches $x_f = 0$ to the place where it started the motion. (i.e. v = -15m/s since it is moving in the opposite direction.)

The final position xf = 0 and the time it takes to the initial position can be obtained from the equation,

$$t = \frac{\Delta x}{v} = \frac{x_f - x_3}{v} = \frac{0 - 75/}{15m/s} = 5s$$

b) The displacement vs time graph, and the corresponding velocity vs. time graph is shown below.

The displacement-time graph will have a parabolic shape in the first interval (speeding up); straight line with a positive slope in the second interval (constant speed); parabola inverted downward in the third interval which indicates the decreasing of speed in magnitude (slowing down); and in the last interval, it is a straight line with a negative slope that indicates the reverse in direction.



One of the three features of the velocity versus time graph is the area under the curve is the displacement. From the lower part of the graph in Figure 3.18, the sum of the area of the regions sum up to give us the displacement.

The area of the triangle in region 1 is: $Area_1 = \frac{1}{2}(10m/s)(5s) = 25m$

The area of the triangle in region 2 is: $Area_2 = (10m/s) \times (4s) = 40\,m$

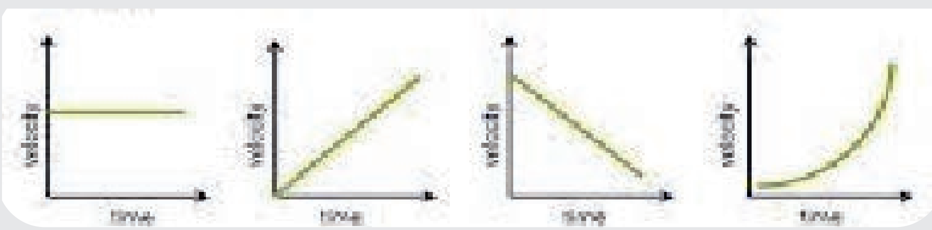
The area of the triangle in region 3 is: $Area_3 = \frac{1}{2}(10m/s)(2s) = 10m$

The area of the triangle in region 4 is: $Area_1 = -(15m/s)(5s) = -75m$

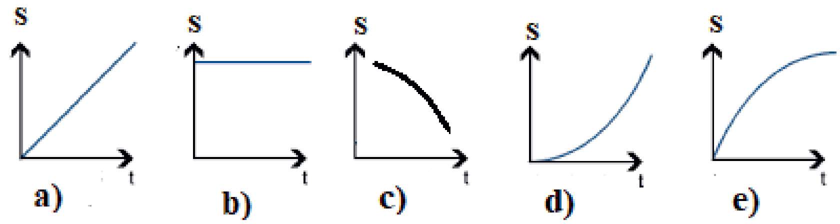
The total area is: $Area_1 + Area_2 + Area_3 + Area_4 = 25m + 40m + 10m - 75m = 0$
This value of the displacement coincides with the value clearly seen in the displacement vs. time graph.

Review Questions 3.4

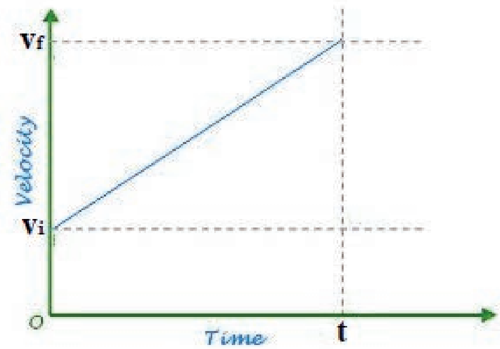
1. Interpret the following the following velocity-time graphs.



2. Interpret the following displacement (s) vs. time graphs.



3. By finding the area of the region between the velocity-time curve and the time axis of the velocity vs, time graph shown below, derive the equation for displacement: $s = \left(\frac{v_i + v_f}{2}\right)t$.



4. Starting from rest a car moves along a straight level road with uniformly increasing velocity for 20 minutes, until its velocity becomes 15 m/s, north. Then it maintained this velocity for half an hour. In the next 10 minutes, its velocity uniformly decreases until it becomes zero. Then it continuous moving in the opposite direction for 20 minutes until its velocity is equal to 20 m/s, south. (a) Draw the velocity vs. time, and displacement vs. time graph of the motion. (b) determine the distance and the magnitude of the displacement covered during the motion.

Activity 3.4:

Phet simulation for motion graphs

Learning Objectives

Examine velocity-time graphs and compare them to position-time graphs.

In addition, compare the slopes of the position-time and velocity-time graphs, as well as the area to the time axis of the velocity-time and acceleration-time graphs.

Procedure:

Use the following links to run the simulation. https://phet.colorado.edu/en/simulation/legacy/moving-man for more information.

Then navigate to https://phet.colorado.edu/en/legacy/moving-man.

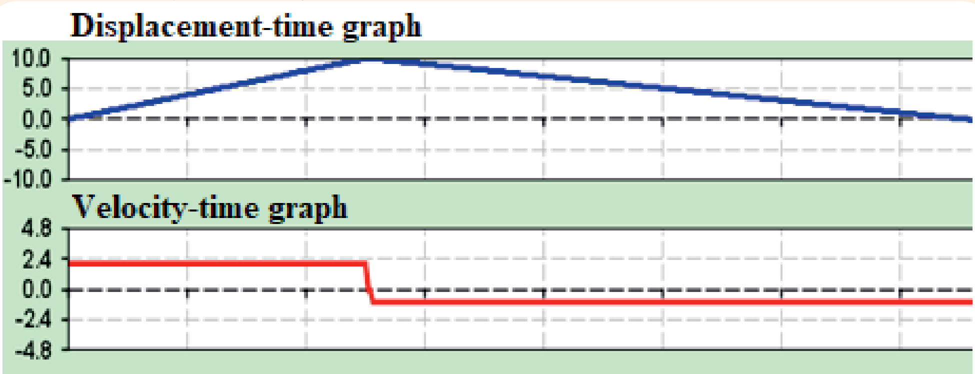
Beginning at rest, a man walks eastward over a straight level road at a constant velocity of 2 m/s for 10 meters. The object then goes continually in the opposite direction at a constant velocity of 1 m/s, which is apparent in the simulation graph below. In the simulation, the motion is represented by a displacement vs. time graph and a velocity vs. time graph. The total required time is 15 seconds.

a. Compute the distance covered in each motion using the formula the area of the region in the graph and compare them.

b. Determine the time required to go east, $t_1 = \frac{s_1}{v_1}$, the time necessary to go west, $t_2 = \frac{s_2}{v_2}$, and compute the total duration?

c. Why is the acceleration 0 in the motion?

d. What are your thoughts on the computed and simulated st values?



3.4 Vertical Motion

By the end of this section, you will be able to:

- *describe what vertical motion is.*
- *solve various problems of vertical motion*
- *describe the motion of objects that are in free fall.*
- *calculate the time, position and velocity of an object given the initial velocity of an object under vertical motion*
- *solve problems related to an object thrown vertically upwards, downwards, and a freely falling bodies.*

Freely Falling Bodies

When things move vertically either up or down under the force of gravity alone, the resulting motion is called **free fall**. The acceleration of a freely falling body is constant and is known as **acceleration due to gravity**, denoted by the letter g. When air resistance is negligible, all objects dropped under the influence of gravity near Earth's surface fall toward Earth with this acceleration. Near the surface of the Earth its magnitude is constant and is equal to 9.8 m/s^2. Therefore, for free fall near the surface of the earth the equations of motion with constant acceleration discussed so far can be applied. The only change in those equations is the replacement of *a* by g, and taking care of directions.

The expression freely falling object doesn't necessarily refer to an object dropped from rest. A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all considered freely falling

1. $\vec{v} = \vec{u} + \vec{g}t$
2. $\vec{y} = \vec{u}t + \frac{1}{2}\vec{g}t^2$
3. $v^2 = u^2 + 2gy$

Note that these equations are vector equations, meaning that we need to take care of the directions of the vector quantities such as velocities, displacements and acceleration.

One way to investigate the free fall of an object is to make a multiflash photo of the object's flight as it falls after being released from rest (Figure 3.11). As seen in the figure the displacement the ball covered in equal intervals of time is increased from time to time uniformly. That means the velocity of the falling ball is increased uniformly through time and hence it is accelerating.

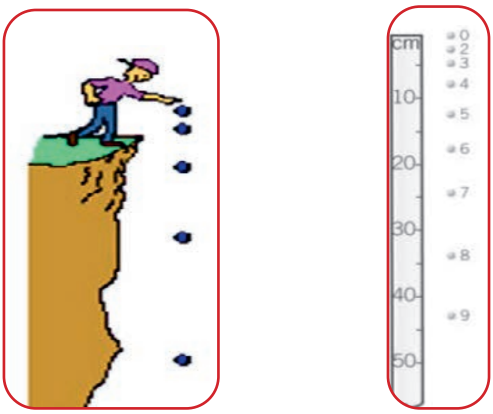


Figure 3.11: Motion of freely falling bodies

For an object thrown vertically upward the initial velocity, $\vec{u}$ is directed up and is taken as positive, but because $\vec{g}$ is always directed down towards the center of the

Earth it is taken as negative. Thus, for an object thrown vertically upward, the above equation may be written in scalar form as follows:

$$v = u - gt \qquad y = ut - \frac{1}{2}gt^2 \qquad v^2 = u^2 - 2gy$$

Here y is the height above the origin at time t, u is the initial velocity in the y-direction at t = 0.

For an object thrown vertically downward, since both $\vec{u}$ and $\vec{g}$ have the same downward directions, the above equations become:

$$v = u + gt \qquad y = ut + \frac{1}{2}gt^2 \qquad v^2 = u^2 + 2gy$$

Example 3.8:

Consider a particle projected vertically upwards with an initial velocity v_0 =20m/s from the top of a 15m building as shown in the figure below. How long does it take for the particle to reach to: A) level A , the initial point, B) level B 5m above the initial point, C) level C, 5m below the initial point and D) level D, the building bottom?

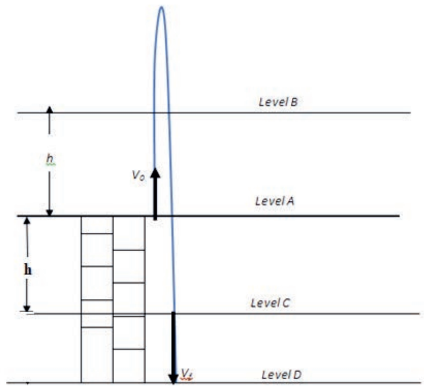


Figure 3.12:

Motion diagram of a particle thrown vertically upward from the top of a building at level A passes through level B reaches to maximum height (h_{ma}), and finally hits the ground at level C.

To find the time of flight at different levels: A, B and C, take the value of the y-coordinate of $y = 0,\ a = -g$

At level A: The kinematic equation for *y* then becomes,

$$0 = 0 + 20t - \frac{1}{2}(10)t^2 \Rightarrow 20t - 5t^2 = 0$$

$$\Rightarrow t = 0, \quad or \quad t = 4s\cdot$$

These two solutions of the time t indicate that the particle appears on the same place at two different instants of time. The former one is the initial condition at the instant the particle was projected. The later one is the one which is the time it takes to cover the entire path to return to its original position.

At Level B: y = +5m $a = -g = -10 m/s^2$

The kinematic equation for t by replacing y= +5m then becomes

$5 = 20t - \frac{1}{2}(10)t^2$ which is a quadratic equation of the form: $5t^2 - 20t + 5 = 0$.

Solving this equation with the general quadratic formula, we get two roots for t. (What are the physical implications of the two roots for t in the solution?)

At Level C: y = - 5m, $a = -g = -10 m/s^2$

The kinematic equation for t by replacing y = -5m then becomes:

$$-5 = 20t - \frac{1}{2}(10)t^2$$

Which is a quadratic equation of the form $5t^2 - 20t - 5 = 0$. The time solution can be obtained from the general quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{(-20)^2 - 4(5)(-5)}}{2(5)}$$

$$t = \frac{-5 + 22.36}{2(5)} = 1.73\,\text{sec}\ or\ t = \frac{-5 - 22.36}{2(5)} = -2.73\,\text{sec}$$

Since, time is a scalar quantity, it can never be negative and the answer will be 1.73 seconds.

Exercise 3.6:

A ball is thrown vertically upwards with a velocity of 10m/s from the balcony of a tall building. The balcony is 15m above the ground and gravitational acceleration is $10 m/s^2$. Find (a) the time required for the ball to hit the ground, and (b) the velocity with which it hits the ground.

Activity 3.5:

Experiment

Measurement of acceleration due to gravity (g) using the freefall meth-od

Objectives

The objectives of this experiment are as follows:

1. To know the meaning of free fall and the conditions under which it occurs.
2. To know the value for the acceleration of gravity (g).
3. Corelate the motion of a free-falling object to the graphical descriptions of its motion.
4. To understand that the acceleration of gravity is independent of the mass of the free-falling object.
5. To test the hypothesis that the acceleration of a freely falling object is uniform,

Apparatus: Millisecond timer, power supply, metal ball, trapdoor and electromagnet.

Diagram

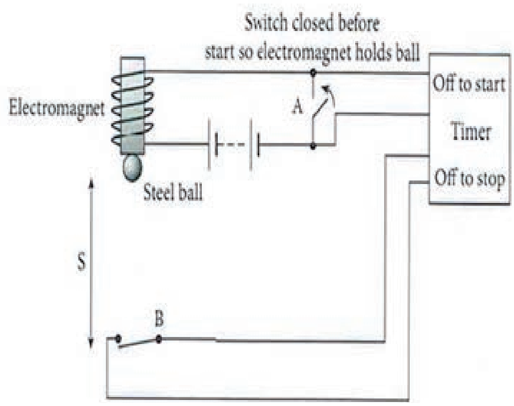


Figure 3.12: Laboratory set up to measure

Procedure

1. Set up the apparatus as shown in Figure 3.21. The millisecond timer starts when the ball is released and stops when the ball hits the trapdoor at B.
2. When switch A is turned off the steel ball starts to fall and the timer starts. The ball falls onto switch B and when it breaks the circuit the timer stops.
3. Measure the distance s using a meter stick.

4. Flick the switch to release the ball and record the time t from the millisecond timer.
5. Repeat for different values of s.
6. Calculate the values for g using the equation $g = \frac{2h}{t^2}$. Obtain an average value for g.
7. Draw a graph of h against t^2 and use the slope to find a value for g (g will be twice the slope).

Precautions

1. For each height h repeat three times and take the smallest time as the correct value for t.
2. Place a piece of paper between the ball bearing and the electromagnet to ensure a quick release.
3. Remember to convert from milliseconds to seconds.

Note

Both points 1 and 2 above are associated with the problem that even though you switch off the power for the electromagnet (and in so doing switch on the timer) it will not lose its magnetism immediately, therefore the ball will not fall straight away. This means that the reading on the timer will always be (slightly) longer than the time for which the ball was dropping.

Purpose

The purpose of this laboratory activity is to measure the acceleration of a falling object assuming that the only force acting on the object is the gravitational force.

Data

h (m)									
t (s)									
t^2 (s^2)									
$g = \frac{2h}{t^2}$									
g_{avg}									

Conclusion: Report your findings.

Activity 3.6:

Drop two balls of different mass (basketball, and tennis ball) from the same height. Which will hit the ground first?

Terminal velocity

When a solid body is inside a liquid or gas, its entire surface is in contact with the particles of the fluid. For this reason, during its motion in a fluid, the body experiences a resistive force. These resistance force which the fluid exerted on the body is called **fluid friction or drag force**.

Thus, a body falling in air moves under the action of two forces (ignoring buoyancy): the downward gravitational force (its weight) and the upward drag force.

At the beginning of the fall the downward gravitational force would be larger than the drag force ($mg > F_d$) and hence it will make an accelerated downward motion, which resulted to an increase in velocity. As the velocity the falling object increases, so does the drag force. At some velocity, the drag force of resistance will be equal to the gravitational pull on the object. At this point the object stops accelerating and continues falling at a constant velocity called the **terminal velocity**. Thus, terminal velocity is defined as the point at which the velocity of a falling object is no longer getting greater. An object at terminal velocity has zero acceleration.

Besides weight, terminal velocity depends on other factors such as shape and cross sectional area of the falling object and the nature of the fluid.

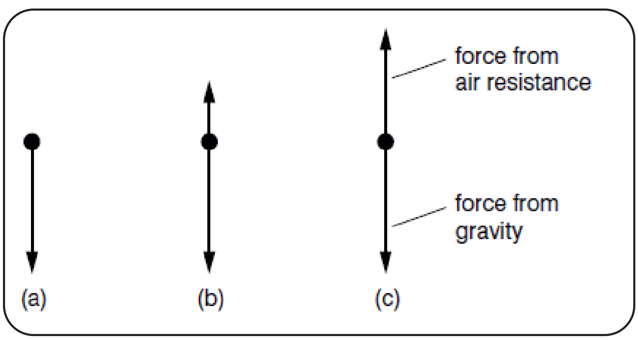


Figure 3.13: A free body diagram showing the forces on a particle at different times during its fall: (a) at the start of the fall; (b) during the fall; (c) at the end of the fall.

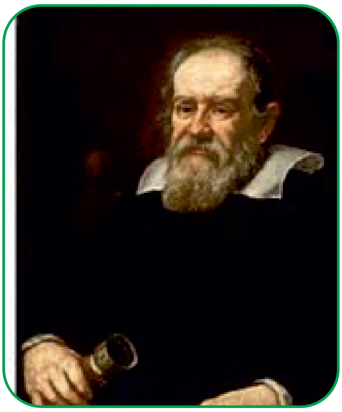
Key term

Terminal velocity is the maximum velocity of a freely falling body where its velocity no more increases, or the acceleration of the motion is zero.

Activity 3.7

Assume a drop of rain is falling from a height of 450m. If the drag force were not there, what would be it velocity on reaching the surface of the Earth. Discuss the effect of this velocity on living things on Earth.

Galileo Galilei and the Motion of Objects



In contrary to Aristotle (384–322 b.c.) view ofmotion, which states that heavier objects fell faster than lighter ones, Galileo Galilei (1564–1642) Italian Physicist and Astronomer, postulated that, regardless of their size and mass all bodies falling through empty space (vacuum) have the same acceleration. He was the first person to determine the correct mathematical law for acceleration: the total distance covered, starting from rest, is proportional to the square of the time.

Galileo also investigated the motion of an object on an inclined plane, established the concept of relative motion, invented the thermometer, and discovered that the motion of a swinging pendulum could be used to measure time intervals. After designing and constructing his own telescope, he discovered four of Jupiter's moons, found that our own Moon's surface is rough, discovered sunspots and the phases of Venus, and showed that the Milky Way consists of an enormous number of stars. He is also known for the assertion that the Sun is at the center of the Universe (the heliocentric system), not Earth.

Activity 3.8: Experiment

Calculating Reaction time

Reaction time is the time a person takes to observe, think and act. Reaction time is the time it takes you to react to a hazard. For example, if a person is driving and suddenly a boy appears on the road, then the time elapsed before he/she applies the brakes of the car is the reaction time. Studies reflect that a person's reaction time is anywhere from one to three seconds. You can measure your reaction time by a simple experiment.

Materials required

You need two people (one dropper and one catcher), plus a meter ruler.

Procedure

The dropper holds the ruler, letting it dangle vertically. The catcher's hand should be about chest high, with the ruler passing between the thumb and fingers as shown in Figure 3.14. The thumb should be horizontal, so that the top of the thumb can be used as a reference. To perform it, have your friend hold the ruler at the very top and you hold your fingers at the zero mark. Your friend then drops it, and you then catch it as fast as you can, as shown below.

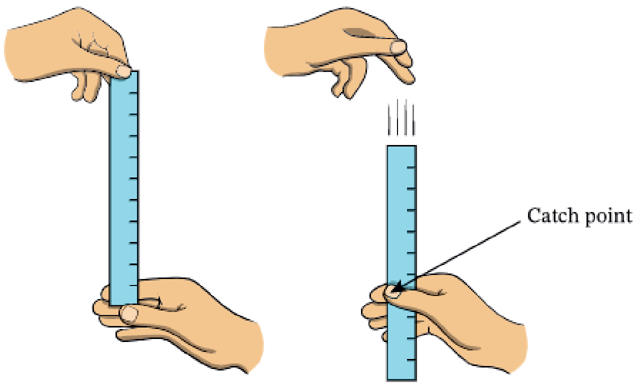


Figure 3.14: Measuring reaction time

The difference in distance between where your fingers initially were, at the zero mark, and where you catch it, the catch point, gives you the total distance traveled. Then, calculate the reaction time using the equation:

$$t = \sqrt{\frac{2d}{g}}$$

Reaction distance is the distance you travel during your reaction time. It depends on the reaction time and speed. It is calculated as:

Reaction Distance = Reaction Time x Speed

$$s = vt.$$

Key term

Reaction time is the time taken for a person to react to a sudden event.

Activity 3.9: Project work

Make a group containing 4 to 5 members in your class. Search internet or books and came up comparing the difference between Aristotle's, Galileo's and Newtons view of motion.

Review Questions 3.4

1. Describe the motion of a freely falling bodies.
2. A ball is thrown vertically upward with a velocity of 20 m/s from the top of a building. The height of the point from where the ball is thrown is 25 m from the ground. (a) how high will the ball rise? (b) how long will it take for the ball to hit the ground?
3. A ball is dropped on the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 sec, what is the average acceleration during contact?
4. If a stone is thrown vertically downward into a well with a speed of 10 m/s, how long will it take it to reach the water surface 60 m below?

3.5 Uniform Circular Motion

By the end of this section, you will be able to:

- *define a uniform circular motion*
- *calculate for angular and tangential displacement, angular and tangential velocity, and centripetal acceleration,*
- *explain uniform circular motion in the horizontal plane,*
- *identify the force required for circular motion to take place*

? Brainstorming Questions

1. When do we say an object is accelerating?
2. Can an object accelerate if it's moving with constant speed?
3. Mention an example in which an object accelerates due to a change in the direction of velocity without a change in its magnitude.

Uniform Circular Motion

Uniform circular motion is a special type of curvilinear motion in which an object travels in a circle with a constant speed but its direction of motion continuously changes. A mass attached to the end of a string and moves in a horizontal circle, the second, minute, and hour hands of a watch, the motion of a turntable, and any point on a propeller spinning at a constant rate are examples that could demonstrate uniform circular motion.

Angular and tangential displacement

While moving in a circular path, the angle made by the radius 'r' of the circle is known as **angular displacement** (θ)**.** It is a vector quantity, hence have a direction. The direction of the circular arrow pointing from the initial position to the final position will indicate the direction of angular displacement. It can be either clockwise or anticlockwise.

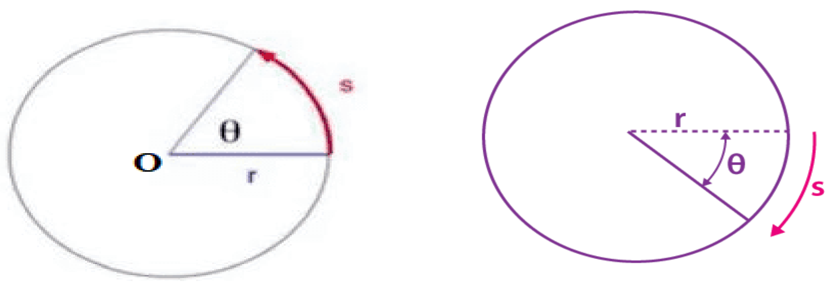


Figure 3.15: Angular displacement

If the arc of the circle in which the body is moving is 's' and the radius of the circle is 'r' then, angular displacement $\theta = \frac{s}{r}$.

After one complete revolution, the rotating object will travel a distance equal to the circumference of the circular path. Which is equal to 2ϖr where r is the radius.

Therefore,

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \, radian$$

Thus, 1rev = 2ϖrad = 360^0.

The unit of angular displacement in the SI system is radian. Since angular displacement is the ratio of these two quantities hence it is dimensionless.

On the other hand, tangential displacement (s) is the distance covered by the rotating object along the curved path (circumference of a circle), see Figure 3.15

Key terms

- *Uniform circular motion is a circular motion with constant speed but continuously changing direction.*
- *Angular displacement is the angle swept by the rotating object.*
- *Tangential displacement the tangential distance on the circumference of the circle.*

Angular velocity and tangential velocity

Angular velocity measures the rate at which angular displacement changes. It is denoted by Greek letter omega (ω) and its standard unit is radians per second.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Angular velocity is a vector quantity. The direction of angular velocity is determined by right hand rule. According to right hand rule, if you hold the axis with your right hand and rotate the fingers in the direction of motion of the rotating body, then thumb will point the direction of the angular velocity.

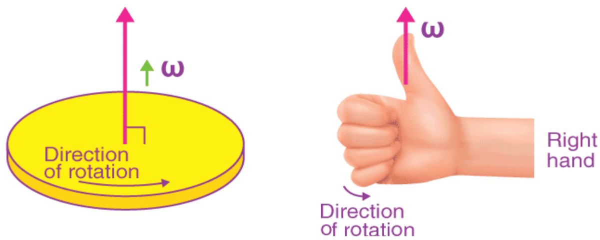


Figure 3.16: RHR for the direction of angular velocity

A tangent is a line that touches a curve at a single point. As the name suggests, tangential velocity describes the motion of an object along the edge of the circle whose direction at any given point on the circle is always along the tangent to that point. It is the linear component of angular velocity at any point along the circular path.

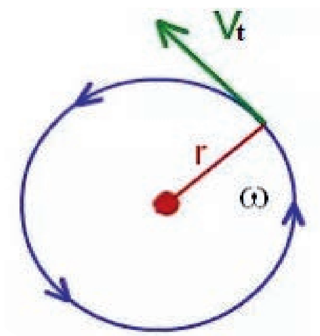


Figure 3.17: Tangential velocity

Tangential velocity is equal to the tangential distance divided by time.

$$v_t = \frac{s}{t}$$

After one complete rotation, the tangential displacement is equal to the circumference of the circle, and time taken for one complete circle is called the period, T. With this the equation for tangential velocity will be:

$$v_t = \frac{2\pi r}{T}$$

The SI unit of tangential velocity is meters per second.

Key term

Centripetal acceleration is an acceleration in circular motion caused by the change in the direction of velocity. Its direction is always towards the center of the circle.

The relationship between angular and tangential velocity

There is a connection between tangential velocity and angular velocity. To find the relationship between them let us start from the equation for tangential velocity:

$$v_t = \frac{s}{t} = \frac{\theta r}{t} = \omega r$$

where r is the radius, and ω (omega) is the angular velocity of the rotating object.

Note that, angular velocity is the same for all points on the rotating object, while tangential velocity depends directly on its distance from the axis of rotation.

Examples 3.9:

If the angular velocity of a wheel is 40 rad/s, and the wheel diameter is 60 cm. Determine the tangential velocity of the wheel.

Solution:

$$\omega = 40\,rad/s,\ d = 60\,cm \Rightarrow r = \frac{d}{2} = 30\,cm = 0.3\,m$$

$$v_t = \omega r = 40\,rad/s \times 0.3\,m = 12\,m/s$$

Centripetal Acceleration: Acceleration in Uniform Circular Motion

We know from kinematics that acceleration is caused by a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, because it always changes it direction of motion. We call the acceleration of an object moving in uniform circular motion the centripetal acceleration, a_c. Centripetal means center seeking or towards the center.

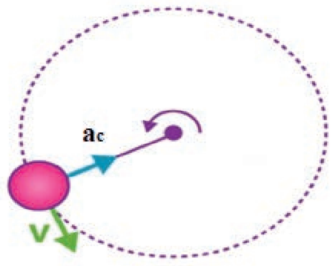


Figure 3.18: Centripetal acceleration

Centripetal acceleration is in the direction of the change in velocity, which points directly toward the center of the circular path. Because ii is directed towards the center of a curved path along the radius it is also called radial acceleration. This direction is shown with the vector diagram in the Figure 3.18.

Key term

Centripetal acceleration is an acceleration in circular motion caused by the change in the direction of velocity. Its direction is always towards the center of the circle.

Is seen first at point P with velocity $\mathbf{v}_i$ at time t_i and then at point Q with velocity $\mathbf{v}_f$ at time t_f, where $|\mathbf{v}_i| = |\mathbf{v}_f|$ that is they differ only in direction, thus $|\mathbf{v_i}| = |\mathbf{v}_f| = \mathbf{v}$. In order to calculate the acceleration, we start with the definition of average acceleration.

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Where Δv can be obtained graphically by the property of parallel shifting of vectors without affecting the direction and the magnitude; so that we can make a parallel shift of the tail of $\mathbf{v_f}$ from Q to the tail of $\mathbf{v}_i$ at P as shown as shown in Figure 3.19.

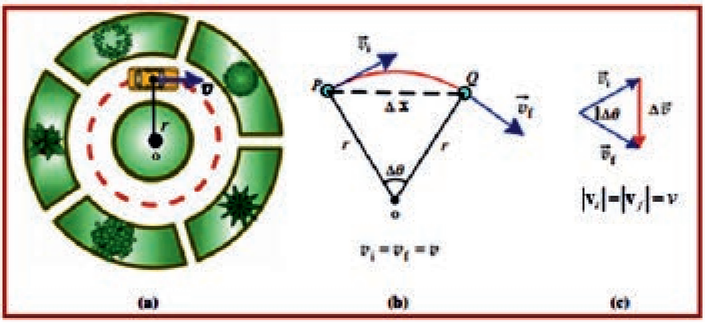


Figure 3.19: (a) Circular motion with constant speed v. (b) Velocity vectors at P and Q. (c) Graphical method of the application of vector subtraction so that, Since $\Delta\theta$ is the same in (b) and (c), the ratio of the sides of the two triangles are equal.

The triangle OPQ in Figure 3.19 (b), which has sides and r, is similar to the triangle ABC of Figure 3.19 (c), which has sides and Δv. Then the pair of corresponding sides are proportional, i.e.,

$$\frac{OP}{AB} = \frac{PQ}{BC} \Rightarrow \frac{\Delta v}{v} = \frac{\Delta x}{r}$$

Which we can obtain the expression for Δv as: $\Delta v = \frac{v}{r}\Delta x$

Substituting this value in the equation for average acceleration we get:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v\Delta x}{r\Delta t}$$

But, $v_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$, and the average acceleration becomes $\frac{v^2}{r}$, the instantaneous acceleration in the direction of Δv which is directed towards the center. Thus, the equation for centripetal acceleration becomes:

$$a_c = \frac{v^2}{r}$$

Since, $v = \omega r$, in terms of angular velocity, the equation for tangential velocity is:

$$a_c = \omega^2 r$$

The SI unit of centripetal acceleration is m/s^2.

Activity 3.10

Forming a group of 3 to 5 students discuss the following questions.

1. State some real-life examples of circular motion.
2. Mention the cases in which the effect of centripetal acceleration could be made to be larger.

Example 3.10

An automobile moves with a constant speed of 50.4 km/h around a circular track that has a diameter of 40 m. (a) what is the angular speed of the automobile? (b) What is the period of the motion?

Solution:

$$v = 50.4\,km/h = 14\,m/s, r = \frac{d}{2} = 20\,m$$

(a) $$\omega = \frac{v}{r} = \frac{14\,m/s}{20\,m} = 0.7/s = 0.7\,rad/s$$

$$v_t = \frac{2\pi r}{T}$$

(b) $$\Rightarrow T = \frac{2\pi r}{v_t} = \frac{2\times 3.14\times 20m}{14m/s} = 8.97\,s$$

Exercise 3.7

By how much would the centripetal acceleration change if the speed of a car changes from 30 km/h to 90 km/h?

Centripetal Force

The force which causes centripetal acceleration is called a **centripetal force**. Without a centripetal force, an object would continue to travel in a straight line and never deviate from its direction. Yet, with the centripetal force directed towards the center of the circle, the object always changing its direction and undergoing a circular motion. Centripetal force is perpendicular to the tangential velocity vector.

From Newton's second law: Centripetal force = mass × centripetal acceleration.

$$F_c = ma_c$$

Example 3.11

A 100 g ball is whirled in a horizontal circle of radius 40 cm. With what maximum speed can it travel if the breaking strength of the rod is 50 N?

Solution:

$$m = 100\,g = 0.1\text{kg},\ \text{r} = 40\,cm = 0.4m,\ F = 50\,N$$

$$F = m\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{50\,N\times 0.4m}{0.1\text{kg}}} = 14.14m/s$$

Key term

Centripetal force is a force that causes centripetal acceleration. It is always perpendicular to the direction of motion of the object.

Applications

A centrifuge is medical laboratory equipment that is used to separate particles suspended in a liquid such as blood or urine according to particle size and density, viscosity of the medium, and rotor speed.

Example 3.12:

(a) Calculate the centripetal acceleration of a point 7.50 cm from the axis of a centrifuge spinning at $7.5\times10^4 rev/\min$.

(b) Determine the ratio of this acceleration to the acceleration due to gravity.

Solution:

$$r = 7.5cm = 0.075m$$

$$v = 7.5\times10^4 rev/\min = 7.5\times10^4 \frac{\cancel{rev}\times\frac{2\pi r}{\cancel{rev}}}{\cancel{\min}\times 60\frac{\sec}{\cancel{\min}}} = 7.5\times10^4\frac{2\pi(0.075m)}{60\sec}$$

$$= 589m/s.$$

(b) In order to compare centripetal acceleration, to that of acceleration due to gravity, we need to compare the centripetal force with the weight of the object. i.e.

$$\frac{F_c}{F_g} = \frac{ma_c}{mg} = \frac{a_c}{g}$$

Substituting values gives:

$$\frac{a_c}{g} = \frac{4.63\times10^6 m/s^2}{9.8m/s^2} = 4.72\times10^5.$$

This last result shows that the centripetal acceleration is 472,000 times larger than the acceleration due to gravity, g. Such high-speed centrifuges are called ultra-centrifuges. The extremely large accelerations involved greatly decreases the time needed to cause the sedimentation of blood cells or other materials in laboratory examinations.

A merry-go round is a popular amusement ride in many kids' play yards. In such play, kids feel more uneasy when the wheels are rotated fast in a short diameter wheel than when the wheels are rotated slow in a long diameter wheel. This is because of the fact that the radial acceleration is very high for short diameter and high-speed rotors.

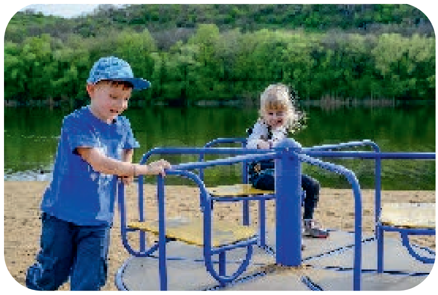


Figure 3.20: In the merry go round play kids enjoy the feeling of the effect of the radial acceleration as they go round.

Exercise 3.8

Suppose the diameter of the merry go round in Figure 3.20 is 4.0 m.

(a) What will be the radial acceleration of the kid if she rotates one rotation every 3 seconds?

(b) Draw schematic circular path that shows the direction of the centripetal acceleration of the circular motion by applying vector subtraction method.

Uniform Circular Motion in a Horizontal Plane

Horizontal circular motion indicates circular motion across a horizontal surface. There are various day-to-day examples such as the motion of a mass attached to the end of a string and made to rotate in a horizontal circle, Figure 3.21. In this case the centripetal force is provided by the tension (T) in the string. Hence,

$$T = m\frac{v^2}{r}$$

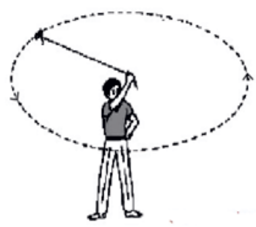


Figure 3.21: Amass attached the end of a string and rotate in a horizontal plane

Example 3.13:

A string cam withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using a string of length 1.0m?

Solution:

$$F = T = 25\,N, m = 1kg, \ell = r = 1.0m$$

$$T = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{25N \times 1m}{1kg}} = 5m/s$$

When a car moves around a roundabout, Figure 3.22, the centripetal force is provided by the frictional force between the road and the tires of the car. Hence,

$$F_f = m\frac{v}{r}$$

$$\Rightarrow \mu F_N = m\frac{v}{r}$$

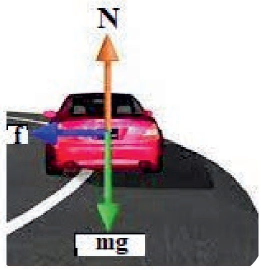


Figure 3.22: A car travelling on a roundabout

Where F_N is the normal force, ad exerts on the car.

Activity 3.11:

It is a common experience to see cars skidding out of a roundabout and collide with anything in the nearby the road. Discuss in group about the causes of such accidents and suggest the remedy. Present your group's opinion to the class and make class level discussion.

Example 3.14

Find the maximum speed of a car with which it can be safely driven along a curve of radius 40 m and the coefficient of friction between tires and road is 0.7. (Take g = 10 m/s²)

Solution:

$r = 10\,m, \mu = 0.2, v = ?$

Since the centripetal force is provided by frictional force,

$$F_f = m\frac{v^2}{r}$$

$$\Rightarrow \mu mg = m\frac{v^2}{r}$$

$$\Rightarrow \mu mg = m\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\mu gr} = \sqrt{0.7 \times 10m/s^2 \times 40m} = 16.73m/s$$

The conical pendulum

Suppose a particle of mass m is tied to a string of length ℓ and then whirled round in a horizontal circle of radius r, with O fixed directly above the center B of the circle, Figure 3.23. If the circular speed of the particle is constant, the string turns at a constant angle θ to the vertical. This is called a conical pendulum.

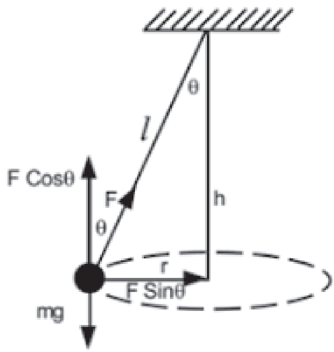


Figure 3.23: A conical pendulum

The horizontal component of the tension in the string (T), points towards the center of the circle, hence it provides the centripetal force.

$$T\sin\theta = m\frac{v^2}{r}$$

The weight (mg) of the particle is balanced by the vertical component of the tension in the string.

$$T\cos\theta = mg$$

Dividing the above equation by the lower one gives:

$$\tan\theta = \frac{v^2}{rg}$$

Motion of a car Round a Banked Road

Suppose a car is moving round a banked road in a horizontal circular path of radius, r, Figure 3.24. If the only forces at the wheels are the normal reaction forces F_N, that is there is no side-slip or strain at the wheels, the force towards the center of the track is :

$$N\sin\theta = m\frac{v^2}{r}$$

The vertical equilibrium gives: $N\cos\vec{E} = mg$

Dividing the top equation by the bottom one gives:

$$\tan\theta = \frac{}{rg}$$

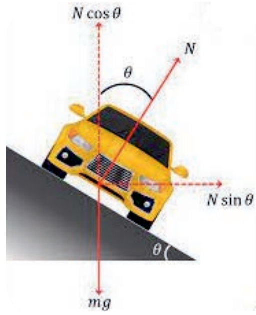


Figure 3.24: Banking of a road

The angle of banking is then,

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Exercise 3.9

Calculate the maximum speed with which a car can be driven safely along a curved road of a radius of 30m and banked at an angle of 30° with the horizontal. (take g = 9.8m/s^2)

Review Questions 3.5

1. What does it mean by uniform circular motion?
2. Define the terms centripetal or radial acceleration and force.
3. What is the direction of centripetal acceleration?
4. How it is possible for a body to be accelerated if its speed is constant?
5. What is (a) the tangential velocity? (b) the angular velocity? and (c) the centripetal acceleration, due to the Earth's rotation, of a body on the equator? Radius of Earth = 6,400 km.
6. The wheel of a bicycle has a radius of 40 cm. Calculate its period of revolution if the bicycle moves with a speed of 20 km/h. How much is the angular velocity?
7. Find the banking angle of the railway track with a radius of curvature of 1500m. If the train's maximum speed is 15m/s. If the distance between the two tracks is 1.8m, calculate the elevation of the outer track over the inner track.

SUMMARY:

- Acceleration is a vector quantity, and is defined as the rate of change of velocity or change in velocity per unit time. The SI unit of acceleration is m/s2.
- Average acceleration: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$
- Instantaneous acceleration, $\vec{a}_{ins}$ or acceleration at a specific instant in time. It is determined by taking the time interval between the two points (Δt) approach zero.

$$a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- A motion with a constant acceleration refers to a motion where, the velocity of the object is changing by equal amount in equal interval of time.
- The equations of motion with a constant acceleration:

$$v_f = v_i + at \quad s = v_i t + \frac{1}{2}at^2$$

$$v_{av} = \frac{v_i + v_f}{2} \qquad s = v_f t - \frac{1}{2}at^2$$

$$s = \left(\frac{v_i + v_f}{2}\right)t \qquad v_f^2 = v_i^2 + 2as$$

$$\text{Braking distance}\left(s_{break}\right) = \frac{v^2}{2a} = \frac{v^2}{2\mu g} \quad \text{Stoppind distance} = vt + \frac{v^2}{2\mu g}$$

- A displacement-time graph shows how displacement changes with time.
- The slope of displacement-time graph gives velocity.
- A velocity-time graph shows how velocity changes with time.
- The slope of a velocity-time graph gives the acceleration.
- The area between a velocity-time graph and the `time' axis gives the displacement of the object.
- Acceleration-time graph shows how acceleration changes with time.
- When things move vertically either up or down under the force of gravity alone, the resulting motion is called free fall. The acceleration of a freely falling body is constant and is known as acceleration due to gravity, denoted by the letter g.
- Equations of freely falling

1. $\vec{v} = \vec{u} + \vec{g}t$
2. $\vec{y} = \vec{u}t + \frac{1}{2}\vec{g}t^2$
3. $v^2 = u^2 + 2gy$

- Terminal velocity is defined as the point at which the velocity of a falling object is no longer getting greater. An object at terminal velocity has zero acceleration.
- Uniform circular motion is a special type of curvilinear motion in which an object travels in a circle with a constant speed but its direction of motion continuously changes.
- While moving in a circular path, the angle made by the radius 'r' of the circle is known as angular displacement (θ).
- Tangential displacement (s) is the distance covered by the rotating object along the curved path (circumference of a circle).
- Angular displacement $\theta = \frac{s}{r}$.
- Angular velocity measures the rate at which angular displacement changes. It is denoted by Greek letter omega (ω) and its standard unit is radians per second.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- A velocity describes the motion of an object along the edge of the circle whose direction at any given point on the circle is always along the tangent to that point.

$$v_t = \frac{s}{t} = \frac{2\pi r}{T} = \omega r$$

- Centripetal Acceleration that acceleration an object moving in uniform circular motion the centripetal acceleration, a_c. Centripetal means center seeking or towards the center.

$$a_c = \frac{v^2}{r} = \omega^2 r$$

- The force which causes centripetal acceleration is called a centripetal force. $F_c = ma_c$

END OF UNIT QUESTIONS

1. Which one of the following pairs of concepts cannot both simultaneously be constant and non-zero for a body?
   A. The speed and velocity
   B. Magnitude of acceleration and acceleration
   C. Total distance and displacement
   D. Velocity and acceleration
2. Can an object can have variable speed but constant velocity? Explain.
3. Can an object can have constant speed but variable velocity? Explain.
4. What do you mean by motion in one and two dimensions? Give examples of each type.

5. Can the displacement be greater than the distance travelled by an object? Give reason.
6. Can the direction of velocity of an object change, while its acceleration is constant? Explain with example.
7. Is it possible for a body to be accelerated without speeding up or slowing down? If so, give an example.
8. To get to school, a girl walks 1 km North in 15 minutes. She then walks 200 m South-west in 160 seconds. What is the girl's average velocity for her walk to school?
9. Very large magnitude of acceleration can be achieved in some devices such as in a gun. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of 6.20×10^{5} m/s^2 for 8.10×10^{-4} s. What is its muzzle velocity (that is, its final velocity)?
10. It is very common for drivers when they enter the express road, they increase their speed. While entering to the express road on its travel from Addis Ababa to Adama , a car accelerates from rest at a rate of 2.40 m/s2 for 12.0 s.

    (a) Draw a sketch of the situation.
    (b) List the knowns in this problem.
    (c) How far does the car travel in those 12.0 s? (d) What is the car's final velocity?

11. A particle moving at 25 m/s in a straight line slows uniformly at a rate of 2 m/s every second. In an interval of 10 s, find: (a) the acceleration, (b) the final velocity, (c) the distance moved.
12. At the instant the traffic light turns green, an automobile starts moving with a constant acceleration of 5.0 m/s2. At the same instant a truck, travelling with a constant speed of 10 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be at that instant?
13. Describe the motion in each interval of the velocity-time graph shown in the figures below.

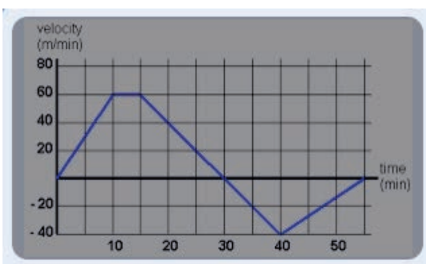


Figure 3.22

14. On its way along a straight line, a car starts from rest and accelerated at a rate of 2.5 m/s2 for 10s. It maintained its speed for further 5 s and decelerated by 0.5m/s2 to come to rest. What is the time taken for the entire motion? How far did it travel? Draw the position vs time, and velocity vs time graph of the motion of the car.
15. On a user's manual of a motorbike you see a specification labeled as "the bike can accelerate to 100km/h from rest in 4seconds." A) what can you say about the maximum average acceleration of the bike? B) How far does it travel in this time?
16. The head quarter of Commercial Bank of Ethiopia having a height of 198m is the tallest building in Ethiopia and one of the top ten tallest buildings in Africa. If the engineer's helmet is suddenly dropped from the roof during the final topping, A) How long does it take for the helmet to reach to the ground? B) If someone needs to throw the helmet up to the top, what minimum initial velocity is needed? Can this initial velocity be achieved by a human being? (neglect air resistance in all cases)

Figure 3.22

17. A stone strikes the ground with a speed of 25 m/s. (a) From what height was it released? (b) How long was it falling? (c) If the stone is thrown down with a speed of 10 m/s from the same height, then what will be its speed just before hitting the ground?
18. Calculate the height of a building if it takes 4.0s for a ball to reach to the ground when it is thrown straight up from the top of the building with an initial velocity of 8.0 m/s. (Sketch the motion diagram.)
19. A kangaroo can vertically jump over an object 2.5m high.
    A. What is its initial vertical speed to reach this height?
    B. How long will it stay in the air if it is jumping on a level ground?

Figure 3.23

20. Calculate the magnitude of the acceleration of a particle moving in a circle of radius r = 0.5 m with a constant speed of 10 m/s.
21. A 0.525 kg ball is attached to a 1.25 m string and swings in a circular path, making 2.00 revolutions per second. The angle of the string is nearly horizontal. Find the tension in the string.
22. A 0.525 kg ball is attached to a 1.25 m string and swings in a circular path. The angle of the string away from vertical is 30.0°. Find the centripetal force acting on the ball and the speed of the ball.

UNIT 4

DYNAMICS

Introduction

Motion is an important element in studying Physics. Naturally, the race of athletes, the flying of a bird, and the jumping of a dolphin out from the water, the trajectory of a soccer ball or a basketball and the like are examples of motion in nature. The motion of a fast driving car, or an airplane or a fast magnetic levitated (Maglev) train, or a supersonic jet, or a satellite orbiting the Earth are all fascinating developments of human beings in the advancement of motion.

.In unit three of this textbook, you studied kinematics: the study of motion that only describes the way objects move-their velocity and their acceleration but not what causes the motions. However, motions have causes and that cause is a force. The study of motion with its cause is dynamics. In this unit, we study about motion, how motion is created by a force, what the impact of force is on moving or stationary objects and the like.

At the end of this unit you should be able to:

- *demonstrate an understanding of the relationship between net force and the acceleration of an object in linear motion*
- *analyze the effect of a net force in quantitative terms, using graphs, free-body diagrams, and vector diagrams*
- *describe the first condition of equilibrium quantitatively.*
- *demonstrate an understanding, in qualitative and quantitative terms, of the concepts of work, energy, energy transformations and power*
- *design and carry out experiments and solve problems involving energy transformations and the law of conservation of energy.*
- *realize that momentum is an inherent property of moving objects*
- *express the relationship between impulse and momentum.*

4.1 The Concept of Force and Newton's Laws of motion

The concept of force

At the end of completion of this section you should be able to:

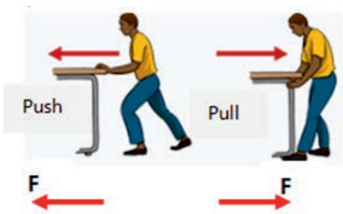
- *discuss what the concept of force means in physics, and why forces are vectors.*
- *define force and discuss its effects on a body.*
- *distinguish between contact forces and field forces.*
- *discuss the four fundamental forces in nature and their real life application.*
- *state newton's laws of motion*
- *demonstrate the cause effect relationship of force and acceleration.*
- *apply newton's second law to determine the weight of an object.*
- *demonstrate the relationship between weight and normal force.*

? Brainstorming

1. How do you define the term force in your own words? Mention some of the natural phenomena that show the action of a force on or between objects in your surroundings.
2. How does force affect every moment in your life?

To understand dynamics, the study of motion and its causes, we need to start with the working definition of force. In everyday language a force is a push or a pull, as when you push or pull a table on the floor or a player kicks a ball, Figure 4.1. The speed and direction of motion of both the ball and the table are determined by the magnitude and direction of the applied force. So force is a vector quantity, just like velocity and acceleration.
The SI unit of measurement of force is newton (N).

$$\text{Where } 1\text{N} = 1\text{kg.m/s}^2$$



(a)

(b)

Figure 4.1 (a) Force exerted on the table (b) Force exerted on the ball

Apart from changing the speed or direction of motion of a body or both, force can also change the shape or size of a body. A tennis ball hit by a racket is deformed to some extent. Such an effect is called deformation.

A force can be either a contact force or a field force. Forces that result from physical contact between two bodies are called contact forces. A girl stretching a spring, a man pulling a box on the floor, and a girl kicking a ball are few examples of contact forces.

On the other hand forces like gravitational attraction between the Sun and the Earth, the electrostatic force between charged particles, and magnetic force between a magnet and magnetic substances that operate at a distance are known as field forces.

The known fundamental forces in nature are all field forces. These are, in order of decreasing strength:

1. the strong nuclear force between subatomic particles;
2. the electromagnetic forces between electric charges;
3. the weak nuclear force, which arises in certain radioactive decay processes; and
4. the gravitational force between objects.

The strong force keeps the nucleus of an atom from flying apart due to the repulsive electric force of the protons. The weak force is involved in most radioactive processes and plays an important role in the nuclear reactions that generate the Sun's energy output. The strong and weak forces operate only on the nuclear scale, with a very short range on the order of 10-15 m.

In this section we will use two new concepts, force and mass, to analyze the principles of dynamics. These principles were clearly stated for the first time by Sir Isaac Newton (1642–1727); today we call them Newton's laws of motion.

Key terms

- *Dynamics: The subdivision of mechanics that deals with the motion of objects in relation to the physical factor that causes the motion, the force.*
- *Force: An external agent of a body that changes state of rest or state of motion of that body.*

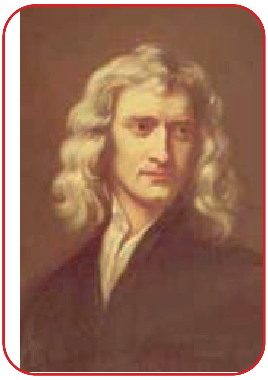


Figure 4.2 Isaac Newton English physicist and mathematician

Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today. (Giraudon/Art Resource)

Laws of Motion

Newton's First Law of Motion

? Brainstorming

Suppose you are in a stationary car. What will happen to you if the car suddenly starts to move forward? Suppose now you are in a moving car. What will happen to you if the driver hits the pedal and the car suddenly brakes to stop?

How do the forces that act on a body affect its motion? To begin to answer this question, let's first consider what happens when the net force on a body is zero. Consider a book placed on your desk. What will happen to the book if you leave it there for a long time? Your response will obviously be 'it will remain at rest'. You would almost certainly agree that if a body is at rest, and if no net force acts on it (that is, no net push or pull), that body will remain at rest. But what if there is zero net force acting on a body in motion.

To see what happens in this case, do the following activity.

Activity 4.1

1. Slide a hockey puck along a horizontal tabletop as described following the three steps:

   (a) Place the puck on the table Figure 4.3 a. Apply a horizontal force and once it is in motion stop pushing it.

(b) Repeat step (a) but now cover the table top with a smooth surface of ice, Figure 4.3b.

(c) Repeat step (a) by putting the puck on an air-hockey table, where it floats on a thin cushion of air, Figure 4.3 c.

Write down your observation in each of the three cases. Is there a difference in distance covered by the puck before it stopped? What do you think is the reason for this?

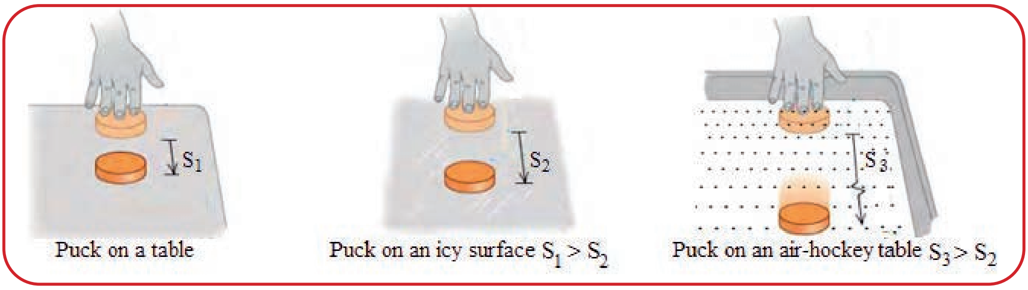


Figure 4.3 A puck pushed horizontally on three surfaces of different nature. Compare the distances.

From the results of the above activity, one can conclude that if we could eliminate friction completely, the puck would never slow down, and we would need no force at all to keep the puck moving once it had been started. Once a body has been set in motion, no net force is needed to keep it moving. We call this observation Newton's first law of motion:

Newton's first law of motion

"An object at rest remains in a state of rest and an object in motion continues in motion along a straight line with constant speed unless an external force acts upon it. "

Newton's first law of motion is also called the law of inertia. The property of a body to remain at rest or to remain in motion with constant velocity is called *inertia*.

Key term

Inertia: The tendency of a body to oppose any change in its state of rest or its state of uniform motion.

It is more difficult to change the state of motion of a truck than that of a small *bajaj* (a three wheel taxi in many parts of Ethiopia). This is because the mass of the truck is much larger than that of the *bajaj*, Figure 4.4. As the mass of the bajaj is small, a single person can pull or push it which is unthinkable for the truck. This property of maintaining state of motion is called the inertia. mass is a measure of imertia.

(a)

(b)

Figure 4.4 Inertia is larger for large masses than for small masses.

Activity 4.2

Material required: Two medium-size cans, tow strings and a horizontal fixed beam from which the cans can be suspended, Figure 4.5.

Procedure

Fill one of the cans with sand and leave the other one empty.

a) Try to set the cans in motion. What difference did you observe?

b) Now pull the cans to one side (to the same height) and release themTry to stop the cans one after the other. What did you experience? Discuss with your classmates about the difficulties you faced in terms of the forces you are required to set the cans in motion (case (a)) or to stop them (case (b)).

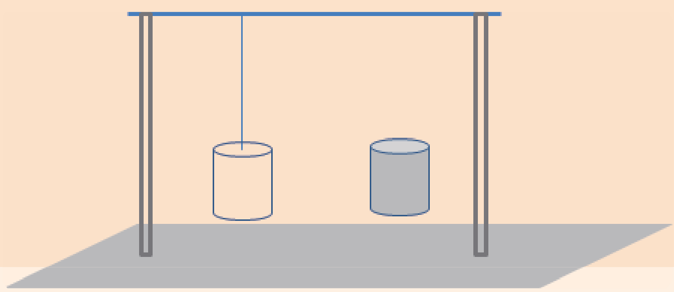


Figure 4.5 Large masses have larger inertia than small masses

Some examples of Applications of Newton's first law

You have heard about the importance of safety belts to protect passengers in a car or in an airplane or in a bus from severe damage during accidents. The probability of facing severe injury to the extent of death when an accident occurs is very high on people who do not fasten seat belts.

The question is: Why Seat belts? What is the physics behind Seat belts?

The answer is **Newton's first law of motion**. If a car is moving with a speed of 80km/h, all passengers in the car are moving with a speed of 80km/h. If the car suddenly stops for reasonable case, the passenger who fastened seat belt remains intact with the body of the car and stops with the car safely, Figure 4.6a. On the other case, the passenger who doesn't fasten seat belt is in a loose state from the body of the car and when the car abruptly stops, according to Newton's first law, the passenger continues its motion with 80km/h and collide to anything in front of him such as to the steering wheel, Figure 4.6b or even may smash againes the windshied of the car which may lead to severe injury and death. Modern cars have airbags on the steering wheel or on the dashboard, Figure 4.7, to protect passengers from severe injury by giving cushion during accidents. However, airbags are designed to emerge automatically only when the car collided with something. Therefore, passengers should always fasten their seatbelts.

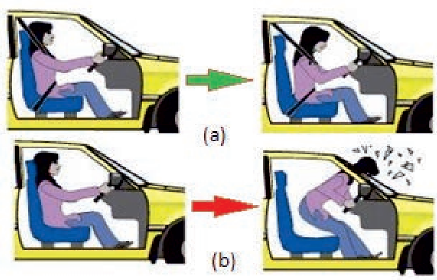


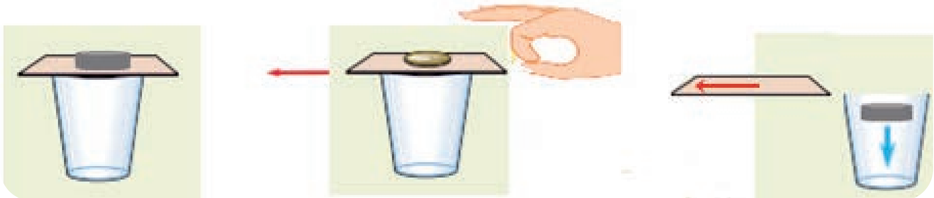
Figure 4.6 A passenger without a seat belt will suffer injury during an accident.

Figure 4.7 Airbag systems in vehicles are designed to deply during vehicle collision

Activity 4.3

Fun with Inertia

- Materials
- Glass tumbler
- Small, stiff piece of cardboard
- Coins
- Ruller



(a)

Figure 4.8 Inertia at rest

Procedure 1: Refer Figure 4.8a.

(a) Put a playing card on top of the plastic cup

(b) Put a coin on top of the card

(c) With a sharp flick, hit the card out from under the coin.

Question: What happened to the coin? Explain your observation interms of Newton's First law of motion.

The motion of a body can only be described relative to other bodies. Bodies with which motion of a body can be compared are known as frames of reference. Frame of reference can be catagorized as Inertial frame of reference and Non-inertial frame of reference.

A frame reference which is at rest or moves with a constant velocity is known as an inertial frame of reference. A stationary car or a car traveling at constant velocity is an inertial frame of reference. A car slowing down for a stoplight, or speeding up after the light turns green, will be accelerating and is not an inertial frame of reference. Such a frame of reference is known as Non-inertial frame of reference. An inertial frame of reference is the type of frame where Newton's laws hold true. If no outside force is exerted on an object, it will continue to stay in state of rest or uniform motion.

Newton's Second Law of Motion

? Brainstorming

Based on your knowledge of kinematics discuss the difference between moving with uniform velocity and having acceleration.

What is the cause of acceleration?

A net external force and a system are the main focus of this section in which the cause effect relation of net force and acceleration will be discussed. The concepts of weight, normal force, and friction force and how they are related to each other and some applications of Newton's law are also the focus of interest in this section.

Newton's second law of motion is closely related to and is a consequence of Newton's first law of motion. According to the first law, a body persists to its state of motion or rest unless an unbalanced force acts on it." The question is: What will happen to the object if an unbalanced force acts on it? The answer was given by Newton after simple experiments and inquiries (refer Figure 4.9.)

a

b

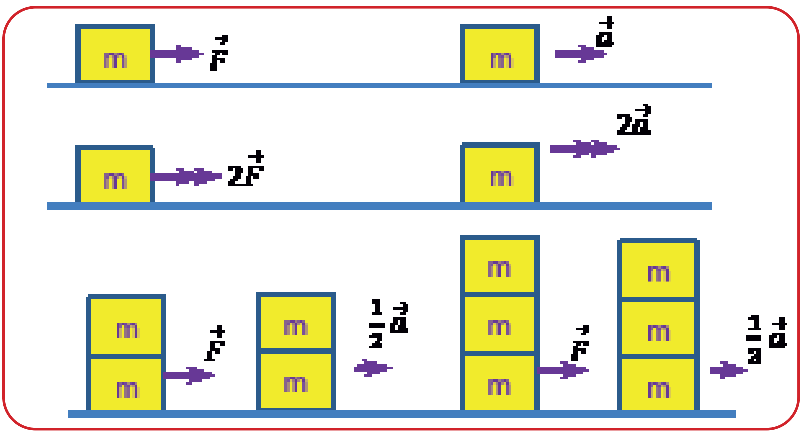


Figure 4.9 Demonstrating Newton's second law of motion. See how the acceleration is affected (a) when the force is changed for a constant mass and (b) when the mass is changed for a constant force

Inquiry I: What will happen if a force F is applied to an object of mass m which is initially at rest on a horizontal smooth surface where friction is negligible?

(Answer): By observation, it moves with some velocity in the direction of the applied force.)

Inquiry II: How does it move as long as the force is maintained? (Answer: The velocity is increasing uniformly through time. That means it is accelerating uniformly).

Inquiry III: What will happen to the magnitude of the acceleration if the force is doubled, or tripled, while the mass is kept unchanged? (Answer: Experimentally as the force is doubled the acceleration is doubled as shown in Figure 4.9a a. The acceleration is therefore directly proportional to the applied force.)

Inquiry IV: What will happen to the magnitude of the acceleration if the mass is doubled, or tripled, while the force is kept unchanged? (Experimentally, for the constant force, as the mass is doubled the acceleration is reduced by half, when the mass is tripled, the acceleration is reduced by one-third and so on as shown in Figure 4.9 b. The acceleration is therefore inversely proportional to the mass.)

The answers for inquiries I through IV with an illustrative demonstration of Figure 4.9 collectively give the full statement of Newton's second law that can be stated as follows.

Newton's Second Law of motion

"When an unbalanced force acts on an object of mass m, the object accelerates in the direction of the applied force with the magnitude of acceleration directly proportional to the net applied force and inversely proportional to the mass of the object."

Mathematically Newton's second law can be expressed as

$$a \sim \vec{F}_{net} \text{ and } a \sim \frac{1}{m}$$

$$\text{We write } a = k\frac{\vec{F}_{net}}{m}$$

The constant of proportionality k is equal to one, so in mathematical terms the preceding statement can be written

$$a = k\frac{\vec{F}_{net}}{m}$$

$$\text{Rearranging in a linear form, } \vec{F}_{net} = ma$$

Key terms

- *Net force: The sum of two or more forces (Resultant force)*
- *Acceleration: The time rate of change of velocity. It could happen when a body changes its speed or direction of motion or both.*
- *System: A collection of objects, elements or components that are organized for a common purpose. Anything outside the system is the surrounding.*

The SI unit of force, newton, in terms of the three basic quantities of measurement in mechanics: length mass and time is expressed as

$$1\,newton = 1kg\frac{m}{s^2} \qquad \text{(SI unit of force in terms the basic units)}$$

Large quantities of forces can also be expressed in kilo newton (kN); 1kN=1000N

Activity 4.4

Group work

Explore the relationship between Force, Mass, and Acceleration.

Problem

If you pull an object with a force, how do force and mass affect the acceleration of the object?

Materials

Dynamics cart with hook, two 200-g standard masses, one 1 kg standard mass, spring scale (0 –5 N) smooth, and a flat surface (about 2 m long), Figure 4.10.

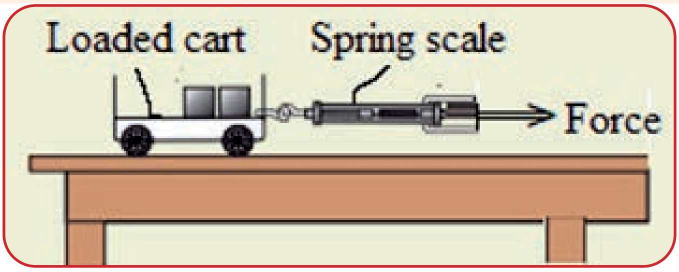


Figure 4.10 Set up of the materials

Procedure

A. Place the 200 g standard masses on the cart and attach the spring scale to the hook on the cart.

B. Pull the spring scale so that the cart starts to accelerate forward. Pull the spring with a constant force of 3 N. Keep the force as constant as possible while pulling the cart and observe the acceleration of the cart.

C. Replace the 200 g masses on the cart with the 1kg standard mass. Then pull the cart applying the same force you used in step B. Observe the acceleration of the cart.

D. Place the 1 kg standard mass on the cart and now pull the cart applying a constant force of 4 N. Observe the acceleration of the cart.

E. Repeat step D but this time pull with a force of 2 N. Observe the acceleration of the cart.

Answer the following questions based on your observation.

1. What can you say from your observation in steps B and C? How did the mass affect the acceleration for a constant force?
2. What can you conclude from what you observed in steps D and E? How did the force affect the acceleration for a constant mass?
3. What did you find difficult in this activity? What other techniques can you suggest to keep the force applied on the spring constant? Discuss with your group members.

Newton's third Law of Motion

? Brainstorming

If you drop your pencil at a certain height, it falls downward due to the force that the Earth exerts on the pencil. Can you say that the pencil in turn exerts force on Earth? Why is it difficult to walk on sand than on a solid ground?

When a body exerts a force on a second body, the second body exerts a force on the first body which is equal in magnitude but opposite in direction. Figure 4.11 shows some of the common action-reaction pairs.

If you hit down a nail with a hammer, the nail hits the hammer back with a force of the same size. A typical application of Newton's third law is the propulsion of a rocket while hot gas is exhausted from the rocket.

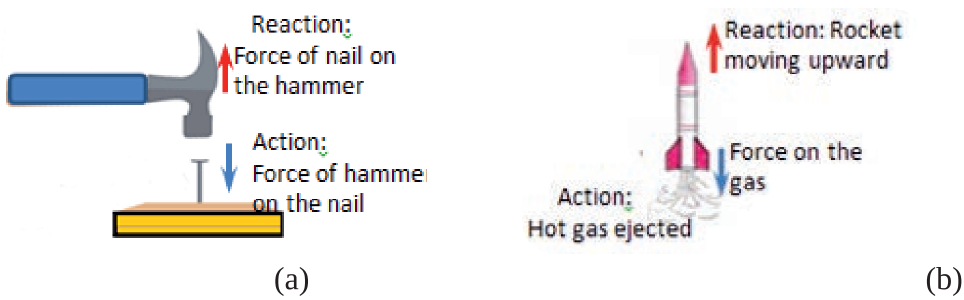


Figure 4.11 Action and reaction pairs. Forces are always in pairs and for every action there is an equal and opposite reaction

Key terms

Action and reaction: During an ineaction between two objects the force one exerts on the other is Action and the force it is acted upon by the other is Reaction

Exercise 4.1

Discribe the action and reaction pairs a) as you walk on the ground,
b) as a player kicks a ball

Newton's third law is popularly expressed as:

"For every action there is always an equal and opposite reaction."

In Figure 4.12, it is clearly seen that the boy is pushing the girl but it is not obvious that the girl is pushing back on the boy. In fact when both the boy and the girl move in opposite directions immediately after interaction, it becomes clear that the boy is affected by the force that the girl exerted on him. Can you identify the action and reaction forces in this case?

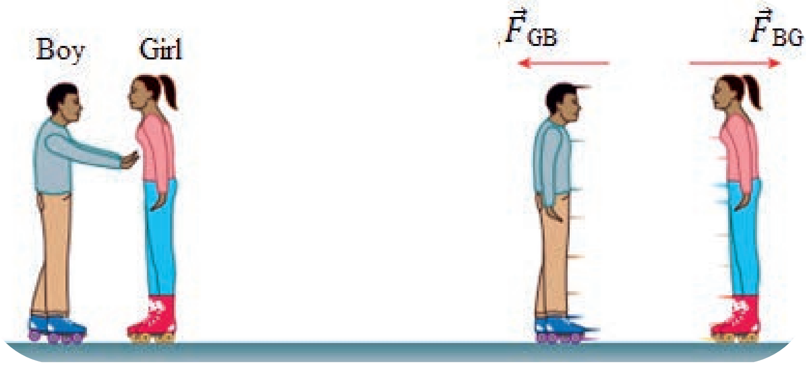


Figure 4.12 Action and reaction are equal and opposite $F^{BG} = -F^{BG}$

Activity 4.5

Discuss with your classmate

Imagine a fat person and a thin person that are running in opposite directions accidentally collide with each other. If the thin person falls backward while the fat person stands still, what can you say about the forces the persons exerted on one another? Give your reason in terms of Newton's third law of motion.

Misconception alert

Action and reaction forces, though they are equal in magnitude and opposite in direction, they act on two different bodies. Therefore they cannot be added to give a zero resultant!!!

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of the Earth. According to Newton's second law, acceleration is the effect caused by a force. Therefore, a falling object, experiences a downward force known as weight of the object denoted by $\vec{W}$. The magnitude of the weight is the product of mass (m) and the value of acceleration due to gravity g. Weight is a vector whose direction is always down towards the center of the Earth.

Mathematically,

$$Weight = (mass)\,(acceleration\ due\ to\ gravity)$$
$$\vec{W} = m\vec{g}$$
$$Near\ to\ the\ surface\ of\ the\ earth,\ the\ magnitude\ of\ \vec{g}\ is\ 9.8m/s^2$$

Question

Is weight of a body the same everywhere? Discuss you answer.

Example 4.1

What is the weight of a 1 kg block on the surface of the moon where acceleration due to gravity is only one-sixth of its value on the surface of the Earth?

Solution

$$Weight\ W = mg_m,\ where\ g_m = \frac{1}{6}(g) = \frac{1}{6}(9.8m/s^2) = 1.6m/s^2$$
$$W = (1kg)\,(1.6m/s^2) = 1.6N$$

An object on the moon weighs one-sixth as much as it does on the surface of the Earth.

This example depicts that the weight of an object is not an intrinsic property, as its value varies from place to place depending on the value of acceleration due to gravity g.

When an object is released in air, it experiences a downward force due to its weight and a drug force due to friction with the air. When the net force acting on the object is assumed to be gravity only, we say that the body is in free fall. The gravitational force $(\vec{F}_g)$ on a freely falling body is thus its weight $(\vec{W})$.

$$(\vec{F}_g) = \vec{W} = m\vec{g}$$

Misconception Alert

In everyday language, it is common to interchangeably use mass and weight. However in science, these terms are different from one another. Mass, measured in kilograms, is the measure of how much matter is in an object and it characterizes the inertial properties of a body. Weight on the other hand is a measure of the force of gravity acting on an object and measured in newton.

Normal Force

If you place your book on a table top as shown in the Figure 4.13, then a downward gravitational force, m, is exerted on the book, but the book is still intact on the table.

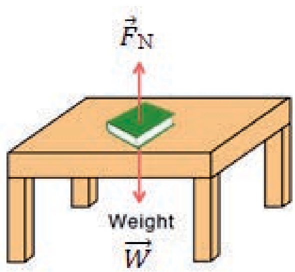


Figure 4.13 Normal force and weight for a book placed on a horizontal surface.

What keeps the book intact on the table? The upward force that supports the book is the Normal Force, denoted by $\vec{F}_N$. The normal force is perpendicular to the surface that supports the book.

Normal force is a type of contact force - two objects or surfaces have to touch for there to be a normal force.

Since the book is in equilibrium, the net force acting on the book is zero. Therefore, on a level surface the normal force is equal to the weight.

$$\vec{F}_g = \vec{w} = m\vec{g}$$

Note that as the name implies normal force is always normal (perpendicular) to the surface. Thus the weight and the normal force are equal only when the object is placed on a level surface. In most cases objects are placed on non-leveled surfaces such as on an inclined plane as shown in Figure 4.14. The normal force is less than the weight by a factor of cos θ. As θ is increased the normal force that supports the object is decreased and it will be zero when θ is 90^0. When θ is 0^0, as in Figure 4.14 the inclined plane becomes a level surface and obviously the normal force will be equal to the weight.

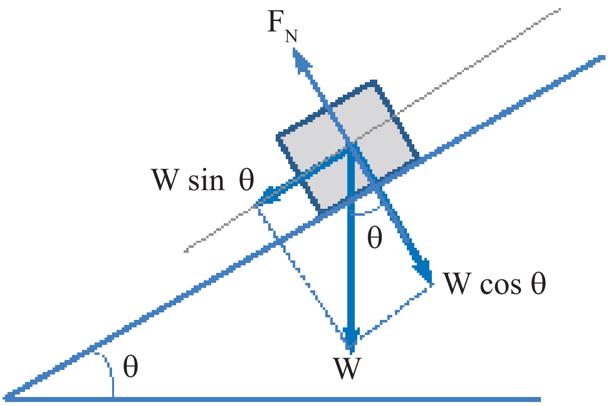


Figure 4.14 A block on an inclined plane

$$F_N = W \cos\theta = mg \cos\theta$$

Key terms

- *Weight: The gravitational pull on an object*
- *Normal forces: A force with which to objects push one another and acting perpendicularly at the surface of contact*

Activity 4.6

Place your book on a horizontal surface, Figure 4.15a. What is the normal force on the block? Now raise one side of the surface slowly as in Figure 4.15b and answer the following questions based on your observation.

a) What is the force that tends to slide the block down the surface?

c) What is the force that keeps the block on the inclined surface?

d) Did the block slide down the plane the moment you started to raise the surface? If your answer is no, what kept the block from sliding at that instant?

e) For what angle of elevation did the block start sliding?

Figure 4.15 A block on a horizontal surface and inclined surface

Example 4.2

If in figure 4.15b, the mass of the block is 6 kg resting on top of a 37^0 inclined plane, determine the weight and the normal force.

Solution:

The weight of the block is: $W = mg = (6\ kg)\ (9.8\ m/s^2) = 58.8\ N$.

The normal force is: $FN = mg\cos\theta = (6\ kg)\ (9.8\ m/s^2)\ \cos 37^o = 47\ N$

Review Questions 4.1

1. What are contact forces and field forces? Give examples of each.
2. List down the four fundamental forces in nature and arrange them in the order of increasing strength.
3. State the three Newton's laws of motion.
4. How are inertia and mass related?

$\vec{F}_{GB}$ $\vec{F}_{BG}$

5. Discuss the advantages of wearing seat belt in a car?
6. What is the advantage of having airbag in a car?
7. Discuss the dependence of acceleration of a body with its mass and net force applied on it.
8. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?
9. Identify the action and reaction forces in the following situations: (a) Earth attracts the Moon, (b) a boy kicks a football, (c) a rocket accelerates upward, (d) a car accelerates forward, (e) a high jumper leaps, and (f) a bullet is shot from a gun.
10. A block of mass m is placed on a horizontal surface as shown in Figure 4.16. What is the normal force in each of the following cases?

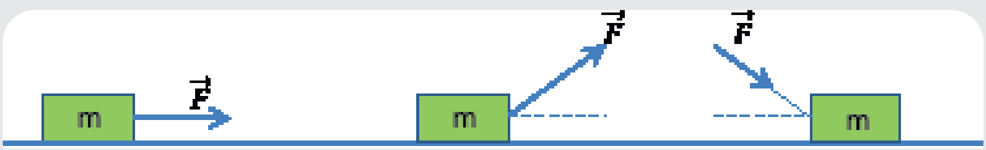


Fig 4.16

11. (a) If gold were sold by weight, would you rather buy it in Addis Ababa or in Alaska?

    (b) If it were sold by mass, in which of the two locations would you prefer to buy it? Why?

4.2 Frictional Force

At the end of completion of this section you should be able to:

- *describe the general characteristics of friction*
- *list the various types of friction*
- *calculate the magnitude of static and kinetic friction, and use these in problems involving Newton's laws of motion*
- *solve more complex acceleration problems involving Newton's laws of motion.*
- *use free-body diagrams to solve problems on Newton's Laws of motion.*

? Brainstorming

What is the role of friction in our daily life?
Can you get rid of friction?
Mention pros and cons of frictional force?

Friction is a type of force that We experience in our everyday activity. Friction, in simple terms is a force that opposes motion. It occurs when two surfaces are in contact with each other and when one surface slides or attempts to slide over the other. Friction occurs due to the roughness of the surfaces in contact. Even if the surface seem very smooth, it is not so at micro or nano scale, Figure 4.17.

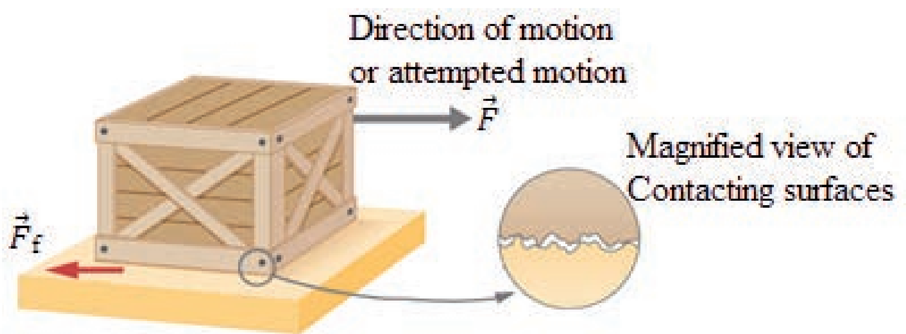


Figure 4.17 Magnified view of surfaces in contact

Friction helps us to move while we walk by preventing us from slipping. Without friction between the tires and the road we couldn't drive or turn the car. Without the frictional force exerted by the air on a body moving through it (air drag), parachutes do not work. Without friction nails would pull out, light bulbs would unscrew effortlessly. See Figure 4.18.

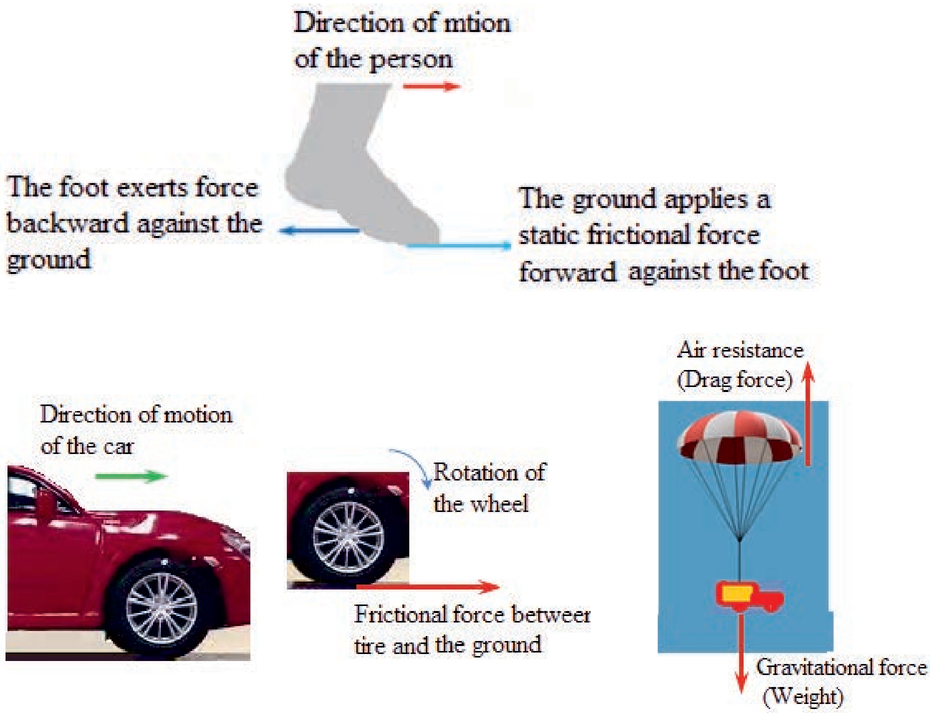


Figure 4.18 Friction force in action (a) Friction on the foot of a walking person (b) Friction on the wheel of a moving car (c) Drag force on a parachute

There are two types of friction:Static friction and Kinetic friction. Static friction is the friction between two surfaces when there is no movement. Suppose you pull a block slightly along a table top, Figure 4.19a. The block will not move with such a small force that you apply. What do you think is the reason for this? The force that keeps the block form sliding is the force of static friction denoted by (F^s) and is directed opposite to the applied force.

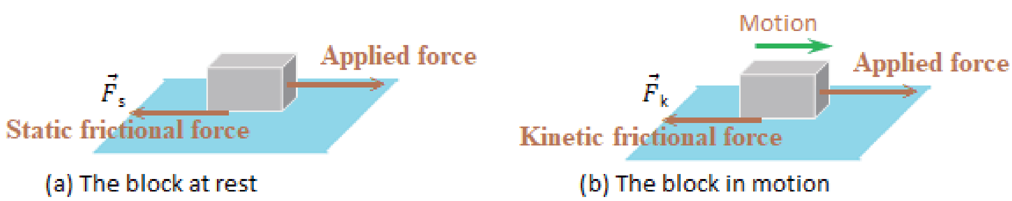


Figure 4.19 Static friction and kinetic friction

Kinetic friction: Kinetic friction is the friction between two surfaces when one of them is sliding over the other. For example, when the block that you push begins to slide over the table, Figure 4.19b, there is force of kinetic friction denoted by $(\vec{F}_k)$ between the bottom surface of the block and the table top.

Key terms

- *Friction: a force that opposes the motion of one surface over another.*
- *Static friction: Friction between two contacting surfaces at rest.*
- *Kinetic friction: Friction between two surfaces while one slides over the other.*

Frictional force is directly related to the normal force and its value depends upon the property of the surfaces in contact. This property of material that resists motion is called the **coefficient of friction (** μ **),** which is defined as the ratio between the friction force and the normal force. Mathematically,

$$Coefficient\ of\ friction = \frac{Frictional\ force}{Normal\ force}$$

$$\text{or symbolically } \mu = \frac{F_f}{F_n}$$

Coefficient of friction is a dimensionless number.

We write $F_s = \mu_s$, for static frictional force, where μ_s is coefficient of static friction. $F_k = \mu_k F_N$, for kinetic frictional force, where μ_k is coefficient of kinetic friction.

The maximum value of static friction

The maximum value of static friction is known as limiting friction and it is the frictional force between the surfaces when the body just begins to slide. Static friction is less than or equal to the product of coefficient of static friction (μs) and the normal force (F_N).

$$F_k \leq \mu_s F^N$$

The maximum value being Fs = μsFN

Once the static friction gives way to kinetic friction, i.e., the block is in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force.

Kinetic friction is less than static friction.

$$\vec{F}_k < \vec{F}_s$$

$$and\ \mu_s < \mu_k$$

Note that

Unlike the kinetic friction force, the static friction force takes on any value between zero and its maximum value of $\mu_s F_N$, depending on the magnitude of the applied force. It could take any value between zero to its limiting value.

$$0 \leq F_s \leq \mu_s F_N$$

Group work

Consider a block of wood placed at rest on a horizontal laboratory table and that a horizontal force is applied on the block to get it moving along the surface. The applied force versus frictional force is plotted as in Figure 4.20. Discuss the graph with your classmates.

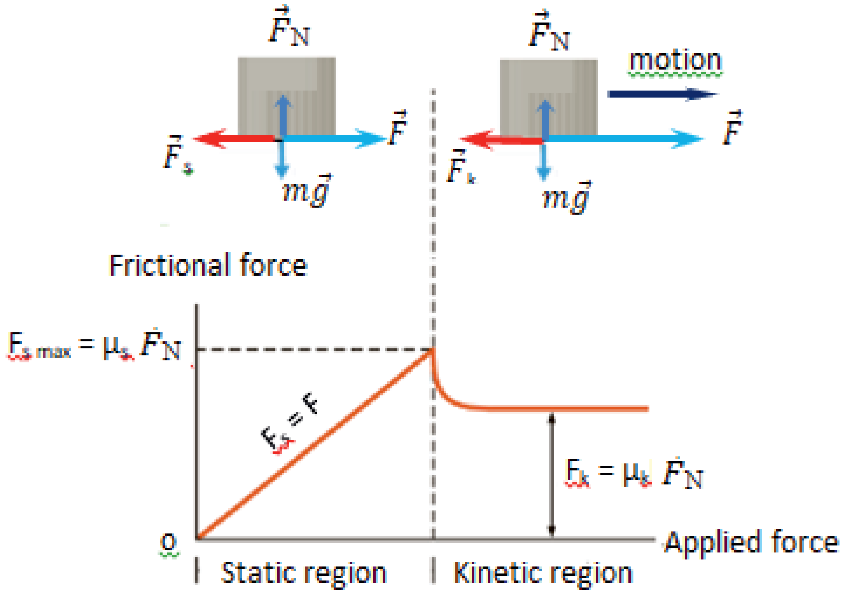


Figure 4.20 Plotting frictional force versus applied force

Activity 4.7

Relating static friction and normal force

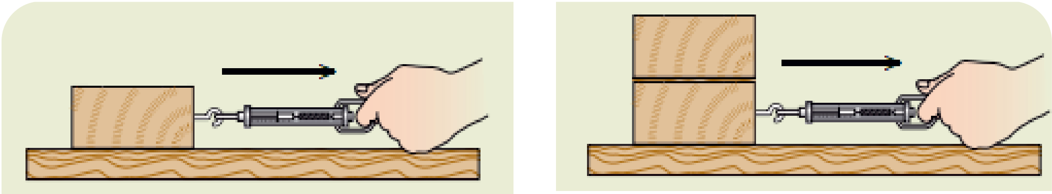
Materials: Four identical blocks of known mass

- A horizontal board
- A spring scale
- A graph paper

Procedure

1. Place one of the blocks on the board.
2. Attach the spring scale to the block, Figure 4.21a. Pull the block with an ever-increasing horizontal force until the block just begins to move. Take the reading of the scale.

3. Increase the mass of the system by placing an identical block over the first block, Figure 4.21b. Repeat step 2 and take the reading of the scale.



(a) (b)

Figure 4.21

4. Repeat step 3 two more times first with three blocks and then with four blocks.
5. Prepare a table and fill in the scale readings and the corresponding masses used.
6. On a graph paper draw a graph of the reading of the scale versus the weight. Draw a line of best fit and calculate the slope of the graph.

Question: What is the relationship between the maximum magnitude of the force of static friction and the magnitude of the normal force? Write this as a proportionality statement.

Example 4.3

An 8 kg block is placed on a horizontal surface. The coefficient of static friction and that of kinetic friction between the block and surface are 0.4, and 0.35, respectively.

A) What is the horizontal force just enough to start moving the block?

B) What horizontal force must be applied on the block to keep it uniformly accelerating at 4 m/s2?

Solution

Given: m = 8 kg, $\mu_s = 0.4$, $\mu_k = 0.3$

A) The block is in a limiting equilibrium, so that the friction force will be $F_s = \mu_s F_N$, but $F_N = mg$ (since the block is on a level surface the normal force is equal to the weight).

Therefore, $F_s = \mu_s mg = (0.4)\,(8\text{ kg})\,(9.8\text{ m/s}^2) = 31.4\text{ N}$

B) As the block accelerates the net force that accelerates the block is the vector sum of the applied force and the kinetic frictional force. That is $F_{net} = F_{app} - F_k$ $F_{net} = ma = (8kg)(4m/s^2) = 32N$, and $F_k = \mu_k mg = (0.3)\,(8\text{ kg})\,(9.8\text{ m/s}^2) = 23.5\text{ N}$

thus, $F_{app} = F_{net} + F_k = 32\text{ N} + 23.5\text{ N} = 55.5\text{ N}$

Table 4.1 Coefficient of static and kinetic friction of some materials

Material	Against Material	Static friction	Kinetic friction
Glass	Glass	0.94	0.4
Rubber	Asphalt (dry)	0.85	0.67
Rubber	Asphalt (wet)		0.58
Rubber	Concrete(dry)	0.90	0.68
Wood	Wood	0.42	0.3
Steel	Aluminum	0.61	0.47
Steel	Copper	0.53	0.36
Steel	Steel	0.74	0.57

Activity 4.8

Group work

Set up an experiment to verify that of smoothness or roughness of a surface affects the frictional fore between objects in contact?

Activity 4.9

In order to minimize accidents, it is advisable to drive slower during rainy season than during dry season. What application of physics can you relate to this traffic advice?

Activity 4.10

Interleave your books to each other until they reach to approximately 3cm in thickness as shown in Figure 4.22. Try to separate the books after interleaving by pulling them apart. Why do you think it is very hard to separate the books by pulling?

Figure 4.22 Interleaving the pages of two books

Did you know?

Natural lubricants include saliva produced in our mouth to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves.

Example 4.4

A 6 kg block is placed on a ramp that makes an angle of 30° above the horizontal, Figure 4.23a. If the block remains at rest,

A) what is the static friction that supports the block from sliding down the ramp?

B) what is the coefficient of static friction between the block and the ramp?

Figure 4.23 A block on a ramp

As shown in Figure 4.23b, the force that tends to move the block down the ramp is mg sinθ and the one that prevented sliding is static friction F_s. The normal force between the ramp and the block is F_N = mg cosθ.

A) Under the condition where the block is at rest we write,

$$F_{net} = 0$$

$$mg\sin\theta - F_s = 0$$

$$F_s = mg\sin\theta = (6\text{ kg})(9.8\text{ m/s}^2)(\sin 30^\circ) = 29.4\text{ N}$$

B) $F_s = \mu_s F_N = \mu_s mg\cos\theta$

$$\mu_s = \frac{F_s}{mg\cos\theta} = \frac{29.4\text{N}}{(6\text{kg})(9.8\text{m}/\text{s}^2)(\cos 30^\circ)} = 0.58$$

See that $mg\sin\theta - F_s = 0$, we have $mg\sin\theta - \mu_s mg\cos\theta = 0$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta = \tan 30^\circ = 0.58$$

Example 4.5

A 6 kg block is placed on a ramp that makes an angle of 30° above the horizontal, Figure 4.23a. If the block remains at rest,

A) what is the static friction that supports the block from sliding down the ramp?

B) what is the coefficient of static friction between the block and the ramp?

Some applications of Newton's Laws of motion

A free-body diagram

In order to easily understand and apply Newton's laws to solve practical problems we need to know the use of a free-body diagram. A free-body diagram is a graphic, dematerialized, symbolic representation of the body. In a free-body diagram, the size of the arrow denotes the magnitude of the force. While the direction of the arrow denotes the direction in which the force acts, Figure 4.24. For the block of example 4.4, what acceleration would the block attain once it is set in motion down the plane? $(\text{Take } \mu_k = 0.5)$

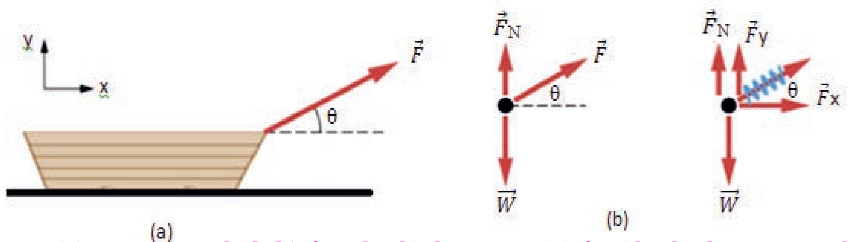


Figure 4.24 (a) A moving sled (b) free-body diagram (c) free-body diagram with force components

Solution:

When the block accelerates down the plane, we write

$$F_{net} = ma$$
$$mg\sin\theta - F_k = ma$$
$$mg\sin\theta - \mu_k mg\cos\theta = ma$$
$$mg(\sin\theta - \mu_k\cos\theta) = ma$$
$$a = g(\sin\theta - \mu_k\cos\theta)$$
$$a = (10 m/s^2)(\sin 30° - 0.5\cos 30°) = 0.67 m/s^2$$

To draw a free-body diagram, we draw the object of interest, draw all forces acting on the object, and resolve all force vectors into x and y components as required. We must draw a separate free-body diagram for each object in the problem. There are many advantages of drawing free-body diagram. It is a medium to convert ideas into a physical model. It also helps in understanding all facts of a problem. It assists in building up suitable mathematical equations between various forces acting upon link.

Key term

Free-Body Diagram: The most important step in solving a problem by means of Newton's second law. It is a drawing that includes only those forces that act directly on the object of interest

Example 4.6

Two blocks of identical materials are connected by a light string on a level surface (Figure 4.25). Assuming no friction between the blocks and the level surface, find the acceleration of the masses and the tension in the connecting string when the string attached to m_1 is pulled to the right by a 36 N force. Use m_1 = 4 kg and m_2 = 8 kg.

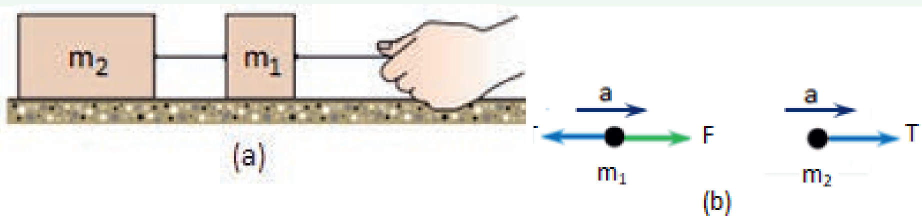


Figure 4.25

Solution:

Given: m_1 = 4 kg, m_2 = 8 kg, F = 36 N

The blocks move together to the right under the action of force F. Writing Newton's second law for each block we have

For m_1, $F - T = m_1a$

For m_2, $T = m_2a$

Adding the above equations we get

$$F = m_1a + m_2a = (m_1 + m_2)a$$

$$a = \frac{F}{m_1 + m_2}$$

Substituting the given values we obtain a = 3 m/s^2.

Using a 3 m/s^2 in $T = m_2a$ we get T = 24 N

Example 4.7

For the system of masses in Example 4.6, find the acceleration of the masses if the coefficient of kinetic friction between each of the blocks and the level surface is 0.25.

Solution

given: $m_1 = 4$ kg, $m_2 = 8$ kg, F = 36 N, $\mu_k = 0.25$
This time the free-body diagram will include frictional forces on the blocks as in Figure 4.26.

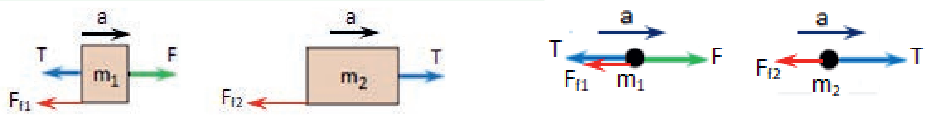


Figure 4.26 Free-body diagram

Writing Newton's second law of motion for each of the blocks as:

$$\text{For } m_1, F - T - F_{f1} = m_1 a$$

$$\text{For } m_2, F - T - F_{f2} = m_2 a$$

Adding these two equations, we get

$$F - F_{f1} - F_{f2} = (m_1 + m_2)a$$

$$F_{f1} = \mu_k N_1 = \mu_k m_1 g = (0.25)(4kg)(9.8m/s^2) = 9.8m/s^2$$

$$F_{f2} = \mu_k N_2 = \mu_k m_2 g = (0.25)(8kg)(9.8m/s^2) = 19.6m/s^2$$

Substituting values, we obtain

$$a = \frac{F - F_{f1} - F_{f2}}{m_1 + m_2} = 0.55\,m/s^2$$

Example 4.8

Two masses ($m_1 = 4$kg and $m_2 = 6$kg), are connected by a light cord that passes over a massless, frictionless pulley as shown in Figure 4.27a. This arrangement is called Atwood's machine and sometimes is used to measure the acceleration due to gravity. Find the magnitude of acceleration of the two masses and the tension in the cord.

Solution:

We construct a free-body diagram for the two masses as shown in Figure 4.27b, and we identify the forces acting on each of the masses separately. m_1 is under the action of the tension force on the string and its own weight while m_2 is under the action of the tension force and its weight.

When the system is released, m_1 moves upward while m_2 moves downward. Newton's second law is applied to m_1 in part (b) of the figure, we find:

$$F_{net} = m_1 a$$
$$T - m_1 g = m_1 a$$

Also, we do the same for m_2 of part (c) of the figure, to get:

$$F_{net} = m_2 a$$
$$m_2 g - T = m_1 a$$

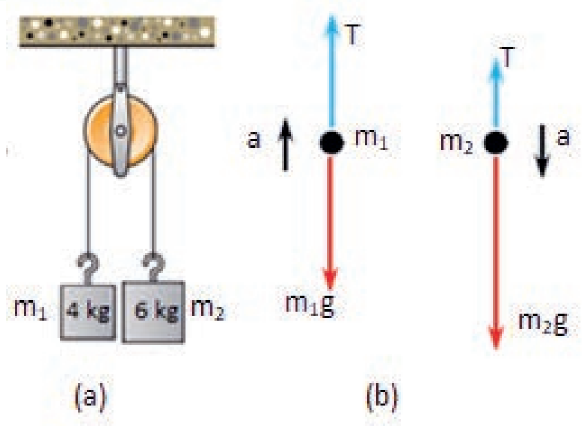


Figure 4.27: a) System of masses $m_2 > m_1$ attached by a rope over a frictionless pulley, b)free-body diagram and direction of acceleration for m_1 c) free body diagram and the direction of the acceleration for m_2

In the above two equations we have two unknowns the acceleration and the tension. Adding the equations $T - m_1 g = m_1 a$ and $m_2 g - T = m_2 a$, T will be cancelled out and we get:

$$m_2 g - m_1 g = m_2 a + m_1 a$$

$$\Rightarrow g(m_2 - m_1) = (m_1 + m_2)a$$

The acceleration is therefore,

$$a = g\frac{m_2 - m_1}{m_1 + m_2}$$

m_2 *and* m_1 in the equation we get $a = 9.8m/s^2\left(\dfrac{6kg - 4kg}{6kg + 4kg}\right) = 1.96\,m/s^2$

Substituting this value of acceleration into one of the equations $T - m_1 g = m_1 a$ we get

$$T = m_1 g + m_1 a = m_1(g + a) = 4kg(9.8m/s^2 + 1.96m/s^2) = 47N\cdot$$

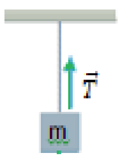
Tension Forces

When an object attached to a string (or a cable) is pulled by means of a pulling force exerted on the string, the force on the string is called a tension force. The tension force acts along the direction of the string and exerts a force both on the object and on the person exerting force on the cable, as illustrated in Figure 4.28. If the mass of the given cable is neglected, the same tension acts on both the object and the person

Figure 4.28 A man pulling a block

Vertical Tension force on a static object

The mass in Figure 4.29 is suspended from a vertical string. As the mass is not moving it is in equilibrium and its acceleration is zero. Applying Newton's second law along the y direction we write

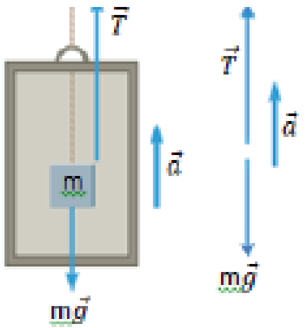


$$F_{net} = ma$$
$$T - mg = ma$$
$$T - mg = 0$$

Figure 4.29 The tension in the string supports the weight

Vertical tension force on an accelerating mass

The string attached to an object is pulled upward under such tension that it accelerates the object upward (Figure 4.30). In this case the tension supports the weight and also accelerates the object upward. Applying Newton's second law along the y axis we write



$$F_{net} = ma$$
$$T - mg = ma$$
$$T - mg = 0$$
$$T = m(g + a)$$

Figure 4.30 An object hangs by a string in an elevator accelerating upward

Exercise 4.2

What would the tension in the string be if the elevator of Figure 4.30

(a) is at rest? (b) accelerates downward?

(c) accelerates upward? (d) decelerates downward?

Review Questions 4.2

1. Walking on horizontal slippery ice can be much more tiring than walking on ordinary pavement. Why?
2. You often hear people say that "friction always opposes motion." Give at least one example where (a) static friction causes motion, and (b) kinetic friction causes motion
3. If you push on a heavy box that is at rest, you must exert some force to start its motion. Once the box is sliding, why does a smaller force maintain its motion?
4. A 25 kg block is initially at rest on a horizontal surface. A horizontal force of 75 N is required to set the block in motion. After it is in motion, a horizontal force of 60 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.
5. If a car is traveling westward with a constant speed of 20 m/s, what is the resultant force acting on it?
6. What will happen to the tension in the string of Figure 4.30, if the cables of the elevator suddenly break and the elevator falls freely?
7. A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator. What can you say about the motion of the elevator if the reading of the scale is (a) equal to mg? (b) greater that mg? (c) less than mg?
8. A block is placed on a smooth inclined plane shown in Figure 4.31. What

force parallel to the surface of the inclined plane is require to move the block up the plane at (a) a constant speed? (b) a constant acceleration of 0.5 m/s2?

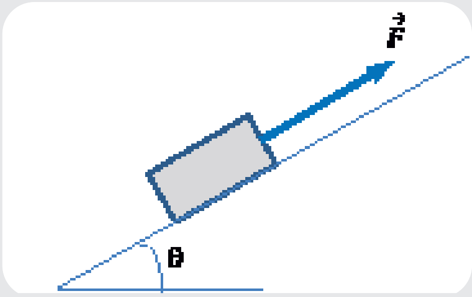


Figure 4.31 A mass on an inclined surface

9. Repeat question 8 if the coefficient of kinetic friction between the block and the inclined surface is 0.25.
10. A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in Figure 4.32. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects in terms of m_1, m_2, g and θ

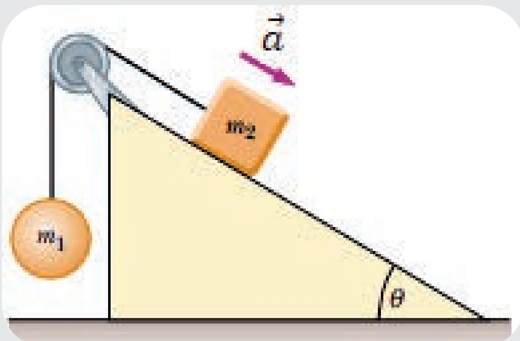


Figure 4.32

11. The driver of a speeding empty truck slams on the brakes and skids to a stop through a distance d. (a) If the truck carried a load that doubled its mass, what would be the truck's skidding distance? (b) If the initial speed of the truck were halved, what would be the truck's skidding distance?

4.3 The First Condition of Equilibrium

At the end of completion of this section you should be able to:

- *define the term equilibrium*
- *explain the first condition of equilibrium*
- *apply the first condition of equilibrium to solve practical problems.*

Brainstorming

What will happen to an object when it is under the action of balanced set of forces?

If an object is either at rest or moves uniformly including translational (linear) as well as rotational motion it is said to be in equilibrium. In this section we shall focus on one dimensional motion. If a body is at rest or moves at a constant speed in a fixed direction (constant velocity) then it is in translational equilibrium. This statement defines the first condition of equilibrium.

If an object is in equilibrium, its acceleration is zero and according to Newton's second law of motion, the vector sum of all the forces acting on it must be zero.

$$\sum \vec{F} = 0 \quad \text{(The first condition of equilibrium)}$$

For n number of forces acting on a object, we write

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n = 0$$

This equation can be written component wise as

$$\sum \vec{F}_x = 0 \text{ and } \sum \vec{F}_y = 0$$

Key term:

Equilibrium: a word that describes a state of rest or uniform motion along a fixed direction of a body.

Example 4.9

A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support, as in Figure 4.33a. The upper cables make angles of 37° and 53° with the horizontal. Find the tension in each of the three cables.

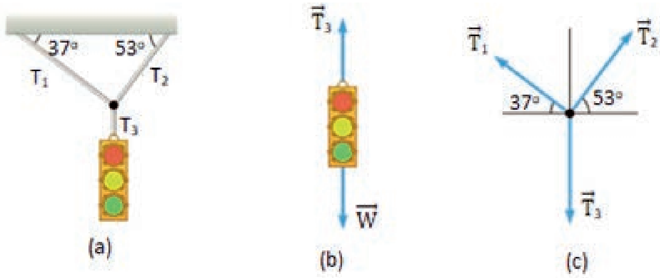


Figure 4.33 A traffic light suspended from two strings Solution

Solution

Drawing a free-body diagram is highly helpful in solving problems involving Newton's laws. As shown in the free-body diagram, the traffic light is under the action of three tension forces, $\vec{T}_1$, $\vec{T}_2$ and $\vec{T}_3$. For it is in equilibrium, we write

$$\sum \vec{F} = 0$$

$$\vec{T}_1 + \vec{T}_2 + \vec{T}_3 = 0$$

But from Figure b, $\vec{T}_3 = \vec{W}$ and we have $\vec{T}_1 + \vec{T}_2 + \vec{W} = 0$

Breaking the forces into their components and applying the first condition of equilibrium we have

$$\sum \vec{F}_x = 0, T_{2x} - T_{1x} = 0 \quad T_2 \cos 53^\circ - T_1 \cos 37^\circ = 0 \quad 0.6T_2 - 0.8T_1 = 0$$

$$\sum \vec{F}_y = 0, T_{2y} - T_{1y} - W = 0 \quad T_2 \sin 53^\circ - T_2 \sin 37^\circ = W \quad 0.8T_2 - 0.6T_1 = 100N$$

Solving for T_1 *and* T_2 *we get* $T_1 = 60N$ *and* $T_2 = 80N$

Activity 4.11

If an object is at rest, it is necessarily in equilibrium. Is this statement correct? Discuss your answer.

Review questions 4.3

1. If only one force acts on an object, can it be in equilibrium? Explain.
2. If an object is in equilibrium, which of the following statements is not true?
   (a) The speed of the object remains constant
   (b) The acceleration of the object is zero.
   (c) The net force acting on the object is zero.
   (d) The object must be at rest.
   (e) The velocity is constant.

3. What is wrong with the statement “Because the car is at rest, there are no forces acting on it”? How would you correct this sentence?
4. Can an object be in equilibrium if it is in motion? Explain.
5. Mass m= 10kg is supported by two strings is in equilibrium as shown in Figure 4.34. Find the tension in the horizontal string. Take $\theta = 53^{o}$

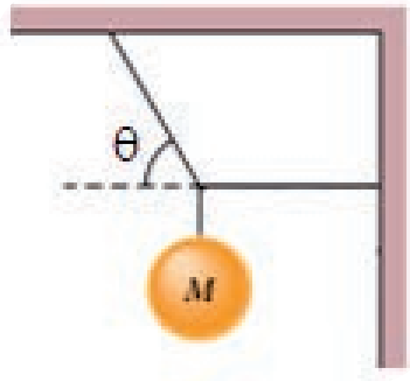


Figure 4.34 A mass in equilibrium

4.4 Work, Energy and Power

By the end of this section, you will be able to:

- *define work in terms of the applied force and the displacement.*
- *calculate the work done by a constant force, by a varying force.*
- *demonstrate by examples that maximum work is done when the applied force is parallel to the displacement vector.*
- *apply work-energy theorem to solve problems.*
- *calculate kinetic energy and potential energy of a body.*
- *differentiate between conservative and non-conservative forces.*
- *explain the law of the conservation of energy.*
- *solve problems by applying the law of conservation of energy.*
- *define power and solve problems involving power.*

? Brainstorming

What do you mean by the term work in your everyday language? When do you consider that you did some work?

Work, in general, is the means of transferring energy from one body to another or a means of transforming energy from one form to another. For instance, if you raise a load from the floor to the top of a shelf, energy is transferred from your muscles to the load and in the process chemical energy is converted into potential energy of the load. Energy, which is defined as the capacity to do work, has important features: it exists in different forms and it is converted from one form to another with the total amount being constant. It cannot be created or destroyed. In an automobile engine, chemical energy stored in the fuel is converted partially to the energy of the automobile's motion and partially to thermal energy. In a microwave oven, electromagnetic energy obtained from the mains is converted to thermal energy of the food being cooked. In these and all other processes, the total energy—the sum of all energy present in all different forms—remains the same. No exception has ever been found.

Work done by a constant force

? Brainstorming

If your friend carried your 35 N bag and walked horizontally through 10 m, did she do some work for you? How about carrying the same bag upstairs to the second floor?

Work has a different meaning in physics than it does in everyday usage. In physics doing work involves applying a force to an object while moving it a given distance. In other words, work is done when a force acts on something that undergoes a displacement from one position to another.

The definition for work W might be taken as W = F S, where F is the magnitude of the force acting on the object and S is the magnitude of the object's displacement. That definition, however, gives only the magnitude of work done on an object when the force is constant and parallel to the displacement, which must be along a line, Figure 4.35. For a more general case work is defined as the scalar product of the force and the displacement produced.

$$W = \vec{F}.\vec{S}$$

$$W = F.S\cos\theta,\ where\ \theta\ is\ the\ angle\ between\ \vec{F}\ and\ \vec{S}$$

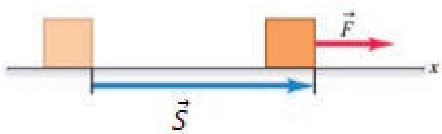


Figure 4.35 A constant force moves the mass through displacement ,

W = F.S cos (0°) = F S

For a force $\vec{F}$ acting at an angle, Figure 4.36, the work done is W = F S $\cos\theta$, which can also be written as

$$W = (F\cos\theta)\,S$$

where $F\cos\theta$ is the component of the force along the direction of the displacement. See that the component of the force along the direction perpendicular to the displacement ($F\sin\theta$) does no work (for $\theta = 90^o$, $\cos 90^o = 0$).

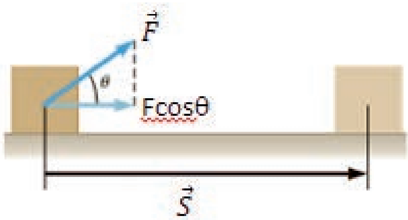


Figure 4.36 Force at angle, W = (F cos θ) S

The SI unit of work is joule, abbreviated as J.

$$1 \text{ joule} = 1 \text{newton.meter}$$

Other unit of work in the (centimeter-gram-second) cgs unit is erg where 1 erg = 1dyne. cm.

$$1\text{joule} = 10^7\text{erg}$$

Example 4.10

What is the work done on a block of mass 10kg, when a force of 20 N 30^0 above the horizontal displaces the block by 5m?

Solution:

The givens are F = 20 N, $\theta = 30^0$ and S = 5m.

We are required to find the work done.

$$W = FS\cos\theta = (20\text{N})\,(5\text{m})\,(\cos 30^0) = (100\text{ Nm})\,(0.866) = 86.6\text{ J}$$

Exercise 4.3

A man pushes a stalled vehicle with a steady force $= (150\hat{i} - 40\hat{j})$ N and the displacement of the car is $= (14\hat{i} + 11\hat{j})$ m. How much work does the man do on the car?

Activity 4.12

Can work done be positive, negative or zero? Specify the conditions in which work can be positive, negative or zero.

Example 4.11

A man pulls a package along a horizontal floor with a force of 400 N at an angle of 30° as shown in Figure 4.37a. A constant frictional force of 100 N acts between the package and the floor. Find the work done by each force acting on the sled and the total work done by all the forces.

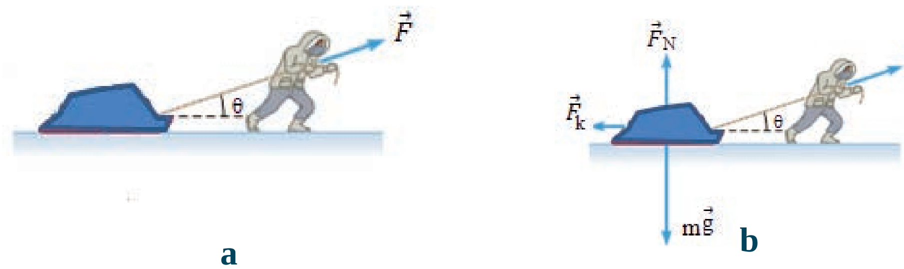


Figure 4.37

Solution

The forces that are acting on the package are shown in Figure 4.37b.

Work done by the applied force

$$W_F = F\,S\cos 30^{\circ} = (400\text{ N})\,(10\text{ m})\,(0.866) = 3464\text{ J}$$

Work done by frictional force

$$W_f = F_k\,S\cos(180^{\circ}) = (100\text{ N})\,(10\text{ m})\,(-1) = -1000\text{ J}$$

The gravitational force (Fg = mg) and the normal force FN are both perpendicular to the displacement and therefore,

$$Wg = WFN = 0.$$

The total work Wtot done on the package by all forces is the algebraic sum of the work done by the individual forces.

$$W_{tot} = W_F + W_f + W_g + WF_N = 3464\text{ J} + (-1000\text{ J}) + 0 + 0 = 2464\text{ J}$$

Activity 4.13

1. Based on your knowledge of dynamics determine the net force acting on the package of Example above to show that the total work done on the package is also equal to the net force on the package times the displacement.

$$W_{tot} = F_{net}\,S$$

Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 4.38 shows an object of mass m that is thrown upward with initial speed v_o and thus with initial kinetic energy $K_o = ½ mv_o^2$. As the object rises, it is slowed down by a gravitational force F_g; that is, the object's initial kinetic energy decreases because F_g does work on the object as it rises. The work done by the gravitational force is

$$W_g = F_g d \cos\theta$$

The force magnitude is mg and we write

$$W_g = mg\,d \cos\theta \text{ (work done by gravitational force)}$$

With $\theta = 180^o$, work done by gravitational force is

$$W_g = mgd$$

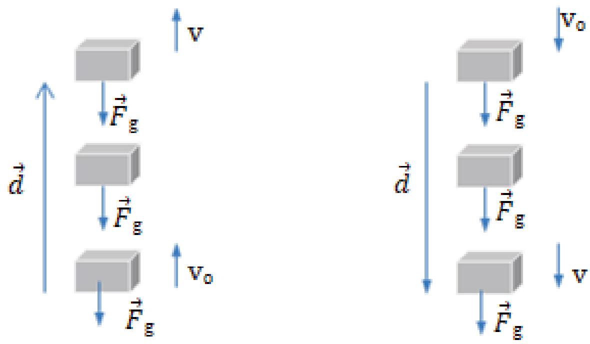


Figure 4.38 Work done by gravitational force: (a) A mass raised through vertical distance d (b) a mass lowered through vertical distance d

The minus sign tells us that during the object's rise, the gravitational force acting on the object transfers energy equal to mgd from the initial kinetic energy that it had at the beginning of its motion.

After the object has reached its maximum height and is falling back down, the angle θ between force and displacement is zero. Thus, the work done by the gravitational force will be $W_g = mgd \cos0^o = + mgd$

The plus sign tells us that the gravitational force now transfers energy in the amount mgd to the kinetic energy of the object.

Example 4.12

A 5 kg bag is raised to a height of 2.5 m above the ground. What is the work done by the applied force and by the gravitational force?

Solution

The work done by the applied force is

$$W_f = F_f\, S \cos 0^o = F_g S$$

The force required to raise the bag is the weight

$$W = mg.$$

Taking vertical distance as height h, we write $W_F = mgh$

$$= (5\text{ kg})\,(9.8\text{ m/s}^2)(2.5\text{ m}) = 122.5\text{ J}$$

The work done by gravitational force is

$$W_g = F_g\, S \cos 180^o = -\,mgh = -\,122.5\text{ J}$$

How would the answer change if the bag were lowered from the same height to the ground?

Example 4.13

Consider a block of mass m is pushed up a rough inclined plane of angle θ by a constant Force F parallel to the incline, as shown in Figure 4.39. The displacement of the block up the incline is d.

a. by the applied force
b. by the force of gravity
c. by the normal force
d. by the kinetic friction

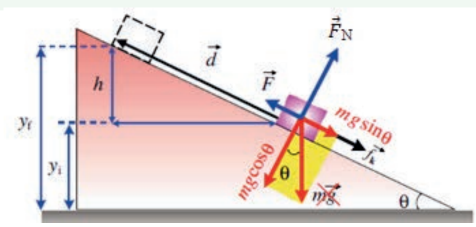


Figure 4.39 A block is pushed up a rough inclined plane with a constant force F parallel to the incline. The normal is perpendicular to the surface. A mark is on mg to show that the weight is decomposed to its parallel and perpendicular components.

Solution:

(a) The work done by the applied force W_F

$$W_F = F\,S \cos 0^o = F\,d \quad \text{(since F is in the same direction as d)}$$

(b) The work done by gravity W_g is: $W_g = (mg \sin\theta)(S)\cos 180^o = -mgd\cos\theta$, ($mg \sin\theta$ is the component of the weight parallel to the displacement and 180^0 is the angle between $mg \sin\theta$ and the displacement)

(c) The work done by the normal force is:

$W_N = F_N S \cos 90^o = 0$ (The normal force is perpendicular to the displacement)

(d) The work done by the kinetic friction is:

$W_f = F_k S \cos 180^o = \mu_k(mg\cos\theta)(d)(-1) = -\mu_k mgd\cos\theta$

Activity 4.14

Suppose you apply a constant force F on a mass to displace the mass through distance S. If you plot the Force-Displacement graph and determine the area of the region enclosed by the graph, what quantity would you obtain?

Work done by a variable force

In general, a force that acts on a body may vary in magnitude and direction as it displaces the body between two points in space. Let us consider a varying force F acting on a body to displace the body along the x axis. In this case using the expression W = F.S will not be appropriate. Instead we calculate the infinitesimal work done by the variable force and add those values to get the total work done. Figure 4.40a, shows an example of the F vs S curve of the case of a variable force acting to produce displacement S.

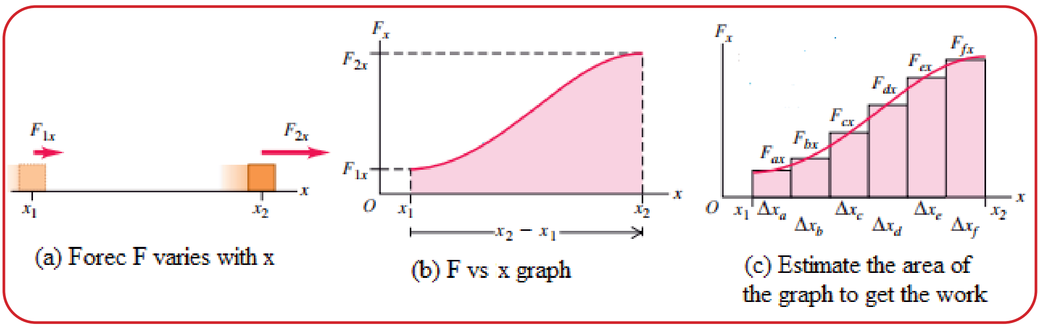


Figure 4.40 Work done by a varying forces = Area under F vs S graph

Key terms

- *Constant force: A force that does not vary with time or position*
- *Variable force: A force that varies with time or position*

Activity 4.15

One of the common examples where varying force acts on a body is the case of a force applied on a spring. Show that the work required to stretch a spring (spring constant k) from initial elongation x_1 to a final elongation x_2 is $W = ½\, k\,(x_2^{\,2} - x_1^{\,2})$.

Activity 4.16

Group work

- Work done by a varying force
- Aim: To determine the work done by a varying force.
- Apparatus:
- Clamp Stand
- Helical Spring of force constant 100 N/m
- Mass Hanger
- Pointer
- Meter Ruler
- Five equal masses each weighing 5 N

Figure 4.41 Setup of apparatus

Method:

(a) Hang a helical spring from a clamp stand, Figure 4.41.

(b) Attach a mass directly to the bottom of the helical spring and record the position of the bottom of the mass hanger relative to a meter ruler. Take that reading as x_1.

(c) Add the second mass to the first and record the new position of the mass hanger relative to the meter rule as x_2.

(d) Repeat step (c) for the third, fourth and fifth masses to the spring and record the position of the bottom of the mass hanger, as x_3, x_4, and x_5, Table 4.2

Table 4.2

F (N)	F_1 =	F_2 =	F_3 =	F_4 =	F_5 =
x (m)	x_1 =	x_2 =	x_3 =	x_4 =	x_5 =

Questions:

(1) Plot the Force (weight) versus extension (elongation) x.

(2) What quantity do you get from the slope of the graph?

Note that according to Hooke's law, the extension of the force is directly proportional to the force applied, $F = k\,x$, where k is the spring constant).

(3) What is the area of the graph?

(4) Compare the result you obtained in step (3) with the result you get by the doing product ½ kx2, which is equal to the work done on the spring.

Kinetic energy

One of the common types of energy that we encounter in our daily activities is kinetic energy. Kinetic energy is the energy a body possesses due to its motion. A body of mass m moving with velocity v is said to have kinetic energy of KE = ½ mv2.

Example 4.14

A 1200 kg car traveling at 30 m/s has a kinetic energy of

$$KE = \tfrac{1}{2}\,(1200\text{ kg})\,(30\text{ m/s})^2 = 540\text{ kJ}$$

Kinetic energy is a scalar quantity.

The work -energy Theorem

The total work done on a body by external forces is related to the body's displacement—that is, to changes in its position. But the total work is also related to changes in the speed of the body.

Consider a particle of mass m, moving with an acceleration a under the effect of a net constant force F along the x axis, Figure 4.42.

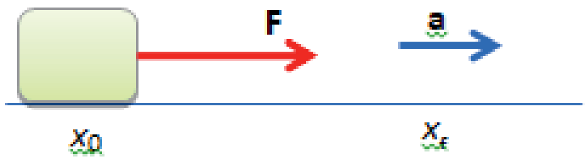


Figure 4.42: The work done by the applied force as it displaces the mass from x_0 to x_f

The work done by the applied force is

$$W = F(x - x_0)$$

We know that the effect of the force in moving an object is the acceleration, thus:

$$W = ma(x - x_0)$$

From kinematic equations, the relation of the acceleration can be expressed in terms of the displacement and the velocity as:

$$v_f^2 - v_i^2 = 2a(x - x_i) \Rightarrow a = \frac{v_f^2 - v_i^2}{2(x - x_i)}$$

Substituting this value of the acceleration into the above equation, the work done will be

$$W = m\left(\frac{v_f^2 - v_i^2}{2(x - x_i)}\right)(x - x_i)$$

Simplifying and rearranging we get

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity $\frac{1}{2}mv^2$ is called the kinetic energy of the moving object having mass m and velocity v denoted by K with SI unit joule (kg.m2/s2).

$$K = \frac{1}{2}mv^2 \quad \text{(Kinetic energy of a moving object)}$$

Therefore, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ is the change in kinetic energy of the moving object as its velocity is changed from vi to vf. Therefore, the work done by a net force is equal to the change in kinetic energy. This principle is called the work-energy theorem. Mathematically,

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = K_f - K_i = \Delta K \qquad \textbf{(The work-energy theorem)}$$

That is, the work done by the net force in displacing a particle equals the change in its kinetic energy.

Potential energy

The energy of a body due to its relative position or shape is known as potential energy

The energy due to the relative position of a body is called Gravitational potential energy while the potential energy due to change in shape of the body is called Elastic potential energy.

Gravitational potential energy of a body of mass m placed at a height above a reference level is given by PE = mgh.

Elastic potential energy of a body such as a spring that is stretched through x is given by

$$\mathbf{PE = ½\, kx^2}$$

The sum of the kinetic energy and potential energy of a body is termed as the total mechanical energy of the body.

$$\mathbf{ME = KE + PE}$$

Suppose you throw a ball vertically upward. We already discussed that as the ball rises, the work W_g done on the ball by the gravitational force is negative because the force transfers energy from the kinetic energy of the ball. We say that this energy is transferred by the gravitational force to the gravitational potential energy of the ball–Earth system.

On its way up, the ball slows (decelerates) and finally stops, and then begins to fall back down because of the gravitational force. During the fall, the transfer is reversed: The work Wg done on the body by the gravitational force is now positive—that force transfers energy from the gravitational potential energy of the ball–Earth system to the kinetic energy of the ball.

For either rise or fall, the change U in gravitational potential energy is defined as being equal to the negative of the work done on the ball by the gravitational for ce.

$$\mathbf{U = -W}$$

Key terms

- *Kinetic energy: Energy due to motion*
- *Potential energy: Energy due to position or shape*

The above conclusion can also apply to a block-spring system shown in Figure 4.43. Initially the block moving to the right hits the spring. The spring force acts toward left causing deceleration of the block doing negative work on the block and transferring energy from the kinetic energy of the block to the elastic potential energy of the spring–block system. The block then begins to move leftward because the spring force is still leftward. The transfer of energy is then reversed—it is from potential energy of the spring–block system to kinetic energy of the block.

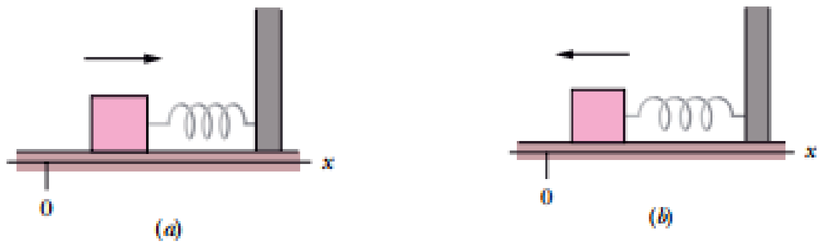


Figure 4.43 (a) As the block moves to the right the spring force does negative work on it (b) As the block moves backward, the spring force does positive work on it.

Conservative and non-conservative Forces

There are generally two kinds of forces namely, Conservative forces and non-conservative forces.

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical).

Gravitational force is an example of a conservative force, and the force that a spring exerts on any object attached to the spring is also conservative.

A force is non-conservative if it does not satisfy properties 1 and 2 for conservative forces. Non-conservative forces acting within a system cause a change in the mechanical energy ME of the system. The work done on a system by a non-conservative force is equal to the change in mechanical energy of the system. Typical example of non-conservative force (also known as dissipative force) is frictional force. W_d is work done by dissipative forces.

$$W_d = \Delta ME, W_d$$

$$f_k d = \Delta KE + \Delta PE$$

Where f_k is frictional force, d is the distance through which frictional force acted.

$$f_k d = (KE_f - KE_i) + (PE_f - PE_i)$$

Where KE = ½ mv^2 , gravitational PE = mgh, and Elastic PE = ½ kx^2

Key terms

- *Conservative force: A force in which work done is independent of the path followed.*
- *Non-conservative force: A force in which work done is path dependent.*

Power

In the previous section we have seen that the quantity of work has to do with a force causing a displacement. Work has nothing to do with the amount of time that this force acts to cause the displacement. Sometimes, the work is done very quickly and other times the work is done rather slowly. If you dig a well using a mattock, it may take several hours. The same amount of work can be done in very few minutes if it is dug by an excavator machine (Figure 4.44). The excavator can complete the work in a shorter time than the same work done by the mattock digger.

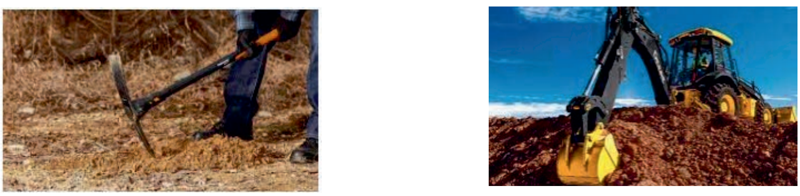


Figure 4.44 A ditch dug by a mattock (less powerful machine) and by an excavator (a very powerful machine)

We express the relation between work done and the time interval involved in doing it, by introducing the concept of power. Since work can vary as a function of time, we first define average power as the work done during a time interval, as:

$$Average\ power = \frac{Work\ done}{Time\ taken}$$

$$Sybmolically\ P_w = \frac{w}{t}$$

The SI unit of power is watt. 1 W = 1 J/s

Metric prefix multipliers of units of power are usually used Such as kilo watt (kW) in rating our home electric stove , mega watt (MW) and giga watt (GW) in rating power generating plants such as the Great Ethiopian Rainnasence Dam (GERD).

Key term

Power : *the rate of conversion of energy or the rate of doing work*

All machines are typically described by their power rating. The power rating indicates the rate at which that machine can do work upon other objects. Thus, the power of a machine is the work to time ratio for that particular machine.
Another non SI unit of power is the horsepower.

1 horsepower = 746 watts

Exercise 4.4

Consider a boy and a girl of equal mass of 50 kg run upstairs to the fourth floor to a height of 20 m above the ground floor.

(a) Compare the work done by each of them.
(b) Compare the power developed by each of them.

Activity 4.17

Discuss

Does the work done in lifting an object depend on how fast it is lifted? Does the power expended depend on how fast it is lifted?

Another expression for power

Using the expression work = Force x displacement

$$\text{Power} = \frac{\text{work}}{\text{time}}, \text{ we get}$$

$$\text{Average power} = \frac{\text{Force x displacement}}{\text{time}}$$

$$\text{Average power} = \text{Force x } \frac{\text{displacement}}{\text{time}}$$

$$\text{Average power} = \text{Force x average velocity}$$

$$P_{av} = F x v_{av}$$

The instantanous power can be written in terms of instantaneous velocity as

P = F v, where v is instantaneous velocity

We can also express instantaneous power in terms of the scalar product as

$$P = \vec{F}.\vec{v}$$

Example 4.15

A 1000 kg elevator car carries a maximum load of 800 kg, Figure 4.45a. As the elevator moves upward a constant frictional fore of 4000 N retards its motion. What minimum power must the motor deliver to lift the loaded elevator car at a constant speed of 2.5 m/s?

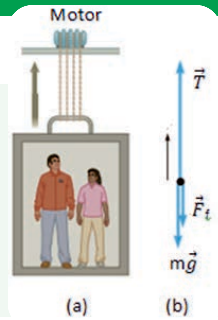


Figure 4.45 Work done on an elevator and its load

As the elevator and its load are moving upward at a constant speed, Newton's second law of motion applied on the system shown in the free-body diagram (Figure 4 45 b), is written as

$$\vec{F}_{net} = m\vec{a} \quad \Rightarrow \vec{T} + m\vec{g} + \vec{F}_t = m\vec{a}$$

T – mg – Ff = ma, for motion at a constant velocity a = 0 and we have,

$$T - mg - Ff = 0$$

$$T = mg + Ff = (1000\text{ kg} + 800\text{ kg})\,(9.8\text{ m/s}^2) + 4000\text{ N} = 21640\text{ N}$$

The power $P = F\,v = (21640\text{ N})\,(2.5\text{ m/s}) = 54100\text{ W} = 54.1\text{ KW}$

Example 4.16

Two forces $\vec{F}_1 = 5N$ and $\vec{F}_2 = 2N$ along the directions indicated are acting on a box that slides horizontally to the right across a frictionless surface, as shown in Figure 4.46. Force F_1 makes an angle $\theta = 60^0$ with the horizontal. Force $\vec{F}_2$ is against the motion. The speed v of the box at a certain instant is 4 m/s. What is the power due to each force that acts on the box at that instant, and what is the net power? Is the net power changing with time?

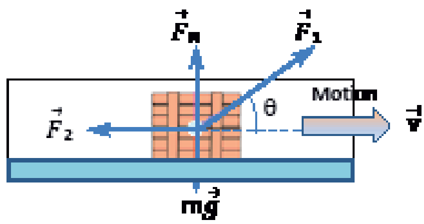


Figure 4.46

Solution:

The weight $m\vec{g}$ and the normal force $\vec{F}_N$ are perpendicular to the velocity v. Thus, their work done is zero, and hence the power due to each of them on the block is zero.

The power delivered by $\vec{F}_1$ is:

$$P_1 = \vec{F}_1.\vec{V} = F_1 \cos 60^\circ = 10W$$

which indicates that the force $\vec{F}_1$ is transferring energy to the box at a rate of 10 J/s.

The power delivered by $\vec{F}_2$ is:

$$P_2 = \vec{F}_2.\vec{V} = F_2 \cos 180^\circ = 8W$$

which indicates that the force $\vec{F}_1$ is transferring energy from the box at a rate of 8 J/s.

The net power is the sum of the individual powers. Thus,

$$P = P_1 + P_2 = 10W + (-8W) = 2W$$

This indicates that the net rate of energy transfer to the box is positive. So, the kinetic energy of the box will increase, and hence its speed. Consequently, the net power will increase with time.

Review questions 4.4

1. Write down two examples in which a force is exerted on an object without doing any work on the object.
2. A bullet has twice the mass of a second bullet. If both are fired so that they have the same speed, which has more kinetic energy? What is the ratio of the kinetic energies of the two bullets?
3. (a) If the speed of a particle is doubled, what happens to its kinetic energy?
   (b) What can be said about the speed of a particle if the net work done on it is zero?
4. Can kinetic energy be negative? Explain.
5. The force acting on a particle varies as in Figure 4.47. Find the work done by the force on the particle as it moves
   (a) from x = 0 to x = 8 m,
   (b) From x = 8 m to x = 10 m, and
   (c) From x = 0 to x =10 m.

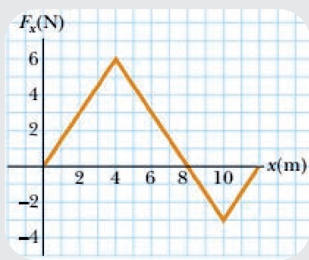


Figure 4.47

6. Elastic potential energy depends on the spring constant and the distance the spring is stretched or compressed. By what factor does the elastic potential energy change if the spring's stretch is (a) doubled or (b) tripled?
7. Calculate the elastic potential energy of a spring with spring constant k = 225 N/m that is (a) compressed and (b) stretched by 1cm.
8. A diver drops from a board 15 m above the surface of water. Neglecting air

resistance, (a) use conservation of mechanical energy to find his speed as he is 5 m above the water's surface, and (b) find his speed as he hits the water.

9. What average power must be supplied to push a 10 kg block from rest to 20 m/s in 5 s when the coefficient of kinetic friction between the block and surface is 0.2? Assume the acceleration is uniform.
10. An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25 m/s in a time of 8 s. What average power must the motor develop to produce this acceleration?

4.5 Conservation of mechanical energy

By the end of this section, you will be able to:

- *define mechanical energy.*
- *state the law of conservation of mechanical energy.*
- *apply the law of conservation of mechanical energy to solve problems*

? Brainstorming

If you drop a ball from the top of the whiteboard, discuss the energy conversion that takes place as the ball falls down.

Suppose a car traveling along the road suddenly brakes to a stop. What kind of energy conversion takes place in the process?

The law of conservation of mechanical energy states that in the absence of dissipative forces like air resistance and friction, the total mechanical energy of an object or system of objects remains unchanged.

$$\Delta ME = 0$$
$$\Delta KE + \Delta PE = 0$$
$$(KE_f\ \ KEi)\ + (PE_f\ \ PE_i) = 0$$
$$KE_i + PE_i = KE_f + PE_f$$

Example 4.17

A 10 kg block is released from rest at the top of a smooth inclined plane 10 m in length, as shown in Figure 4.48. Find the speed of the block as it reaches the bottom of the inclined plane.

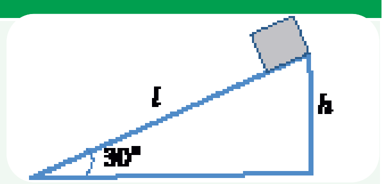


Figure 4.48

Solution

There is no friction between the block and the inclined surface. Applying the law of conservation of mechanical energy we write

$$\Delta KE + \Delta PE = 0$$

$$KE_i + PE_i = KE_f + PE_f$$

The block starts from rest so that $KE_i = 0$, and at the bottom of the incline $PE_f = 0$

$$PE_i = KE_f$$

$$mgh = ½\, mv^2$$

$$v = \sqrt{2gh},\ h = l\sin 30^\circ = (10m)(0.5) = 5m$$

$$Therefore\ v = \sqrt{2(9.8m/s^2(5m)} = 9.9m/s$$

Example 4.18

Repeat the question in Example 4.17 if the inclined plane is rough and the coefficient of kinetic friction between the block and the surface is 0.2.

Solution

In the presence of dissipative forces the work done by these forces equals the change of mechanical energy of the system.

$$Wd = \Delta ME$$

$$-f_k d = (KE_f - KE_i) + (PE_f - PE_i)$$

$$Taking\ KEi = 0,\ \ PEf = 0\ we\ have$$

$$-f_k d = KE_f - PE_i$$

$$-(\mu_k N) \ = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2}mv^2 = mgh - (\mu_k N)\, l$$

$$= \frac{1}{2}mv^2 = mgh\ \ (\mu_k mg\cos(30^\circ))\, l \quad = mg[\, h - (\mu_k \cos(30^\circ)\, l]$$

$$v^2 = 2g[\, h - (\mu k\cos(30^\circ)\, l\,] \ = \ 2g[\, h \ \ (\mu k\cos(30o)$$

$$= 2(9.8\ m/s^2)[\, 5\ m - (0.2)(0.866)(10m)(10\ m)] = 64\ m/s$$

$$v = 8\ m/s$$

Example 4.19

A 2 kg block sliding over a smooth horizontal surface with a constant speed of 8 m/s hits a horizontal spring whose one end is fixed as shown in Figure 4.49. If the maximum compression of the spring just before the block comes to rest is 20 cm, what is the spring constant of the spring?

Figure 4.49

Solution

Given

Speed of the block v = 8 m/s, compression of the spring x = 20 cm = 0.2 m.

Required is spring constant k

Initially the spring has no compression, $x_1 = 0$, $PE_i = 0$ and after being compressed through $x_2 = 0.2$ m, it possesses elastic potential energy $PE_f = \frac{1}{2} kx=_2{}^2 \frac{1}{2} kx^2$

The block has initial $KE_i = \frac{1}{2} mv_i^2 = \frac{1}{2} mv^2$ and it stops after compressing the spring $v_f = 0$ and $KE_f = 0$.

In the absence of dissipative forces we write

$$\Delta KE + \Delta PE = 0$$

$$KE_i + PE_i = KE_f + PE_f$$

$$KE_i = PE_f$$

$$KE_i \text{ of the block } = PE_f \text{ of the spring}$$

$$\frac{1}{2} mv^2 = \frac{1}{2} kx_2^2$$

$$k = \frac{mv^2}{x^2} = \frac{2(8m/s)^2}{(0.2)^2} \; 3200 N/m.$$

Review Questions 4.5

1. A ball is thrown straight up into the air. Where will it attain its minimum kinetic energy? At what position is the gravitational potential energy of the ball–Earth system a maximum?
2. A 1 kg ball is released from the top of an 18 m high tower. Neglecting air resistance, where above the ground will the ball have its kinetic energy twice its potential energy at that location?
3. A particle of mass m = 8 kg is released from point A and slides on the

frictionless track shown in Figure 4.50. Determine

(a) the particle's speed at points B and C and

(b) the net work done by the gravitational force in moving the particle from A to C.

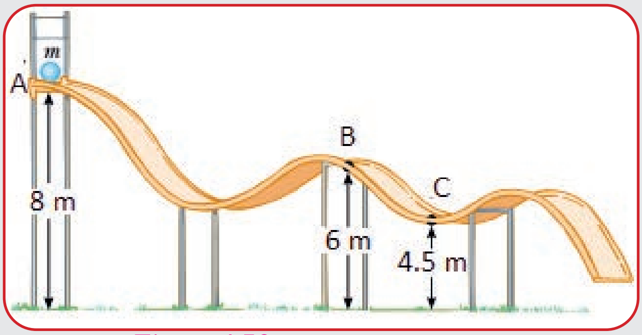


Figure 4.50

4. A 5 kg block is set into motion up an inclined plane shown (Figure 4.51) with initial velocity of 10 m/s. How far up along the plane does the block go before coming to rest assuming?

   (a) no friction between the block and the surface of the inclined plane?

   (b) the coefficient of friction between block and the surface is 0.25?

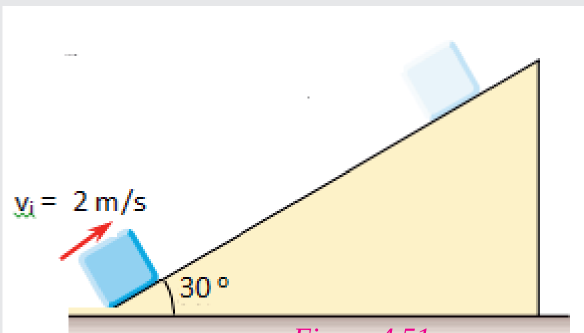


Figure 4.51

5. A 0.50 kg block slides across a tabletop with an initial velocity of 20 cm/s and comes to rest in a distance of 50 cm. Find the average friction force that retarded its motion.
6. Suppose a car traveling at a speed v skids a distance d after its brakes lock. Estimate how far it would skid if it were traveling at speed 2v when its brakes locked.

4.6 Impulse and Linear Momentum

By the end of this section, you will be able to:

- *discuss impulse and linear momentum*
- *relate impulses to collisions*
- *apply the impulse-momentum theorem to solve problems*
- *differentiate elastic and inelastic collisions*
- *state the law of conservation of linear momentum*
- *explain the meaning and usefulness of the concept of center of mass*
- *calculate the center of mass of a given system*
- *calculate the velocity of the center of mass*
- *apply the law of conservation of linear momentum to solve problems involving collision in one dimension.*

? Brainstorming

Suppose you are riding a bicycle at a speed of 15 m/s. How different is the effort you are required to exert when you want to stop in 5 second or when you want to stop in 9 seconds? Give your answer in terms of the force you apply on the brakes and the time taken to stop.

Linear momentum is the property of a moving object that describes how hard it is to set a body in motion or to stop it. It is hard to stop a fast moving object than a slow moving object. Similarly, it is hard to stop a larger mass than a smaller mass provided that they move with the same speed. The Linear momentum of a body is defined as the product of its mass and its velocity.

Mathematically, Momentum = mass x Velocity

$$\text{Linear mumentum denoted by } \vec{P} \text{ and}$$
$$\vec{P} = m\vec{v}$$

Momentum is a vector quantity directed along the direction of the velocity and its SI unit is kgm/s.

Exercise 4.5

How would the momentum of an object change if

a. the mass is doubled but the velocity remains the same?
b. the velocity is reduced to one-third of its original magnitude?
c. the direction of the velocity changes from South to East?

Example 4.20

What is the linear momentum of a 0.5 kg match box that is moving at a constant speed of 6m/s toward the positive x direction?

Solution:

$Given;\ m = 0.5kg \qquad \vec{V} = 6\ m/s + x\ direction$

$Required = ?$

$Using\ the\ equation\ P = m\vec{v}$

The magnitude of the momentum is P = mv = (0.5kg) (6m/s) = 3kgm/s.
The momentum takes the direction of velocity and we have $\vec{p}$ = 3 kg m/s, toward + x direction.

Example 4.21

If the box in the above example is acted by an external force and its velocity is increased from 6m/s to 15m/s along the same direction, what is the change in momentum?

Solution:

$The\ given\ are\ m = 0.5kg,\ v_i = 6m/s,\ v_f = 15m/s$

$Required\ \Delta\vec{P} = ?$

$$\begin{aligned}The\ change\ momentum\ \Delta P &= \Delta(m\vec{v}) \\ &= m(\vec{v}_f - \vec{v}_i) \\ &= (0.5kg)(15m/s\ south - 6m/s\ south) \\ &= 1.5kg\ m/s,\ south\end{aligned}$$

Example 4.22

A 1.5 kg ball that was moving along a smooth horizontal floor at 5 m/s toward East hits a hard surface and rebounds with the same speed along the same line. What is the change in momentum of the ball?

Solution

Given

$$Mass\ m = 1.5\ kg\ ,$$
$$v_i = 5\ m/s,\ East,\ \ \vec{v}_f = 5\ m/s, West$$
$$\Delta\vec{P} = \Delta\,(m\vec{v})$$
$$= m\,(\,\vec{v}_f - \vec{v}_i) = \ (1.5\ kg)\,(5\ m/s, West - 5\ m/s,\ East)$$
$$= (1.5\ kg)\,(5\ m/s, West + 5\ m/s, West)$$
$$= (1.5\ kg)\,(10\ m/s, West)\ \ = 15\ kgm/s, West$$

Changing the momentum of a body require application of a force. According to Newton's second law, velocity of a moving object changes when a non-zero net force acts on it. Starting from the common version of Newton's second law we define an equivalent expression as follows.

$$\vec{F} = ma = m\frac{\Delta\vec{v}}{\Delta t} = m\left(\frac{mv_f - mv_i}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}\ \text{(Newton's second law in terms of momentum)}$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle.

Rearranging the above equation we have

$$\Delta\vec{P} = \vec{F}\ \Delta t$$

Activity 4.18

Group discussion

According to the equation $\Delta\vec{P} = \vec{F}\Delta t$, a rapid change in momentum requires a large net force, while a gradual change in momentum requires less net force. How is this used in the design of automobile safety devices such as air bags that help to reduce the possibility of injury of the driver?

$$\vec{J} = \vec{F}\ \Delta t$$

The product of the net force and the time interval, $\vec{F}\Delta t$, is known as Impulse of the net force, denoted by $\vec{J}$.

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval. This statement defines the Impulse-Momentum theory.

Key term

- *Momentum: describe the effort require to get a body moving or to stop it.*
- *Impulse: is change of momentum*

Example 4.23

A 2 kg ball that was moving along a smooth horizontal floor at 2.5 m/s toward East hits a hard surface and rebounds with a speed of 2.3 m/s along the same line. If the time of interaction between the ball and the wall is 0.20 s,

(a) What is the change in momentum of the ball?

(b) What is the impulse imparted to the ball?

(b) What is the net force exerted on the ball by the surface?

Solution

Given

$Mass\ m = 2\ kg$,

$\vec{v}_i = 2.5\ m/s,\ East,\quad \vec{v}_f = 2.3\ m/s, West,\ \ t = 0.2\ s$

$$(a)\quad \Delta\vec{P} = m\Delta\vec{v} \quad = m(\vec{v}_f - \vec{v}_i)$$

$$= (2\ kg)(2.3\ m/s, West \quad 2.5\ m/s,\ East)$$

$$= (2\ kg)(2.3\ m/s, West + 2.5\ m/s, West)$$

$$= 9.6\ kg\ m/s, West$$

$(b)\ Impulse = Change\ of\ momentum$

$$\vec{J} = \Delta\vec{P} \quad = 9.6\ kg\ m/s, West$$

$(c)\ From\ \vec{J}\ =\ \vec{F}\Delta\ t\ , we\ have$

$$\vec{F}\ = \frac{\vec{J}}{\Delta t} = \frac{9.6kgm/s\ west}{0.2s}\quad 48\ kg\ m/s2\ , West = 48\ N, West$$

Exercise 4.6

To improve the safety of a motorist, modern cars are built so the front end cramps upon impact. A 1500 kg car is traveling at a constant speed of 10 m/s due East. It hits a rigid wall and comes to rest in 0.25s.

a. Determine the impulse provided to the car.
b. What is the average net force exerted on the car?
c. For the same impulse, what would be the average net force exerted on the car if it had a rigid bumper and frame that stopped the car in 0.04 s?

Activity 4.19

Suppose you are standing on a table top that is 1 m high above the floor and that you jump off the table and land on the floor. It is common experience that your feet experience an upward force of the floor. What action do you take at the instant you land on the floor in order to minimize the effect (magnitude) of the force on your feet?

Example 4.24

The force that an average football player can exert on a ball is 1200N. Such a football striker shoots a penalty from a penalty spot 11m from the goal line. The average mass of a soccer ball is 400g. If the time of contact between the ball and the foot is 0.02s, what is the velocity of the ball as it leaves the spot?

Solution:

The given are: magnitude of the force F = 1200 N, $\Delta t = 0.02s$ m = 400g = 0.4kg.

Required: v =?

Reasoning: The ball is accelerating from rest to a velocity $\vec{v}$ as long as the force is applied on it for 0.02 s. After it leaves the foot of the striker, the only forces are gravity and air resistance. As the ball accelerates from $\vec{v}_i=0$ to $\vec{v}_f = \vec{v}$, the momentum of the ball is changed from Pi to Pf.

Therefore using the equation, $\vec{F}\Delta t = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

Since the ball is initially at rest on the penalty spot, $\vec{p}_i = 0$ and $\vec{p}_f = m\vec{v}$.

Thus, $v = \frac{(120\text{N})(0.01\text{s})}{0.4\text{kg}} = 30$ m/s.

Exercise 4.7

What is the acceleration of the ball in the above example while it was in contact with the striker's foot? How long does it take for the ball to pass the goal line? (Assume the ball obtained a slight air clearance on its path to the goal line so that friction can be neglected.)

One way to calculate the impulse provided to an object is to graph the net force on the object as a function of the interaction time as in Figure 4.52. The area of the net force versus time graph is equal to the magnitude of the impulse provided.

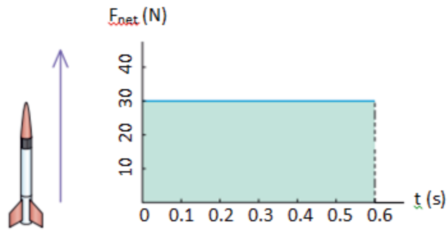


Figure 4.52 Magnitude of net force (F_{net}) versus time graph of a model rocket. The area under the graph equals the impulse provided to the rocket.

In reality, many interactions occur for a very short period of time that accurate measurement of the net force is difficult. The net force increases from zero to a very large value in a short period of time like in the case of the baseball bat hitting the ball, Figure 4.53. The area bounded by the dotted line equals the impulse provided by the bat.

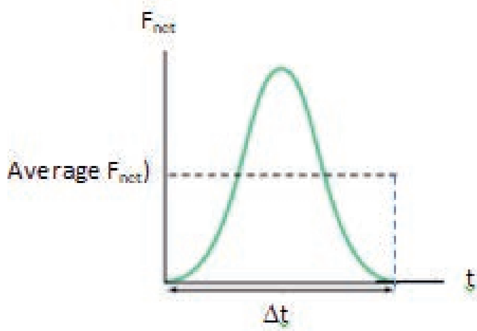


Figure 4.53 the bat exerts non-linear net force on the ball. The product (average net force) x (time interval) gives the impulse.

Exercise 4.8

Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 209 km/). What is the average force exerted on the 0.057 kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure 4.54, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5 ms.

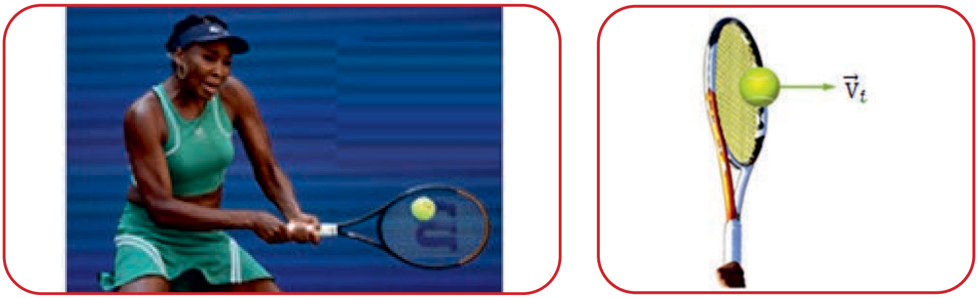


Figure 4.54 The tennis ball acquires a final velocity of 209 km/h (58 m/s).

Law of Conservation of Linear Momentum

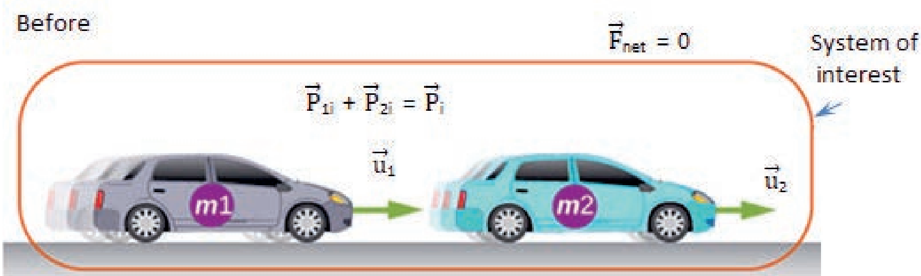
If two bodies collide or push each other apart and no forces act except for each one pushing on the other, the total momentum of the two bodies does not change. That is the initial total momentum is equal to the final total momentum.

Mathematically, $\vec{p}_i = \vec{p}_f$

For two bodies in motion having initial and final velocities each,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Consider what happens if the masses of two colliding objects are more similar such as two identical cars for example, one car bumping into another, as shown in Figure 4.55. Both cars are moving in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.



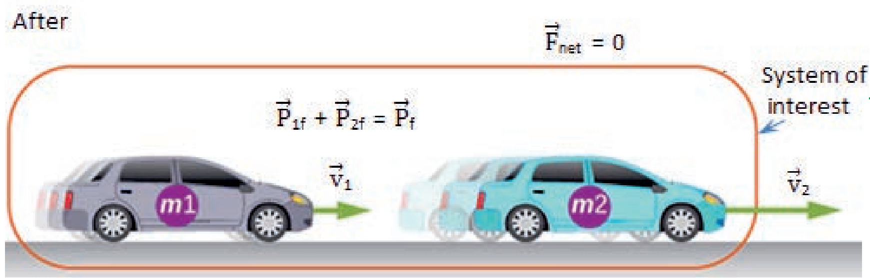


Figure 4.55. Car of mass m1 moving with a velocity of $\vec{u}_1$ bumps into another car of mass m2 and velocity $\vec{u}_2$ that it is following. As a result, the first car slows down to a velocit of $\vec{v}_1$ and the second speeds up to a velocity of $\vec{v}_2$. The momentum of each car is changed, but the total momentum $\vec{P}_{tot}$ of the cars is the same before and after the collision (if you assum friction is negligigle)

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta\vec{P}_1 = \vec{F}_1\Delta t$$

where $\vec{F}_1$ is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision).

Similarly, the change in momentum of car 2 is

$$\Delta\vec{P}_2 = \vec{F}_2\Delta t$$

where $\vec{F}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that

$$\vec{F}_2 = \vec{F}_1, \text{ and so } \Delta P_1 = \Delta P_2$$
$$m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$$
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$\vec{P}_i = \vec{P}_i \ (\text{The law of conservation of momentum})$$

Collisions in One Dimension

The term collision is used to represent an event during which two particles come close to each other and interact by means of forces. The time interval during which the velocities of the particles change from initial to final values (time of interaction) is assumed to be short so that the interaction forces are assumed to be much greater than any external forces present.

In the previous section we have discussed that the total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision.

In contrast, depending on the type of collision, the total kinetic energy of the system of particles may or may not be conserved. Collisions are classified as Elastic and

Inelastic depending on whether or not kinetic energy of the system is conserved.

A collision in which both total momentum and total kinetic energy are conserved is known as Elastic collision. Elastic collisions occur between atomic and subatomic particles. Collisions between two billiard balls are some of the practical collisions that are only approximately elastic.

A collision in which total momentum is conserved but total kinetic energy is not conserved is known as inelastic collision. When the colliding objects stick together after interaction, the collision is said to be completely inelastic.

Activity 4.20

Discuss

Momentum of an isolated system is conserved in all collisions. Kinetic energy of an isolated system is conserved only in elastic collisions. Why?

Elastic collision

Consider two masses m_1 and m_2 moving along the same line in opposite direction as shown in Figure 4.56. If the masses make a head-on elastic collision, how do we determine the velocities of each of the masses after collision?

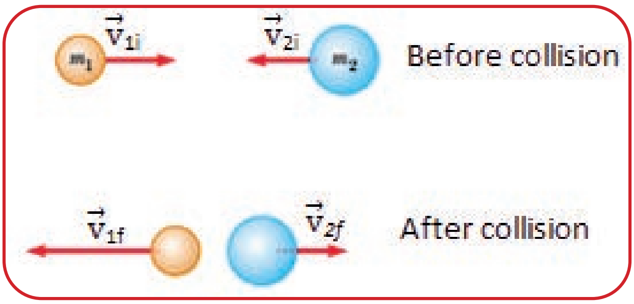


Figure 4.56 Two masses making an elastic head-on collision

In an elastic collision both kinetic energy and momentum are conserved and we have

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

As the masses are moving along the same line in opposite directions, velocities directed to the right can be taken positive and velocities directed to the left can be taken negative. With this consideration we write

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

And for the conservation of kinetic energy, we write

$$\frac{1}{2}m_1\vec{v}_{1i}^{\,2} + \frac{1}{2}m_2\vec{v}_{2i}^{\,2} = \frac{1}{2}m_1\vec{v}_{1f}^{\,2} + \frac{1}{2}m_2\vec{v}_{2f}^{\,2},$$

$$m_1\left(v_{1i}^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_{2i}^2\right)$$

$$m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right)$$

$$\frac{m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right)}{m_1\left(v_{1i} - v_{1f}\right)} = \frac{m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right)}{m_2\left(v_{2f} - v_{2i}\right)}$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$$

This equation, in combination with the equation of the law of conservation of momentum can be can be used to solve problems dealing with elastic collisions. See that $(v_{1i} - v_{2i})$ is the relative velocity of the masses before collision and $(v_{1f} - v_{2f})$ is the relative velocity of the masses after collision.

Activity 4.21

Give real life examples of elastic collisions,

- inelastic collisions and
- perfectly inelastic collisions.

Example 4.25

Consider two masses m_1 of 4 kg and m_2 of 6 kg moving along the same line in opposite direction as shown in Figure 4.56, with velocities of 2 m/s and 1 m/s respectively, along the directions indicated. If the masses make an elastic collision, what are the velocities of each of the masses after collision?

Given

$m_1 = 4$ kg, $v_{1i} = +2$ m/s

$m_2 = 6$ kg, $v_{2i} = -1$ m/s

For an elastic collision both momentum and kinetic energy are conserved and we write

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(4\,kg)(2\,m/s) + (6\,kg)(-1\,m/s) = 4v_{1f} + 6v_{2f}$$

$$4v_{1f} + 6v_{2f} = 2$$

$$2v_{1f} + 3v_{2f} = 1$$

$$\textit{And also} \quad v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$$

$$2\,m/s - (-1\,m/s) = v_{2f} - v_{1f}$$

$$v_{2f} - v_{1f} = 3$$

Solving the simultaneous equation

$$2v_{1f} + 3v_{2f} = 1 \text{ and}$$
$$v_{2f} - v_{1f} = 3,$$

we get

$$v1f = -\frac{8}{5}\ m/s \text{ and } v_{2f} = \frac{7}{5}\ m/s$$

The final results show that after collision m1 moves to the left with a speed of $\frac{8}{5}$ *m/s and* m_2 *moves to the right with a speed of* $\frac{7}{5}$ *m/s.*

Key terms

- *Elastic collision: a collision in which both kinetic energy E and momentum are conserved*
- *Inelastic collision: a collision in which momentum is conserved but kinetic energy is not.*

Example 4.26

Two gliders A and B with masses of 0.5 kg and 0.3 kg, respectively, move toward each other on a frictionless air track (Figure 4.57 a). After they collide (Figure 4.57 b), glider B has a final velocity of 2 m/s toward the positive x direction (Figure 4.57 c).

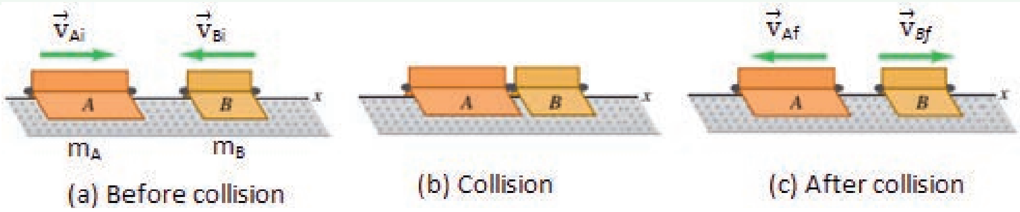


Figure 4.57 Gliders on an air track

(a) What is the final velocity of glider A?
(b) How do the changes in momentum and in velocity compare?

Solution

Given

$m_A = 0.5$ kg, $v_{Ai} = 2$ m/s
$m_B = 0.3$ kg, $v_{Bi} = -2$ m/s, $v_{Bf} = 2$ m/s

(a) Applying The law of conservation of linear momentum,

$$m_A V_{Ai} + m_B V_{Bi} = m_A V_{Af} + m_B V_{Bf}$$

$(0.5 \text{ kg})(2 \text{ m/s}) + (0.3 \text{ kg})(-2 \text{ m/s}) = (0.5 \text{ kg})(v_{Af}) + (0.3 \text{ kg})(2 \text{ m/s})$

$1 \text{ kg m/s} - 0.6 \text{ kgm/s} = (0.5 \text{ kg})(v_{Af}) + 0.6 \text{ kg m/s}$

Solving for v_{Af}, we get $v_{Af} = -0.4$ m/s

(b) Change of velocity of glider A is

$$\Delta v_A = v_{Af} - v_{Ai} = (-0.4 \text{ m/s}) - (2 \text{ m/s}) = -2.4 \text{ m/s}$$

Change of velocity of glider B is

$$\Delta v_B = v_{Bf} - v_{Bi} = (2 \text{ m/s}) - (-2 \text{ m/s}) = +4 \text{ m/s}$$

See that Δv_A ' Δv_B

Change of momentum of glider A is

$$m_A(\Delta v_A) = (0.5 \text{ kg})(-2.4 \text{ m/s}) = -1.2 \text{ kgm/s}$$

Change of momentum of glider B is

$$m_B(\Delta v_B) = (0.3 \text{ kg})(+4 \text{ m/s}) = 1.2 \text{ kgm/s}$$

The gliders experience equal and opposite forces and according to Newton's second law the one with the smaller mass will have acceleration of larger magnitude. By the impulse–momentum theorem, the gliders experienced equal and opposite impulses and therefore equal and opposite changes in momentum.

Inelastic collision

Inelastic collisions are of two types. When the colliding objects stick together after the collision, the collision is called perfectly inelastic. When the colliding objects do not stick together, but some kinetic energy is lost the collision is called inelastic.

For a perfectly inelastic collision where the masses stick together after collision we use

$$v_{1f} = v_{2f} = v, \text{ and}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v + m_2 v$$

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$, v is the common velocity of the masses after collision.

Example 4.27

A 700 g pistol loaded with a 180 g bullet rests on a smooth table, Figure 4.58. If the bullet suddenly goes off with a speed of 300 m/s, with what speed will the pistol recoil?

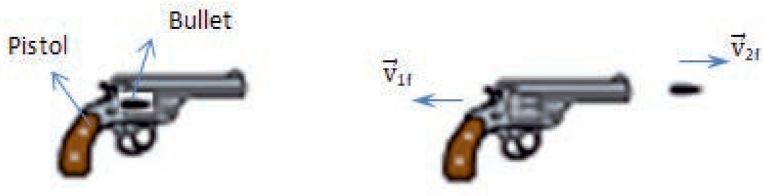


Figure 4.58 Recoil velocity of a Pistol

Solution

Given:

Mass of the Pistol-bullet system before interaction $m = 700\ g + 180\ g = 880\ g = 0.88\ kg$

Initial velocity of the system $\vec{v}_i = 0$

Final velocity of the bullet $\vec{v}_{2f} = 350$ m/s, forward

To find the final velocity with which the pistol recoils just after interaction we apply the law of conservation of momentum as

$$\vec{p}_i = \vec{p}_f$$

$$m_1\ \vec{v}\ 1_i + m_2 v_{2i} = m_1 \vec{v}_{1f} + m_2\ \vec{v}_{2f}$$

$(m_1 + m_2)\ i = m1 \vec{v}_{1f} + m2\ \vec{v}_{2f}$

$M \vec{v} i = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$

Substituting the given values we write

$0 = (0.7\ kg)(\ 1f) + 0.18\ kg)(300\ m/s)$

Substituting the given values we write

$0 = (0.7\ kg)(v_{1f}) + 0.18\ kg)(300\ m/s)$

Rearranging this equation and solving for v_{1f}, we obtain

$\vec{v}_{1f} = -77$ m/s

The minus sign indicates that as the bullet goes off forward, the pistol recoils backward.

$\vec{v}_{1f} = 77$ m/s, backward

Activity 4.22

A physics student (mass 60 kg) carrying a 2 kg book is standing in the middle of a frozen pond. What can he do in order to go toward the shore of the pond?

Example 4.28

Perfectly inelastic collision

A fully loaded dump truck with a mass of 25000 kg is moving at 72 km/h on a level paved road. On its way, it suddenly collides head on with a 150 kg bajaj which was moving at 54 km/h in the opposite direction. What will be the final velocity of the truck-bajaj system after collision?

Given: $m_1 = 25000\text{kg}$,

$v_{1i} = 72\text{km/h} = 20\text{m/s}$

$m_2 = 150 \text{ kg}$

$v_{2i} = 54 \text{ km/h} = 15\text{m/s}$ (initially moving in opposite direction to the track's motion)

Required: $\vec{v}_f = ?$

Solution:

By conservation of linear momentum for a head-on collision,

Total momentum before collision = Total momentum after collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Since the dump truck and the bajaj stick and go together to have a common final velocity, we write $\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$.

Therefore: $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$

$$(25000 \text{ kg})(20 \text{ m/s}) + (150 \text{ kg})(-15 \text{ m/s}) = (25000 \text{ kg} + 150 \text{ kg})v_f$$

Solving for the final velocity we get $v_f = 19.7 \text{ m/s}$

The value is positive indicating that after collision the truck-bajaj system moves along the initial direction of motion of the truck.

Example 4.29

Ballistic pendulum

Consider a ballistic pendulum, a type of pendulum used to determine the speed of bullets before electronic timing devices were invented (Figure 4.59).

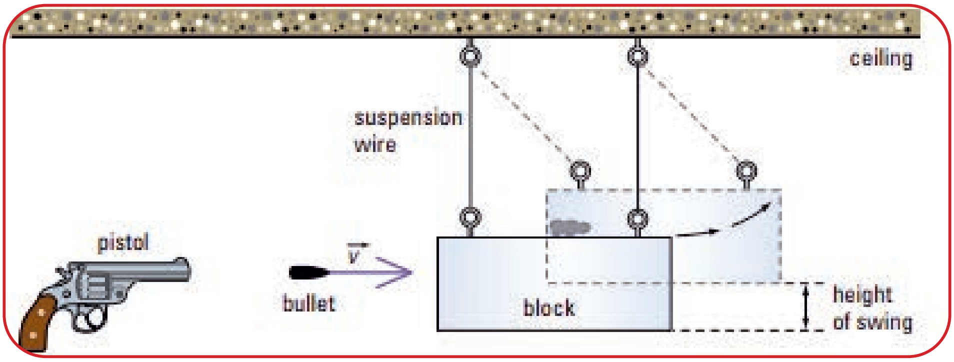


Figure 4.59 A Ballistic pendulum

A stationary block of wood is suspended from the ceiling by light ropes. When a bullet is fired at the block, the bullet becomes embedded in the wood upon impact. Just after interaction the pendulum-bullet system starting from rest, rises up to a certain height of swing before stopping. Derive an expression for the velocity of the bullet in terms of mass of its own mass, mass of the block, g and swing height.

Solution

On impact some of the kinetic energy of the bullet is transferred to the pendulum-bullet system so that the system will have speed v_f just after interaction. As the pendulum-bullet system swings upward, its kinetic energy is converted to gravitational potential energy.

Taking mass of bullet $= m_1$,

mass of the block $= m_2$

velocity of bullet before impact $v_{1i} = v$,

initial velocity of the block = 0,

swing height = h

The law of conservation of momentum can be written as

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)\ f \text{ (perfectly inelastic collision)}$$

$$m_1v = (m_1 + m_2)\ f$$

$$vf = \frac{m_1v}{m_1 + m_2}$$

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)\ f \text{ (perfectly inelastic collision)}$$

$$m_1v = (m_1 + m_2)\ v_f$$

$$v_f = \frac{m_1 v}{m_1 + m_2}$$

This is the velocity with which the pendulum-bullet system starts just after interaction.

Starting off with speed v_f the system rises up to swing height h so that the kinetic energy of the pendulum-bullet system is changed to gravitational potential energy of the system.

$$\tfrac{1}{2}(m_1 + m_2)\ vf^2 = (m_1 + m_2)\ gh$$

$$vf^2 = 2gh$$

$$\left(\frac{m_1 v}{m_1 + m_2}\right)^2 = 2gh$$

Rearranging the equation we write the speed of the bullet as $v = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$

Activity 4.23

In the solution for Example 4.29, we have assumed some of the initial kinetic energy of the bullet being transferred to the pendulum-bullet system emphasizing a loss of kinetic energy in the process. How do you explain this?

Example 4.30

A 14.9 g bullet from a pistol strikes a 2 kg ballistic pendulum. Upon impact, the pendulum swings forward and rises to a height of 21.9 cm. What was the velocity of the bullet immediately before impact?

Solution

Given

mass of the bullet $= m_1 = 14.9\ g = 0.0149\ kg$

mass of the block $m_2 = 2\ kg$

initial velocity of the block $= 0$,

swing height $= h = 21.9\ cm = 0.219\ m$

Using the given values in the equation $v = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$, we get the velocity of the bullet just before hitting the block as $v = 280$ m/s

Exercise 4.9

On a frictionless, horizontal air table, puck A (with mass 0.25 kg) is moving toward puck B (with mass 0.35 kg), which is initially at rest. After the collision, puck A has a velocity of 0.012 m/s to the left, and puck B has a velocity of 0.65 m/s to the right.

(a) What was the speed of puck A before the collision?
(b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

Center of Mass

The principle of conservation of momentum can be restated in a useful way by using the concept of center of mass. A mechanical system can be either a particle or a group of particles. Center of mass of an object or system of objects is a place where the total mass of the system assumed to be concentrated. We shall see that the center of mass of the system moves as if all the mass of the system were concentrated at that point. When a resultant force acts on a system of objects or particles, the force is assumed to have acted at the center of mass and the center of mass moves with an acceleration governed by Newton's second law of motion.

Key terms

Center of mass: a point where the total mass of a body or system of objects is as sumed to be concentrated

Suppose we have several particles with masses m_1, m_2 and so on. Let the coordinates of m_1 be (x_1, y_1), be those of m_2 be (x_2, y_2) and so on. We define the center of mass of the system as the point that has coordinates (x_{cm}, y_{cm}) given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots + m_ny_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

Example 4.31

Three masses on $m_1 = 300$ g, $m_2 = 500$ g and $m_3 = 200$ g are place on the xy plane at coordinates (-2, 2), (2,3) and (3,-1), respectively. If distances are in centimeters, find the coordinates of the center of mass of the system.

Solution

Given m1 = 0.3 kg at (-2, 2) , m2 = 0.5 kg at (2, 3), m3 = 0.2 kg at (3, -1)

$$xcm = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(-0.6)+(1)+(0.6)}{0.3+0.5+0.2} = \frac{1}{1} = 1,$$

$$ycm = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(0.6)+(1.5)+(-0.2)}{0.3+0.5+0.2} = \frac{1.9}{1} = 1.9$$

Motion of Center of the Mass

What will happen to the center of mass when the system of particles moves? The center of mass of a system of particles is the point that moves as though all of the system's mass were concentrated there and all external forces were applied there. Figure 4.60 shows the position of the center of mass (the dot) of the baseball bat flipped in the air. The center of mass follows parabolic path while the other parts of the bat follow complicated path.

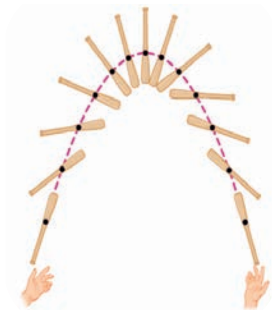


Figure 4.60 The motion of a baseball bat flipped in the air and the motion of its center of mass.

Knowing the x and y coordinates of the center of mass (or the x and y components of its position vector) we can determine the time rate at which the position vector changes to determine the velocity of the center of mass.

The coordinates of x and y components of the center of mass are respectively,

$$(vcm)x = \frac{m_1v_{x1} + m_2v_{x2} + m_3v_{x3} + \ldots + m_nv_{xn}}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$(vcm)y = \frac{m_1v_{y1} + m_2v_{y2} + m_3v_{y3} + \ldots + m_nv_{yn}}{m_1 + m_2 + m_3 + \ldots + m_n}$$

When writing this equation to the single vector equation of the velocity of the center of mass, we have

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

If the sum $m_1 + m_2 + m_3 + \ldots m_n$ represents the total mass of the system M, we have

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n$$

$$\vec{p}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n$$

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

Conservation of Momentum Applied to Rockets

The motion of a rocket, Figure 4.61, can be explained using Newton's third law. When the rocket engines burn the fuel and eject the gas backwards, the rocket propels upward (action–reaction pairs). The law of conservation of momentum can also be used to explain why a rocket can accelerate even in a vacuum. When the engines of a rocket burn fuel, the escaping exhaust gas has mass and considerable speed. When a rocket is in outer space, external friction is negligible. So the rocket-exhaust gas system is an isolated system. For a rocket-gas system, the equation for the conservation of momentum is

$$\Delta\vec{P}_{rocket} + \Delta\vec{P}_{gas} = 0$$
$$\Delta\vec{P}_{rocket} = -\Delta\vec{P}_{gas}$$

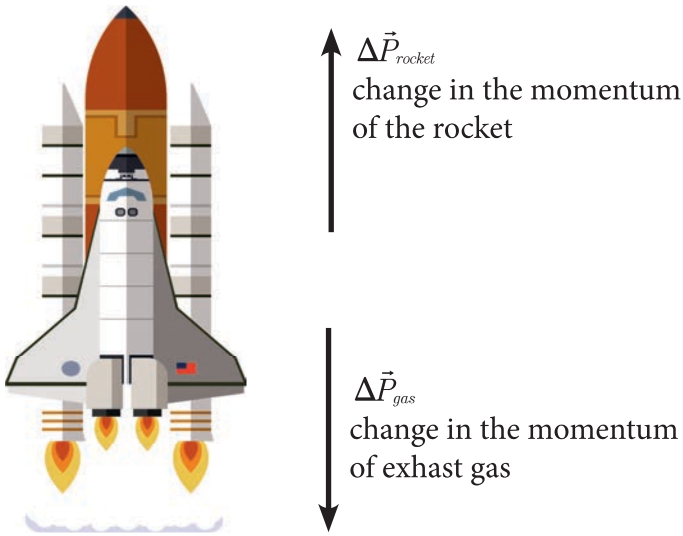


Figure 4.61 Rocket propulsion

Review Questions 4.6

1. A car has the same kinetic energy when it is traveling south at 20 m/s as when it is traveling northwest at 20 m/s. Is the momentum of the car the same in both cases? Explain
2. A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why?
3. An apple falls from a tree and feels no air resistance. As it is falling, which of these statements about it are true? (a) Only its momentum is conserved; (b) only its mechanical energy is conserved, (c) both its momentum and its mechanical energy are conserved, (d) its kinetic energy is conserved.
4. What is the effect on impulse if (a) the time interval is doubled?
   (b) the net force is reduced to one-third of its original magnitude?
5. A golfer hits a long sending 50 g golf ball toward East. The graph of the net force as a function of time for the collision between the golf club and the ball is as shown in Figure 4.62.

(a) What is the impulse provided to the ball?

(b) What is the velocity of the ball at the moment the golf club and ball separate?

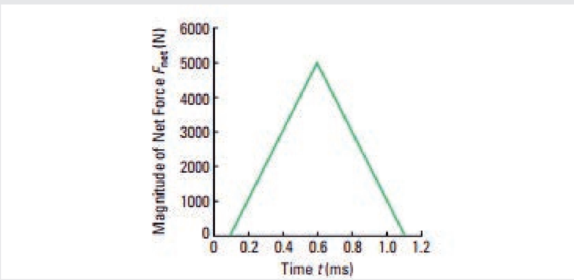


Figure 4.62 Force vs time graph

6. Two billiard balls A and B that are traveling toward each with speeds of v and 2v respectively, If ball A is traveling tward East and B toward West, what are the finalocities of the two balls. Assume an isolated system.
7. A 0.16 kg billiard ball traveling at 0.5 m/s, due North strikes a stationary 0.18 kg snooker ball and rebounds at 0.23 m/s due South. The snooker ball moves off at 0.463 m/s due North. Ignoring the possible rotational effect, determine if the collision is elastic?
8. A 75 kg hunter in a stationary canoe throws a 0.72 kg spear at 12 m/s in the forward direction. The mass of the canoe is 10 kg. What will be the velocity of the canoe and the hunter immediately after the spear is thrown?
9. A basketball player A and her wheelchair have a combined mass of 58 kg.

She moves at 0.6 m/s toward East and pushes off a stationary player B while jockeying for a position near the basket. Player A ends up moving at 20 m/s toward West. The combined mass of player B and her wheelchair is 85 kg. What will be player B's velocity immediately after interaction Figure 4.63)?

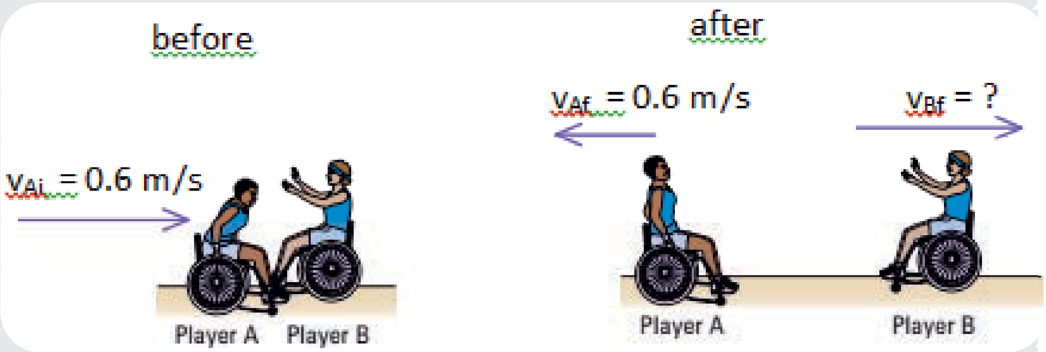


Figure 4.63

10. The center of gravity of an object may be located outside the object. Give a few examples for which this is the case.

SUMMARY OF UNIT 4:

- Newton's laws of motion can be applied in numerous situations to solve problems related to motion.
- Newton's first law also known as the law of inertia tells us that in the absence of unbalanced force a body at rest remains at rest and a body in motion keeps on moving with the same velocity.
- Newam's second law describes what happens to a body if it is ncted upon by mn unbalanced force. The acceleration of a body is directly proportional to the net force and inversely proportional to the mass of the body. A body accelerates in the direction of the net force on it.
- Newton's third law tells us the fact that forces are in pairs and that action and reaction are equal and opposite.
- Friction is an example of contact type of force. It arises when a hody moves or attempts to move on another body. Friction depends on the nature of the surfaces in contact and the normal force.
- Static friction is greater than sliding (kinetic friction).
- The normal force on an object is not always equal in magnitude to the weight of the object. It depends on the orientation of the surface on which an object is placed.
- A body is in equilibrium is the net force acting on it is zera.
- When a body is in equilibrium, it is either at rest or moves at a constant speed in a fixed direction. Itx acceleration will be zero.
- For problems involving two or more forces, draw free-body diagrams, resolve the forces into their components and apply Newton's laws
- The net work done W_{net}, is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is k= $\frac{1}{2}mv^2$
- The work-energy theorem states that the network W_{net} on a system changes its kinetic energy.
- Gravitational potential energy of a mass m placed at height habove a reference is PE=mgh
- One of the common examples of work done by a varying force is work done on a spring.
- Elastic potential energy of a spring that is stretched or compressed through x is $PE = \frac{1}{2}kx^2$
- Linear momentum is defined as the product of a system's mass and its velocity.
- In symbols, linear momentum $\vec{P}$ is defined to be $\vec{P} = m\vec{v}$ where m is the mass of

the system and is its velocity.

- The SI unit for momentum is kgm/s.
- Newton's second law of motion when expressed in terms of momentum it take the form $\vec{F}_{net} = \dfrac{\Delta \vec{P}}{\Delta t}$
- Center of mass of a body is a point where the total mass of the body is assumed to be concentrated.
- Power is the rate of doing work or the rate of transfer of energy.
- When a certain work is done in a given interval of time, the power delivered is given by Power = Work/time.
- When a force is applied on an object, the velocity is changed, the power delivered at that instant of time is given by the dot product of the applied force and the velocity vector as $\vec{P} = \vec{F}.\vec{v}$
- When several forces are acting on a body, the net power delivered is the sum of the power delivered by each of the forces.
- Positive power means energy is transferred to the object by the force.
- Negative power means energy is transferred from the object by the force.

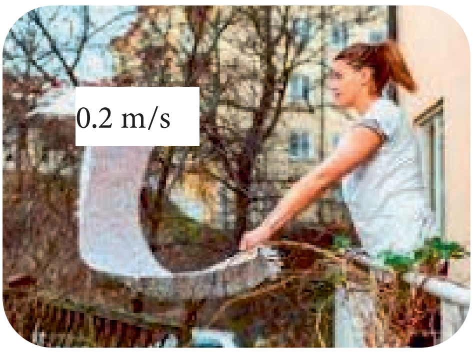
END OF UNIT 4 QUESTIONS:

1. A ball of mass m is suspended by a string from the ceiling, the Earth pulls down the ball with a force of magnitude equal to the weight of the ball. Consider this to be the action force, what is the action force?
   a. The string pulling upward on object with a force of magnitude equal to the weight of the ball
   b. The ceiling pulling upward on the string with a force of magnitude equal to the weight of the ball.
   c. The ball pulling upward on the earth with a force of magnitude equal to the weight of the ball.
2. It is easier to pull a rolling bag on a road than to push. This is because when the rolling bag is pulled,
   a) the vertical component of the force acts in the direction of gravity and weight thereby weight of the roller is increased and friction increased.
   b) the vertical component of the force acts in opposite direction to the weight, thereby weight of the roller is decreased and friction decreased.
   c) the premise is wrong, because pulling a roller cannot be easier than pushing.

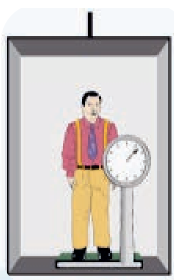
3. The speed of a freely falling object is proportional to its mass or weight.
   a. True   b. False
4. How is inertia of an object affected by its mass?
5. List down the various effects of force.
6. Which of the following statement is NOT true?
   a. Rate of change of momentum is called force.
   b. Action reaction pairs of force are equal in magnitude and opposite in direction.
   c. Action reaction pairs of force act on two different bodies.
   d. Action reaction pairs of force can be cancelled to each other.

7. Any two equal and opposite forces constitute action and reaction pairs. Is this statement correct? Explain.
8. A soccer player starts from rest and accelerates forward, reaching a velocity of 8 m/s in 2.5 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70 kg, and air resistance is negligible. (This problem is solved by integrating the concepts of kinematics to dynamics.)

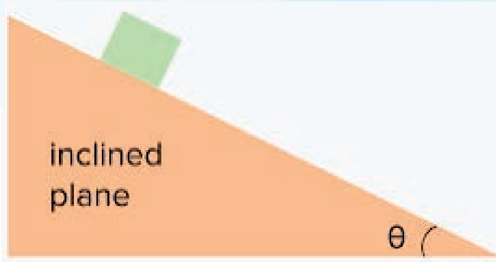
9. What is the use of airbags in cars?
10. Give reason why dust is re moved from a carpet by shaking?



11. Why is it advised to tie a rope on the luggage to a luggage rack while you travel by the bus?
12. The figure at the right shows a 75 kg man standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s2, and (b) if the elevator moves upward at a constant speed of 1 m/s.
13. Define the term equilibrium and state the first condition of equilibrium.



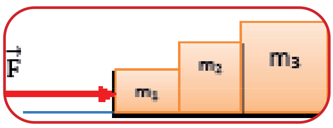
14. If a block is at rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes



a. Draw the free body diagram and show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}(\mu_s)$.

b. This angle is called the critical angle, what will be the acceleration of the block when the angle of the inclination is greater than the critical angle? Is the acceleration dependent on the mass? Is the acceleration dependent on the mass if the surface is frictionless? For what angle would the block begin to slide if the surface were frictionless?

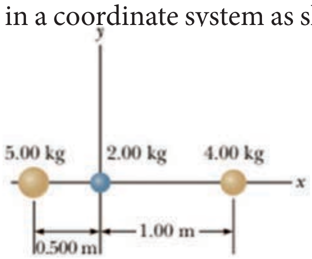
15. Three blocks are in contact with each other on a frictionless horizontal surface as in the figure below.



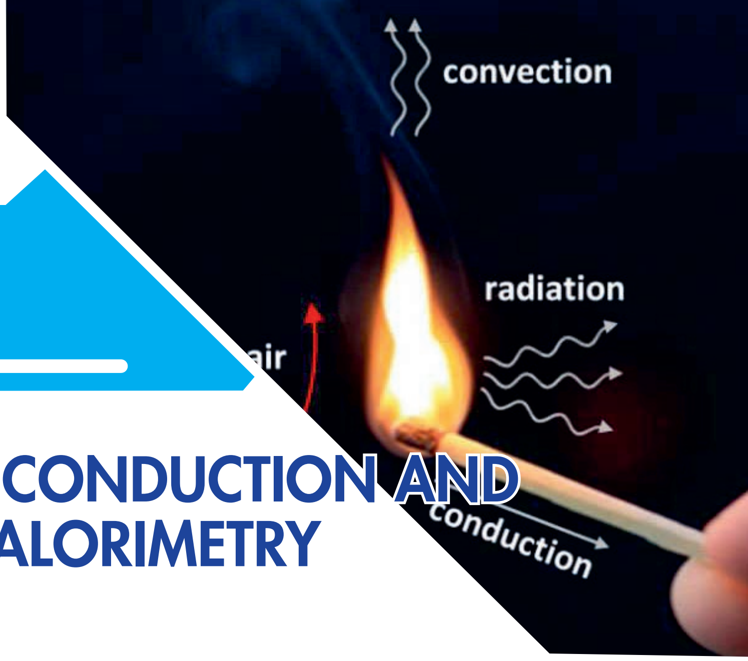
16. A horizontal force $\vec{F}$ =18N is applied to m_1. If m_1 is 1kg, m_2= 2 kg, and m_3 = 3kg, Draw a separate free body diagram for each block and find:
   a. The acceleration of the block.
   b. The resultant force on each block.
   c. The magnitude of the contact forces between the blocks.
17. State the law of conservation of momentum.
18. Compare the momentum of a 78 kg footballer running at a speed of 8.0m/s to that of a hard kicked 0.4kg soccer ball at a speed of 25m/s. Which one has a larger momentum?
19. Does a large force always produce a larger impulse on an object than a smaller force does? Explain.
20. If the speed of a particle is doubled, by what factor is its momentum changed? By what factor is its kinetic energy changed?
21. If two particles have equal kinetic energies, are their momenta necessarily equal? Explain.

22. What is the momentum of a 200 kg mass if it has kinetic energy of 1600 J?
23. By what factor must the speed of a moving body change in order to double its kinetic energy?
24. Suppose that you push on the 30 kg box as shown in the figure with a constant force of 120 N through a distance of 0.80 m, and that the average opposing friction force is 5 N.
    a. Calculate the net work done on the box.
    b. Solve the same problem as in part a, this time by finding the work done by each force that contributes to the net force.
    c. Using the work energy concept, find the velocity of the box at the end of the push

25. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases?
26. Three objects are located in a coordinate system as shown. Find the center of mass.



UNIT 5



HEAT CONDUCTION AND CALORIMETRY

Introduction

Everywhere you look including yourself, energy transfer taking place in different forms. One of such energy transfer methods is heat transfer. Heat energy is very essential to society. From everyday cooking to some of modern-day luxuries, such as refrigerators and Jet engines are using the transfer of heat energy at the heart of their working principle. Heat is one of the mechanisms to transfer internal energy so that it can be usable.

Heat is often misunderstood to be the same as temperature. In this unit, you will be discussing differences and relationships between the two. Apart from that, you will also learn about thermal expansion caused by the change in temperature, the quantity of heat transfer due temperature changes, and change of phase along with internal energy and the work done by a system.

At the end of this unit, you should be able to:

- *gain comprehensive knowledge on basic properties of fluids, thermal properties of matter, phase changes, calorimetry*
- *quantitatively and qualitatively describe about heat capacity, specific heat capacity, and thermal expansions.*
- *solve calorimetry problems.*
- *use specific heat and heat of transformations in the practical application of calorimetry.*

5.1 The Concept of Heat

At the end of this section you will be able to:

- *explain the concept of heat*

Brainstorming:

Get motivated by the following questions

What is temperature?

Although they are highly related, heat and temperature are completely different quantities. Heat is a measure of change, never a property possessed by an object or system. Therefore, it is classified as a process variable. However, temperature is a fundamental physical quantity which is to the average kinetic energy of particles of the substance.

It should also be noted that work and heat are closely related. Both can change the temperature of a substance, and heat can be turned into work and work can be turned into heat. This equivalence is the basis of how heat engines power modern society.

What is heat?

When a hot cup of tea is left in a room in contact with the enveloping air, you know that the cup of tea will gradually cool down over time. So what is happening over the course of time to cause the tea to cool down?

Figure 5.1: Spontaneous transfer of heat is from hot to cold

The fact that the tea lowers its temperature is a sign that the average kinetic energy of its particles is decreasing - the tea is losing energy. The energy that is lost by the tea is being transferred to the colder surroundings. In general, if two objects at different temperatures are brought in thermal contact, energy flows spontaneously from the hotter object to the cooler one. The spontaneous transfer of energy from hotter region to cooler region is called heat. . Since heat is energy, its SI unit is joules (J). When temperatures of the tea and that of the surrounding air balance out, we say thermal equilibrium is reached. At thermal equilibrium, the two regions may exchange energy, but no net heat transfer occurs.

Heat, internal energy and thermodynamic work

Another aspect of the definition of heat is that a body never contains heat. That is, we cannot say "stored heat energy" as we cannot say "stored electric current". Heat is not something you get it sitting down. Rather, heat can be identified only as it crosses a boundary.

Key terms:

- *Internal energy is equal to the sum of internal kinetic energy due to molecular motion (translational, rotational, and energy of vibration) and internal potential energy due to molecular attractive forces.*
- *Heat is energy which is in transit from one body to another as the result of a difference in temperature.*
- *Thermodynamic work is the quantity of energy transferred from one system to another.*

When an object is heated, its temperature rises, since the average kinetic energy of its particles increases. In other words, the energy stored in the substance is increased. This energy stored in the object is what we call internal energy. When the tea is cooling down, it is releasing its internal energy in the form of heat. Internal energy is equal to the sum of internal kinetic energy due to molecular motion (translational, rotational, and energy of vibration) and internal potential energy due to molecular attractive forces.

Heat and work are two different ways of transferring energy from one system to another. Heat is the transfer of thermal energy between systems, while work is the transfer of mechanical energy between two systems. If heat is supplied to an enclosed gas, the temperature of the gas increases by then increasing its internal energy. If the gas is closed by a movable piston as shown in Figure 5.2, work is done by the expanding gas in raising the piston up. Thus, the heat supplied is converted partly into internal energy and partly into work done. We can also increase the internal energy of the gas by pushing the piston down and compressing the gas.

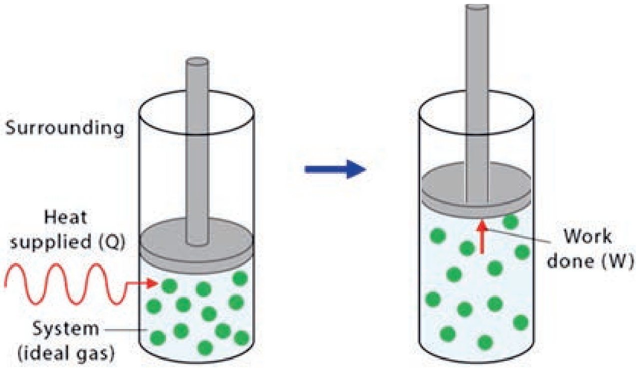
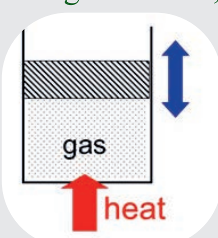


Figure 5.2: Relationships between heat, work and internal energy

Review Questions 5.1:

1. Explain the concept of heat and describe effects of heating.
2. What are the factors that cause heat flow between two systems which are in thermal contact?
3. Is there any heat exchange between two systems in thermal contact if they are in thermal equilibrium?
4. Is it possible to reduce net heat flow to zero? What about internal energy? Is there any relationship between internal energy and temperature?
5. Consider two cylinders, A and B, of equal volume, which are filled with equal amount of gases of the same type but A is at higher temperature than B.
   a. Can you say A has greater heat than B?
   b. Can you say that A has greater internal energy than B?
   c. If A and B are brought in thermal contact, which one loses and which one gains internal energy? What is the direction of heat flow? Does the measure of the average kinetic energy of molecules in B increase or decrease?
6. In the figure shown, heat is supplied to the gas confined in a cylinder.



   a. What would happen to this energy after it enters into the cylinder?
   b. If work is done on the piston and displaces it up, what would happen to the internal energy; decrease or increase?
   c. If the internal energy remains the same while heat is flowing in to the system, what happened to the energy supplied?
7. What are similarities and differences between heat and work?

5.2 Heat transfer mechanisms

At the end of this section you will be able to:

- *explain the concept of heat conduction*

? Brainstorming

This unit is motivated by the following discovery questions:

1. When you touch one end of a metal rod whose other end is on a fire, you feel warm. How is that happened?
2. When you are outside of your house and enjoying the morning sun you feelwarm. How is that happened? When you insert your hand over the steam of boiling water, you feel warm. How is that happened?
3. What types of objects feel cold and what type of objects feel warm in your house? Justify your answer and write them down.

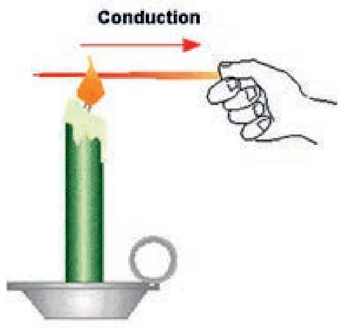
Have you ever touched a hot pan and burned your fingers, or have you ever warmed yourself in front of a campfire on a cold night? These are all related to the movement of heat and how it gets transferred between substances and locations. Heat transfer mechanisms are the ways by which thermal energy can be transferred between objects, and they all rely on the basic principle that kinetic energy or heat wants to be at equilibrium. Heat transfer occurs in three different ways: conduction, convection, and radiation.

I. Conduction

If one end of a metal rod is over a flame and a person is holding the opposite end with a bare hand, as shown in Figure 5.3, the heat will quickly transfer through the metal to the person's hand and burn him/her. Such direct transfer of heat from one object to another by collisions of neighboring particles is called conduction. In conduction, no actual drift of molecules in the direction of heat flow is happening.

Key terms:

- *Conduction is the process by which heat energy is transmitted through collisions between neighboring atoms or molecules.*
- *Convection (or convective heat transfer) is the transfer of heat from one place to another due to the movement of fluid.*
- *Radiation heat transfer occurs via electromagnetic waves. Unlike conduction and convection, radiation does not need a medium for transmission*



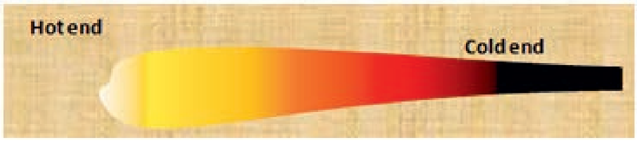
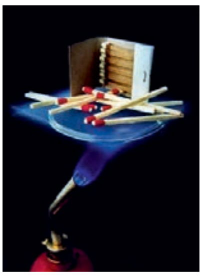


Figure 5.3: Heat transfer by conduction

Materials such as metals that conduct heat very well are called good conductors. Some substances are very bad at transferring heat, and these are called insulators. Good insulators are, therefore, very poor conductors. Do you know why your metal pots have wooden handles?



Do you know...?

The photo shows Aerogel, an extremely good thermal insulator, between a blowtorch and matches. The Aerogel blocks all of the heat from the blowtorch and prevents the matches from burning. Source: https://www.ucalgary.ca/

The reason is that the metal pots are good conductors of heat, and wood is bad heat conductor. So, metal pots have wooden handles so that persons do not feel hot while holding them

Houses are usually insulated with poor conductors such as rock, wool or fiberglass. Imagine how cold a house would be if there was no protection from the cold outside during winter, or how hot it would be if it were not protected from the heat outside in the summer! Interestingly, though, insulators do not actually totally stop the flow of heat, just slow the transfer considerably.

Activity 5.1:

1. Suppose you have a piece of metal and a piece of wood in your room. Why does the metal feel colder than the wood if they are both at room temperature?
2. You know that rate of heat transfer depends on the properties of the medium.

a. What do you think on impact of size or shape on rate of heat transfer?

b. Two regular metal rods of different cross-sectional area and equal length are put in to a fire, as shown in Figure 5.4. If you are holding on the cold side, after some time you feel the temperature increase in your hand. Discuss in groups which one of the metals transfer energy to your hand sooner. Why is that?

3. If you want to have best insulating wall, which factors should you consider?

Figure 5.4: Conducting slab

II. Convection

Convective heat transfer is the transfer of heat between two bodies by currents of moving gas or fluid. Convection happens in all fluids like the air in your environment, water in a sea and so on. When a fluid is heated, the molecules at the bottom gain energy and begin to move faster (See Figure 5.5). As they move faster, they begin to spread apart, which makes the fluid less dense. But instead of being confined to a small space, the fluid's warmer molecules begin to move upward, while the cooler, denser portion of the fluid begins to sink.

Once the cooler molecules reach the bottom where the heat is, they get an increase in energy again. They rise back up to the top, while the cooler molecules sink back to the bottom. The cycle continues as long as there is a heat source at the bottom. Unlike conduction, in convection, heat is carried from the bottom to the top being carried by molecules. That is, molecules travel with the heat.

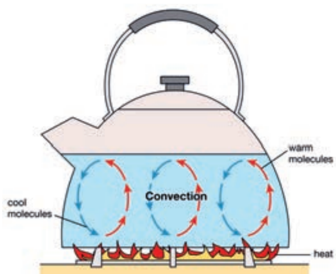


Figure 5.5: Heat transfer by convection

III. Radiation

When you sit in front of a campfire, you are warmed by its heat. Hover your hand over a hot light bulb, and you will feel the heat coming off it, even without touching it. If you stand out in the sunshine, you will quickly get warmed by its rays. Such heat transfer mechanism is called radiation.

Heat transferred by radiation is called radiant heat. Like light, radiant heat is radiant energy, and does not necessarily require a medium to carry it. This form of energy transfer is facilitated through a type of electromagnetic radiation.

Activity 5.2

1. Let there be hot coffee, hot water, hot milk, hot tea, hot porridge. Discuss and rank from hottest to coldest. You may design and carry out investigative work to scientifically support your claim.
2. In hot areas what do people do to cool themselves? (Think of in terms of clothing housing etc.)

Review Questions 5.2:

1. What would happen to boiling water if you close the cover tightly? Try to describe it in terms of convention and conduction.
2. What is the difference between conduction and convection?
3. Does heat transferred less or more if the area is big?
4. Name the general mode of heat transfer in: Solids, liquids and gases.
5. A person prefers to sit by a fire during the cold winter months. Which heat transfer mechanism gives him the most heat?
6. What would be the effect if air were as good conductor as iron?
7. Three types of heat transfer mechanisms are indicated in the same phenomenon, as shown in Figure 5.6. Do you think these are correct designations? Explain.

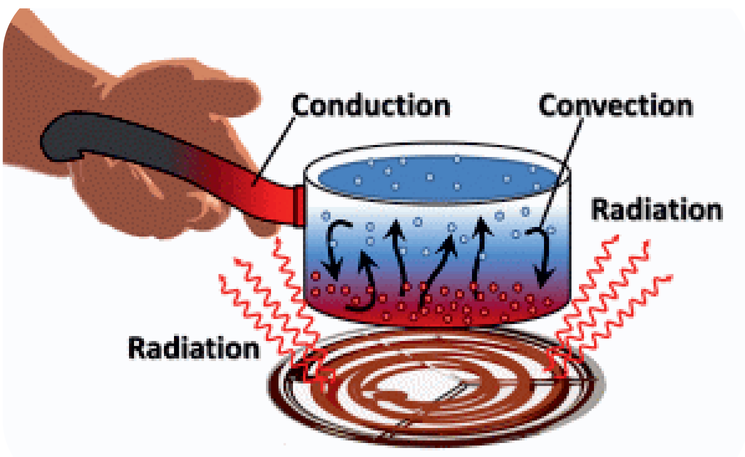


Figure 5.6: Summarizes the three heat transmission mechanism

5.3 Heat Capacity and Specific Heat Capacity

After completing this section, you should be able to:

- *differentiate between heat capacity and specific heat capacity*
- *list the specific heat capacity of various materials*

Brainstorming!

Investigate in groups why water is added in the radiator of a car? Why different additives are added in the radiator water? Your investigation can be done through literature search asking knowledgeable person.

Heat capacity

Different substances respond to heat in different ways. For example, if a metal plate is placed in the bright sun on a hot day, it may become quite hotter than the surrounding. An equal mass of water under the same sun exposure will be cooler than the metal. This means that water has a higher **heat capacity**. Some objects gain and lose heat quickly, some slow and some intermediate. Therefore, we need to define a quantity called heat capacity.

Heat capacity is defined as the amount of heat required to change the temperature of a given amount of matter by 1°C or by 1K. The mathematical form of this definition is given by;

$$C = \frac{Q}{\Delta T}$$

where Q is heat transferred to the object, ΔT is change in temperature of the object and C is heat capacity of the object. The SI unit of C is joule per kelvin (J/K); where 1 J/K=1 J/ °C .

Consider two tankers, 10 kL and the other 10L, are filled with pure water at the same temperature. If the two tankers were subjected to the same input of heat energy, the 10L water would certainly rise in temperature more quickly than the water in the 10kL tanker.

Key terms:

- *Heat capacity is the amount of heat required to raise the temperature of an object by 1 , or 1 K.*
- *The specific heat capacity of a substance is the amount of energy required to raise the temperature of 1 kilogram of the substance by 1 , or 1 K.*

This suggests that the heat capacity of an object depends not only on the property of the object but also on its mass. Because of its much larger mass, the 10 kL water has a larger heat capacity than the 10L water. Therefore, we require defining a quantity, specific heat capacity, which depends only on properties of the substance.

Specific heat capacity

Specific heat capacity is defined as the amount of heat absorbed or released by 1kg of an object to change its temperature by 1o C. The specific heat capacity, c, of a substance of mass, m, and heat capacity, C, can be defined as

$$c = \frac{C}{m}, \text{ or, } c = \frac{Q}{m\Delta T}$$

The SI unit of specific heat capacity is J/kg.°C, or J/kg.K.

Specific heat is a property that is specific to a given type of matter, and substances vary in their specific heat. Table 5.1 holds specific heat capacity of some substances.

Table 5.1: Provides specific heats of some substances at 25 °C and atmospheric pressure.

Substance		Specific heat capacity J/kg.°C
Solids	Aluminum	900
	Iron	448
	Lead	128
	Brass	380
	Glass	837
	Carbon, graphite	709
	Copper	385
	Gold	129
	Silver	233
	Ice (-5°C)	2090
Liquids	Alcohol (ethyl)	2400
	Mercury	140
	Water (15°C)	4186
Gas	Steam (100°C)	2010

Metals tend to have low specific heat. On the other hand, water has very high specific heat compared to most other substances. That is why; water is commonly used as a coolant for machinery because it is able to absorb large quantities of heat.

Example 5.1:

Calculate the heat energy transferred if the temperature of 600 g of lead is raised from 20 °C to 100°C.

Solution: $c_{Pb} = 128 J/kg.K$, $m = 600g$, $\Delta T = 100°C - 20°C = 80°C = 80K$

The amount of heat absorbed by the lead is

$$Q = c_{Pb} m \Delta T = 128\,J/kg.K \times 0.6\,kg \times 80\,K$$
$$Q = 6144.0\,J$$

Review Questions 5.3

1. What does a higher specific heat capacity mean in terms of heat energy transfer?
2. Concrete has a higher specific heat than soil. Use this fact to explain why a city has a higher average temperature than the surrounding countryside at night.
3. How many joules of energy are required to raise the temperature of 100g of gold from 20°C to100°C? (Specific heat of gold is 129 J/kg.°C)
4. Why are coastal climates much more moderate than inland climates?
5. Which heats faster, land or water? You will answer this question by taking measurements carried out to study differences in specific heat capacity.
6. A 0.80 kg aluminum pan on a stove is used to heat 0.20 L of water from 25.0 °C to 85.0 °C. (Hint: Mass of 1 L of water = 1 kg of water, around 4 °C)
   a) How much heat is required?
   b) What percentage of the heat is used to raise the temperature of the pan?
7. Calculate the temperature increase of 50 kg of steel falling from a height of 50 m if 50% of its kinetic energy is converted into heat and assume no heat flow to the surrounding ?

5.4 Thermal expansion

After completing this section, you should be able to:

- *solve problems associated with thermal expansion*

Brainstorming:

1. In the construction of ball bearings (shown in Figure 5.7b) the outer metal ring and the inner metal ring rotate smoothly due to the solid iron balls between them and oil lubrication. Once the iron balls are put in to place, they will be there permanently. Have you ever wondered how the iron metal balls get in to the ball bearings? Discuss with your group and come up with your opinion. Your opinion must be supported by reason.
2. What is the reason for the bending of the railroad, shown in Figure 5.7a?

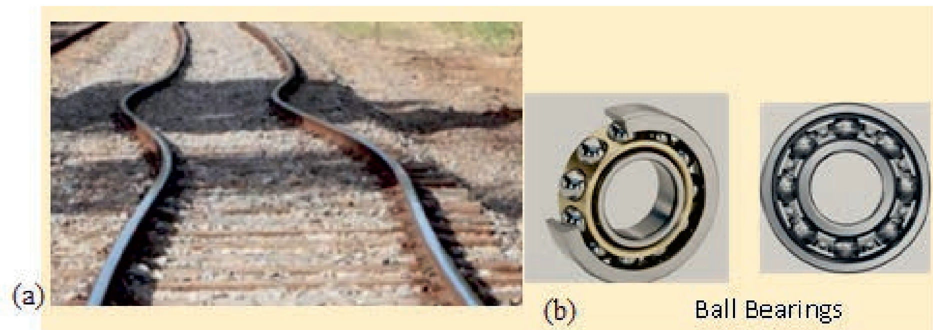


Figure 5.7: (a) Effect of thermal expansion, (b) application of thermal expansion

Thermal expansion is the change in size of matter in response to a change in its temperature. When a substance is heated, the average kinetic energy of its constituting particles is increased.

Key term:

Thermal expansion is the change in size of matter in response to a change in temperature

Thereby, the particles start to vibrate and move more and usually maintain a greater average separation. Except in some cases, most substances expand when heated and contract when cooled. Thermal expansion is also affected by the properties of the substance. For example, solids expand very less as compared to liquids and gases. Gases expand much more than liquids and solids. This happens because intermolecular force of attraction is very strong in solids, intermediate in liquids and very weak in gases. All factors that affect thermal expansion will be treated below.

Thermal expansion can be classified in to three: linear expansion, surface (area) expansion and volume expansion.

Linear expansion

If the change in length is along one dimension (length) over the volume due to a change in temperature, it is called linear expansion. Suppose an object has an initial length of L_0 at some initial temperature T_0. Let the length be changed to L when the temperature is changed to T. Experiments show that if the change in temperature (ΔT) is not too large, the change in length (ΔL) is directly proportional to ΔT and L_0. Then, the change in length is defined as

$$\Delta L = L_0 \alpha \Delta T$$

where,

- $\mathbf{\Delta T = T - T_0}$ is the change in temperature,
- $\mathbf{\Delta L = L - L_0}$ is the change in length,
- $\boldsymbol{\alpha}$ is the linear coefficient of thermal expansion, which has the unit $°C^{-1}$, or K^{-1}, and
- L_0 is the original length.

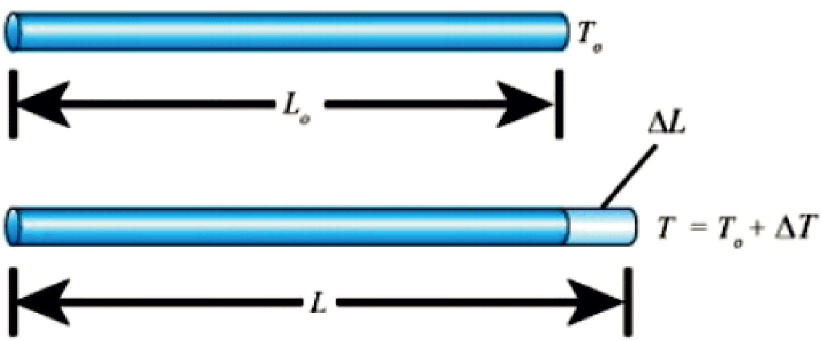


Figure 5.8: Linear expansion

Each substance has a unique linear expansion coefficient; i.e., different substances expand in different amounts for the same temperature change. For example, the coefficient of linear expansion of Aluminum is $2.3 \times 10^{-5} K^{-1}$ means that if the temperature of 1 m long Aluminum is increased by 1 K, its length increases by 2.3×10^{-5} m. By the same change in temperature, the same length of copper wire increases by 1.7×10^{-5} m. This implies that the same change in temperature on different materials of the same length produces different changes in length.

Table 5.2: Linear expansion coefficients of some solids

Material	$\alpha\times10^{-6}\,(^{o}C^{-1})$
Aluminum	24
Brass and Bronze	19
Copper	17
Glass	9
Lead	29
Steel	11
Concrete	12

Example 5.2:

A steel bridge is built in several segments, each 20 m long. The bridge was constructed when the temperature was 20 °C. If a gap of 4 cm is left between neighboring segments what would be the maximum temperature that the bridge can manage before buckling?

Solution:

$T_0 = 20\ °C$, $L_0 = 20.0$ m, $\Delta L = 4$ cm $= 4\times10^{-2}$ m, $\alpha = 1.1\times10^{-5}\,K^{-1}$

The gap is filled from the two sides.

The expected rise in temperature is

$$\Delta T = \frac{\Delta L}{\Delta L_0} = \frac{4.0x10^{-2}m}{1.1X10^{-5}k^{-1}x20.0m} = 180\ K = 180^{\circ}C$$

The temperature at which the bridge may buckle is

$$T = T_o + \Delta T = 20^{\circ}C + 180^{\circ}C = 200^{\circ}C$$

Such big change in temperature may occur if there are exceptional moments such as flowing volcano over the bridge.

Area expansion

The same method as linear thermal expansion can also be applied for area expansion. Thus, consider the plate of surface area A_0 at a temperature T_0, as shown in Figure 5.9. For a small change in temperature of ΔT, the change in area ΔA, is given by

$$\Delta A = A_0 \beta \Delta T$$

where â is called the coefficient of surface (area) expansion, and it can be measured by the unit $°C^{-1}$, or K^{-1}.

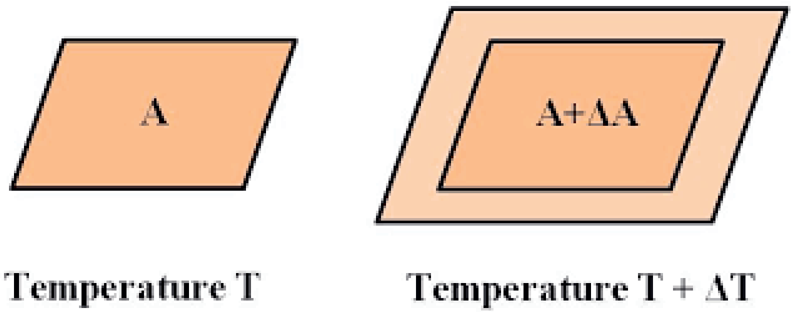


Figure 5.9: Area expansion

Relationship between α and β

Suppose a square plate of side length L_o and surface area A_o, is experienced a change in temperature of ΔT, then the new area in terms of the new length L can be written as:

$$A = L^2 = L_0^2(1 + \Delta T)^2 = A_0(1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$$

Since α is to the order of 10^{-5}, then ignoring the third term containing α^2 ($\sim 10^{-10}$) doesn't affect the result and we get:

$$A = A_0(1 + 2\alpha\Delta T)$$

Comparing this with the original definition, we find that $\beta = 2\alpha$. This is a very good approximation, so we do not need table of values for β as we can easily find it by doubling the value of α.

Example 5.3:

The surface area of one side of a circular brass plate at temperature 100 °C, is 3850 cm^2. To put this plate inside a cylinder, you need to cool it to 0 °C. What is the base area of the cylinder?

Solution:

$T_0 = 100\ ^0C$, $T = 0^0C$, $A_0 = 3850\ cm^2$

The area of the plate at T = 0 °C is obtain as

$$A = A_0(1 + 2\alpha\Delta T)$$
$$A = (3850cm^2)[1 + 2 \times 1.9 \times 10^{-5}K^{-1}(0^\circ C - 100^\circ C)]$$
$$A = 3835.37\ cm^2$$

In order for the plate to pass into the cylinder, either side of its surface area should be reduced to below $3835.37\ cm^2$.

Volume Expansion

When the length, the width and the thickness of the object are equally significant, as shown in Figure 5.10, we need to consider the expansion of the object in three dimensions. In this case, all the three lengths experience significant changes in length with a change in temperature.

Peculiarity of water expansion

The peculiarity of water in its response to temperature changes is that its volume decreases as its temperature increases between 0 °C to 4 °C. The density of water has maximum value at 4 °C.

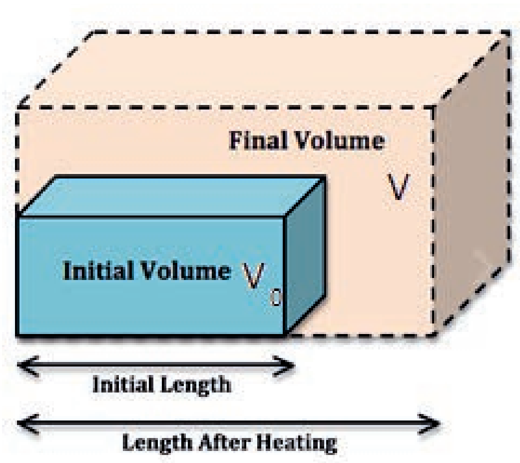


Figure 5.10: Volume expansion

Change in volume ΔV of an object subjected to a small temperature change ΔT, is given by

$$\Delta V = \Delta V_c \Delta T$$

where γ is the coefficient of volume expansion.
The final volume, V, is, then

$$V = V_0(1 + \gamma \Delta T)$$

Both β and γ have the same units as α; that is, °C^{-1}, or K^{-1}.

Relationship of γ, with α and β

Using a method similar to what we have done in deriving relationship between β and α, we find the volume of a cube of length L to be:

$$V = L^3 = L_0^3(1 + \alpha \Delta T) = V_e(1 + 3\alpha \Delta T)$$

Evaluating all the above discussions, we find that:

$$\gamma = 2\beta = 3\alpha$$

Table 4.3: Coefficient of volume expansion of solids and liquids

Material	Volume Expansion Coefficient (β) X 10^{-4} °C^{-1}
Alcohol	1.12
Benzene	1.24
Mercury	1.82
Air	3.67

Example 5.4:

1, A 200 liter aluminum cylindrical reservoir is filled with Ethyl alcohol (γ = 112 x 10^{-5} K^{-1} at the temperature of 20°C. What volume of alcohol will overflow if the system is heated to 100°C.

Solution:

$\alpha_{Al} = 2.31 \times 10^{-5} K^{-1}$, T_0 = 20 °C, T = 100 °C, V_0 =200 lit

The two have the same initial volume, but will have different volumes when their temperature is changed even with the same amount. The difference in their volumes for an increase in temperature ΔT, is equal to the difference between the volume of the alcohol,

$V = V_0(1+\alpha\Delta T)$, *and that of the cylinder*, $V_{Al} = V_0(1+3\alpha_{Al}\Delta T)$. *Thus*

$$V_{lcy} = V_1 - V_{cy} = V_0\Delta T(\gamma_1 - 3\alpha_{Al})$$

Substituting values ans simplifying the result, we obtain

$$V_{lcy} = (200 lit)(80^{\circ}C)(112\times10^{-5})K^{-1} - 3\times2.31\times10^{-5}K^{-1})$$

$$V_{lcy} = 16.81\ lit$$

2. While working in the laboratory, you fill water in a glass flask that holds exactly one litter at temperature 10 ^{0}C. You heat the flask, raising the temperature of the water and flask to 30 ^{0}C. Which material expands more; the glass or the water? The information you need for deciding is the coefficient of expansion.

$$(take\ \gamma_w = 207\text{x}10^{-3}K^{-1}\ for\ water\ and$$

$$\gamma_{g=}3\alpha = 3\text{x}9\text{x}10^{-6}K^{-1} = 2.7\text{x}10^{-5}K^{-1} for\ glass)$$

Solution:

The glass flask and the water both expand when heated. The water expands more, as the result some water spills out. Let ΔV_w be the increase in the volume of water; ΔV_g is the increase in the volume of glass; and ΔV_s is the volume of water spilled. Then,

$$\Delta V_s = \Delta V_w - \Delta V_g$$

$$\Delta V_s = \beta_w V\Delta T - \beta_g V\Delta T = (\beta_w - \beta_g)V\Delta T$$

$$\Delta V_s = (207\times10^{-6}K^{-1} - 27\times10^{-6}K^{-1})\times10L\times(30^{\circ}C - 10^{\circ}C)$$

$$\Delta V_s = 36mL$$

Real and apparent coefficients of cubic expansion

The formula used to calculate change in volume for a change in temperature of a solid can be applied for liquids. However, why we want to discuss volume expansion for liquids is that liquids are treated with their containers and consequently the expansion of the container affects the result for liquids.

While measuring the coefficient of cubic expansion of a liquid, the result is not a real value, rather it is an apparent value since the container itself is affected by the change in temperature. Therefore, the actual coefficient of volume expansion is given by $\gamma_r = \gamma_a + \gamma_c$ where γ_r is the real value, γ is the apparent value and γ_c is the coefficient of cubic expansion of the container.

Example 5.5:

Mercury of volume 10000 cm^3, which is filled in a brass container, is heated from 10 °C to 80 °C. What will be the real and apparent volume expansions of mercury in the brass container?

$$T_0 = 10^\circ C, T = 80^\circ C, \gamma_{m,r} = 18 \times 10^{-5} K^{-1}, \gamma_b = 5.7 \times 10^{-5} K^{-1}$$

The real expansion of mercury is

$$\Delta V_{m,r} = V_0 \gamma_{m,r} \Delta T = (10^4 cm^3) \times (18 \times 10^{-5} K^{-1})(80^\circ C - 10^\circ C)$$

$$\Delta V_{m,r} = 126 cm^3$$

The apparent coefficient of volume expansion of mercury in brass container,

$$\gamma_{app} = 18 \times 10^{-5} K^{-1} - 5.7 \times 10^{-5} K^{-1} = 12.3 \times 10^{-5} K^{-1}$$

The apparent volume expansion of the mercury becomes

$$\Delta V_{app} = \gamma_{app} V_0 \Delta T = 12.3 \times 10^{-5} K^{-1} (10^4 cm^3)(70k)$$

$$\Delta V_{app} = 86.1 cm^3$$

Thermal expansion of gases

Gases expand more at ordinary temperature than solid and liquid. In the case of gases, both temperature and pressure determine the change in volume of the gas. Increasing the temperature leads to increase in the volume of the gas while the increase in the pressure leads to decrease in volume. In this case, you cannot use the method used in liquids and solids to determine the volume expansion coefficient of a gas.

Applications of thermal expansion in solids

? Brainstorming

1. Discuss and present the applications of thermal expansion in your locality.
2. Explore how mercury thermometer is calibrated. What property is used for calibration and why is the mercury made to rise up a capillary tube ? What would be the shape of the metals look like after heating up? Describe the shapes

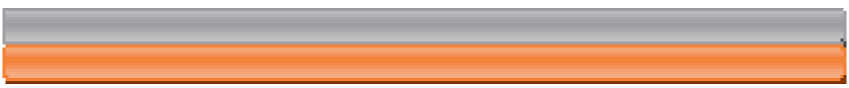


Figure 5.11: Two different metals attached together

3. Follow the demonstration by your physics teacher and explain Gravesend's ring -and -ball experiment to describe thermal expansion of solids. Experiment on thermal expansion of gases by putting an inflated balloon on a hot water.

Some applications of thermal expansion:

- **Riveting two metal sheets tightly:** As the rivet cools it contracts and makes the joint between the two metal sheets tighter, as shown in Figure 5.12.

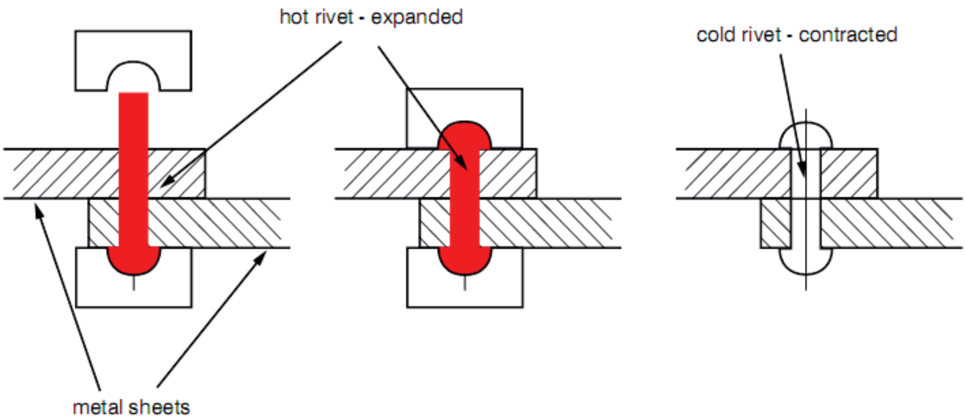


Figure 5.12: Riveting metal sheets

The bimetallic strip: A bimetallic strip is made out of two metals of different coefficients of linear expansion bonded together. For example iron and brass bonded together. The coefficient of linear expansion (α) of iron (1.1×10^{-5} K^{-1}) is less than that of brass (1.9×10^{-5} K^{-1}). When the strip is heated, the brass expands faster than the iron and as a result the strip bends towards the iron side, as shown in Figure 5.13.

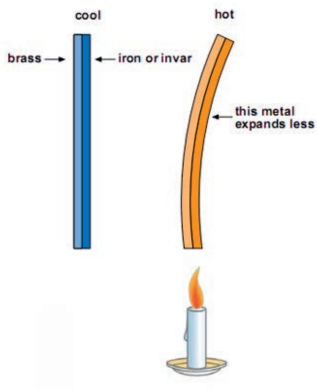


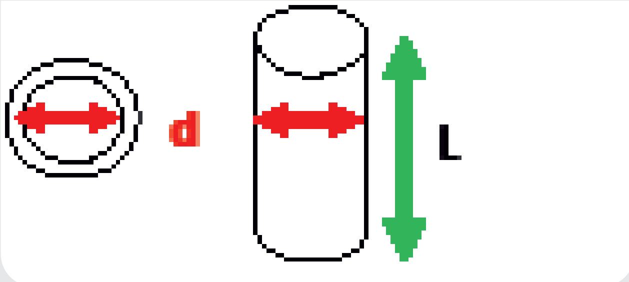
Figure 5.13: Bimetallic strip

A bimetallic strip can be used as a switch in an electric circuit that turns on and off according to the temperature. This switch is what we call thermostat.

- **Removing a stopper:** If you find it difficult to remove the stopper from a glass bottle, just heat its neck slightly. Now the neck of the bottle expands and the stopper comes out easily.
- **Liquid thermometers:** The expansion of mercury or alcohol in a liquid thermometer is an application of the thermal expansion of liquids.
- **Boiling of liquids:** When liquid is heated, the molecules of the liquid expand and the liquid changes its state.
- **Sealed bottles:** Medicine bottles are left out with some space at the top so that the medicine does not spill off after expansion.

Review Questions 5.4

1. A steel cork of a soft drink bottle at normal temperature is too tight to open it. Which method helps to open it easily; heating or cooling?
2. Does heating a thin, circular ring make it wider or narrower?
3. A segment of steel railroad track has a length of 60 m when the temperature is 0.0°C. What is its length when the temperature is raised to 40.0°C?
4. What would happen if, upon heating, the glass of a thermometer expands more than the liquid inside?
5. Why do we use mercury in a glass thermometer while the glass undergoes thermal expansion with it?.
6. When a metal ring and a metal sphere with equal radius are both at room temperature, the sphere does not pass through the ring. After the ring is heated, the sphere can pass through the ring. Why?
7. The diameter of a piston is 10 cm. To what temperature should a steel ring of diameter 9.9 cm be heated so that the piston can pass through the ring?
8. Suppose a 300 L steel barrel is filled with Ethyl alcohol at a temperature of 10°C. How much Ethyl alcohol over flows if the system is heated to 100 °C?
9. Consider three metal rods, each has a length of 40.0 cm at a temperature of 0°C. When the temperature on them is increased by 100 °C, the length of the one is increased by 0.08 cm, the second one by 0.05 cm. The third rod which is made of combinations of the first two metals, extends by 0.06 cm. Find the length of each portion of the composite bar.
10. Suppose that a steel hoop could at 0.0 degree Celcius be constructed around the earth's equator, just fitting it at a temperature 25°C. What would be the thickness of space between hoop and the earth if the temperature of the hoop were increased by 5 °C. $\left(\alpha_{steel} = 1.2 \times 10^{-5}\,°C^{-1}\right)$
11. Two brass rods and one steel rod of equal length L_0 and equal cross-sectional are joined rigidly at their ends. All rods are in the state of zero tension at 0° C. Find the length of the system when the temperature is increased to T. (Take $\alpha_{steel} = \alpha_s$, $\alpha_{brass} = \alpha_b$)
12. The length of a copper wire at 0°C is 100 cm and it is is shorter by 0.05 cm from a steel wire at the same temperature. To what value should the temperature on the two wires be increased so that the two wires attain the same length?
13. Calculate the change in area if the surface area of a container with a glass cover is 0.05 m2 and $\alpha = 9 \times 10^{-6}\,°C^{-1}$ when it is exposed to a temperature difference of 96 °C.



14. Calculate the volume difference of a 1.00 m^3 of water when heated from 10 °C to 90°C. $\alpha_{water} = 2.1 \times 10^{-4}$ °C^{-1}.

5.5 Change of phase

After completing this section, you should be able to:

- *discuss the phenomena of change of phase*

Brainstorming

You know that heat is a transfer or flow of energy due to temperature difference.

1. Is there a situation in which heat transfers without bringing change in the temperature of the substance?
2. Have you ever noticed that melting or boiling undertakes without any change in temperature while heat is being transferring? What happened to the heat supplied?

Anything that has mass is made up of matter – an all-encompassing word for atoms and molecules that make up our physical world. We describe this matter as existing in states (sometimes referred to as phases). Most people are familiar with three states of matter – solids, liquids and gases. The term phase is used to describe a specific state of matter, such as a liquid, solid or gas.

Most substances can exist in different states. For example, water exists in liquid state, water as solid state, such ice, and water as gaseous state such as steam or water vapor. All these states or phases are the result of being at different temperatures. Phases of matter are inter-changeable. That is, matter can be changed from one state to another by adding or taking heat from the object.

What is phase change?

What is important about phases of matter is that matter undergoes changes in phase. When a substance is supplied with heat, it either changes its temperature or changes its internal energy. When internal energy is changed as heat is supplied the substance undergoes phase change. At phase change the heat supplied is used to change the molecular structure to break the bonds between molecules keeping the temperature constant. The transition from one phase to another is called phase change or phase transition, for instance, solid to liquid (melting), liquid to gas (evaporation).

Latent heat

The energy absorbed or released by a substance during a change in its physical state (phase) that occurs without changing its temperature is called **latent heat**. The amount of heat absorbed or released by an object of mass, m, is given mathematically by

$$Q = mL$$

where L is called the latent heat of the substance; i.e., $L = L_f$ for heat of fusion and $L = L_v$ for heat of vaporization. Its SI unit is joule per kilogram (J/kg).

Latent heat depends on the nature of the phase change as well as on the properties of the substance.

Key terms

- *Latent heat of fusion is the amount of heat absorbed by 1kg of a substance from solid to liquid - leaving the temperature of the system unaltered.*
- *Latent heat of vaporization is the amount of heat absorbed by 1kg of the substance to change its phase from liquid to gas.*

The word "latent" means "hidden", so why is the term hidden? The reason why it is called latent heat is that it cannot be measured by a thermometer. The energy involved in a phase change is used to break bonds, and hence there is no increase in the kinetic energies of the particles (therefore, no rise in temperature). An important point that we should consider is that at a given pressure, any phase change undertakes at a specific temperature.

Latent heat of fusion (L_f) is defined as the amount of heat absorbed by 1 kg of a substance to change its state from solid to liquid without any change in temperature. The temperature at which fusion takes place is called the melting point of the substance. For example, the melting point of water is $0\,^{\circ}C$. Since fusion and solidification of a given substance are the exact opposite processes, they have equal in magnitude, but opposite in sign.

Heat of vaporization (L_v) is defined as the quantity of heat transferred to change 1 kg of a substance from liquid state to gaseous state keeping the temperature constant. The temperature at which a liquid substance changes to steam is called boiling point. For example, the boiling point of water is 100 °C.

The temperature versus heat graph, in Figure 5.14, shows temperatures at which phase changes take place.

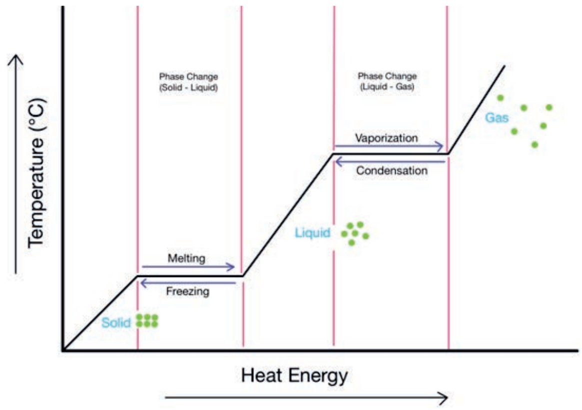


Figure 5.14: Temperature versus heat diagram

Table 5.4: Latent heat of fusion (L_f) and latent heat of vaporization (L_v)

Substance	Melting Point (°C)	L_f (J/kg)	Boiling point (°C)	L_v(J/kg)
Oxygen	-218.79	1.38×10^4	-182.97	2.13×10^5
Ethyl alcohol	-114	1.04×10^5	78	8.54×10^5
Water	0.00	3.33×10^5	100.00	2.26×10^6
Lead	327.3	2.45×10^4	1750	8.70×10^5
Copper	1 083	1.34×10^5	1187	5.06×10^6

Example 5.6

Suppose the temperature of 4 kg of water is increased from -50 °C to steam at 100 °C, at standard pressure. Assume the specific capacity of ice is constant, 21 J/g.K. How much energy is absorbed by the water? Draw the temperature versus heat added graph.

Solution:

$c_i = 2100\ J/kg.K,\ L_f = 3.36\times10^5\ J/kg,\ c_w = 4200\ J/kg.K, T_i = -50°C,$

$T_m = 0°C,\ T_b = 100°C,\ L_v = 32.25\times10^6\ J/kg,$

The heat energy absorbed by the ice to increase the temperature from $T_i = -50°C$ to melting point $(T_m = 0°C)$ is,

$$Q_i = mc_i(T_m - T_i) = 4kg \times 2100 J/kg.K \times [0°C - (-50°C)] = 420000\ J$$

The energy absorbed by the ice at 0°C to water at 0°C,

$$Q_f = mL_f = 4kg \times 3.36 \times 10^5 J/kg = 1344000J$$

The energy absorbed by the water to increase its temperature from 0°C to boiling point

$$(T_b = 100°C),$$

$$Q_w = mc_w(T_b - T_m) = 4kg \times 4200J/kg.K \times (100°C - 0°C) = 1680000\ J$$

The energy absorbed to change water at 100°C to steam at 100°C,

$$Q_s = mL_v = 4kg \times 32.25 \times 10^6\ J/kg = 1.29 \times 10^8\ J$$

The total energy absorbed by the water is, then

$$Q = Q_i + Q_f + Q_w + Q_s$$

$$Q = 420000\ J + 1344000J + 1680000\ J + 129000000\ J$$

$$Q = 132444000\ J$$

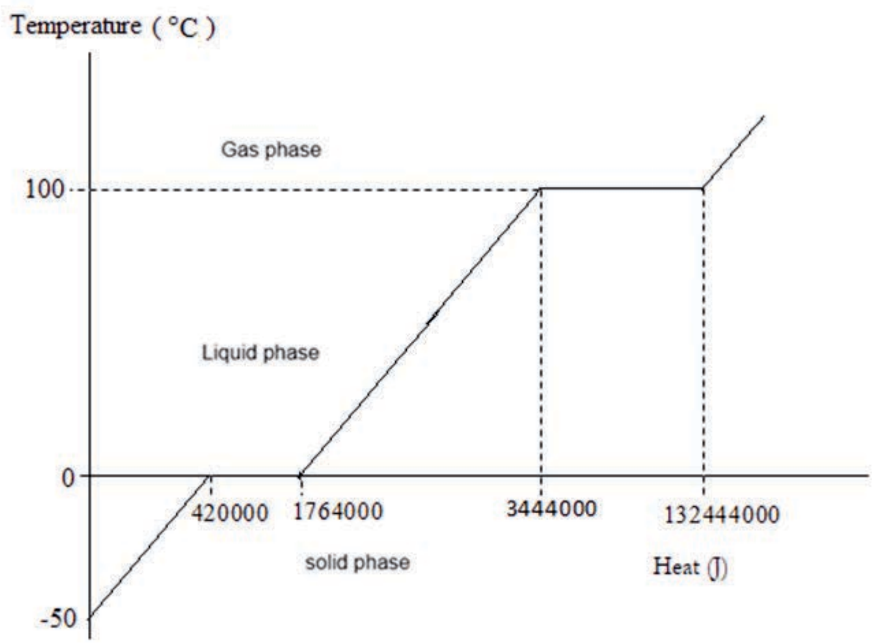


Figure 5.15. illustrates how the temperature varies when we add heat continuously to a specimen of ice with an initial temperature below 0^0C (point A).

Phase diagram

Phase diagram is a graphical representation of the physical states of a substance under different conditions of temperature and pressure. The commonly known phase diagram has pressure on the y-axis and temperature on the x-axis. As you can see in Figure 5.16, there are three major curves for boiling, freezing or melting or fusion, and sublimation on the phase diagram. As you cross the lines or curves on the phase diagram, a phase change occurs.

At several points on the graph, the temperature and pressure combinations will create a state of phase equilibrium between two phases. This is illustrated by a line along those temperature-pressure combinations called, "phase equilibrium lines." At every point along that line, there is equilibrium of both phases.

Key terms

- *The triple point represents the combination of pressure and temperature that facilitates all phases of matter at equilibrium.*
- *Critical Point is the point in temperature and pressure on a phase diagram where the liquid and gaseous phases of a substance merge together into a single phase.*

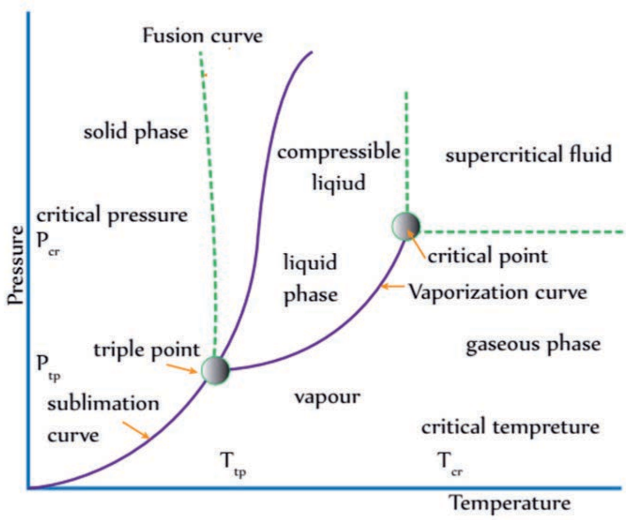


Figure 5.16: Phase diagram

The triple point: The triple point is a point at which the three lines on the pressure-temperature graph converge, offering an equilibrium state where all the three phases of solid, liquid and gas exist simultaneously. For example, the single combination of pressure and temperature at which liquid water, solid ice, and water vapor can coexist in a stable equilibrium occurs at exactly 273.1600 K and a partial vapor pressure of 611.657 Pa.

Sublimation:

Sublimation is the term for a phase change that proceeds directly from the solid phase to the vapor phase without first making the transition to the liquid state.

Critical point (or, critical state): The critical point occurs where the critical temperature and critical pressure of a substance meet. Above this point clear phase boundaries cease to exist. For water the critical point is around 647 K and 22 MPa. The properties of the gas and liquid phases merge together giving only one phase at the critical point. Beyond the temperature of the critical point, the merged single phase is known as a supercritical fluid. A substance cannot form a liquid above its critical temperature, regardless of the applied pressure. Above the critical temperature, the molecules have enough kinetic energy to overcome the intermolecular attractive forces.

The critical point is different to the boiling point of a substance. The boiling point of a substance is usually meant to mean the boiling point at standard atmospheric pressure. However, the actual boiling point of a substance depends on the surrounding pressure. From the phase diagram the substance has a number of boiling points following the curving blue line. The lower the pressure, the lower is the boiling point. The critical point occurs only at one specific temperature and pressure.

Table 5.5: Liquid–vapor critical temperature and pressure for selected substances

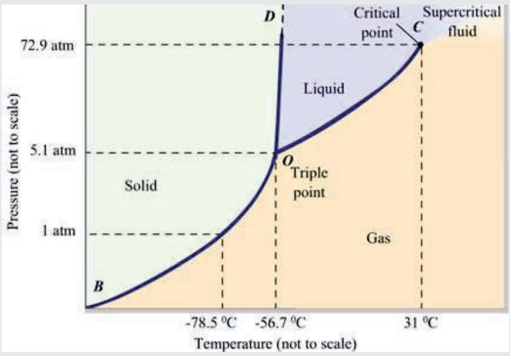
Substance	Critical temperature	Critical pressure (absolute)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)

Review Questions 5.5

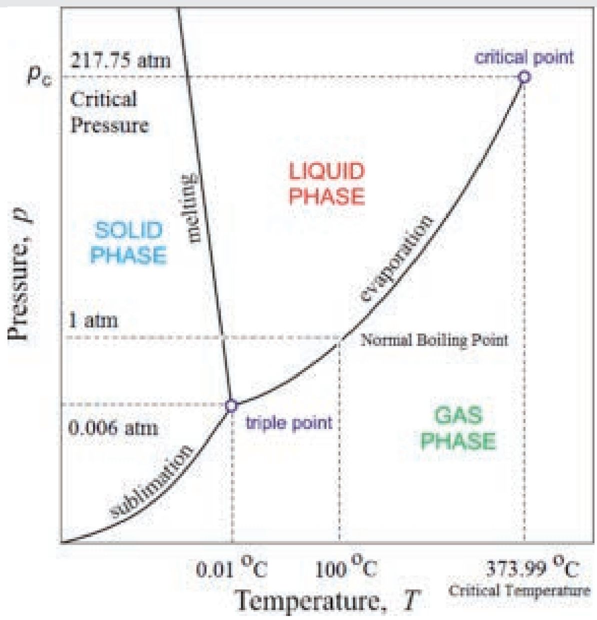
1. Why does the melting point of ice get lower at higher pressures?
2. Define critical temperature. Does it vary with pressure? Is it the same as boiling point?
3. Does the boiling point increase indefinitely with increased pressure?
4. Collect the triple points of water, carbon dioxide, mercury, and zinc, in the internet. Compare temperatures of these at triple point with the corresponding melting points at standard pressure.
5. Is it possible to compress steam to form a liquid if the temperature is higher than 374°C?
6. Calculate the heat energy required to melt 10 g of copper at its melting point. (Lf for copper = 209 000 J/kg).
7. Calculate the heat energy released to cool 0.1kg of aluminium from 2,470°C

(boiling point of Al) of aluminium gas to solid aluminium at 660.3°C (melting point of Al) at standard pressure.

8. The diagram shows the phase diagram of CO_2.
   a. At what temperature and pressure can the solid, liquid and vapor phases of CO2 coexist in equilibrium?
   b. At what temperature does carbon dioxide undergoes change in state at standard pressure? What type of state change does it undergo?
   c. What are the critical temperature and pressure for CO2?



9. The diagram below shows the phase diagram of water.
   a. At what temperature and pressure can the solid, liquid and vapor phases of water coexist in equilibrium?
   b. Is water solid, liquid or gas at 0.01 °C under 1 atm?



5.6 Calorimetry

After completing this section, you should be able to:

- *solve problems associated with Calorimetry*
- *investigate a phenomena related with heat capacity from every day experience.*

? Brainstorming

You have learned specific heat and heat capacity and later specific latent heat. Discuss how you can measure the specific heat and latent heat of a substance.

Calorimetry is the measurement of the quantity of heat exchanged between substances. The experimental approach of measuring heat capacities and the heat changes during chemical and physical processes is called calorimetry. A calorimeter is a device used to measure specific heat capacity and specific latent heat of solids or liquids.

5.6.1. Measuring specific heat capacity

There are several methods we can use to determine the specific heat capacity of a substance, but remember that to calculate this value we will always need to know:

- the mass of the substance,
- the amount of heat energy supplied,
- its starting temperature, and
- the final temperature.

All of these slightly different approaches of specific heat measurement methods involve heat exchange. This is done by heating one material and bringing it in thermal contact with a relatively colder body so that they exchange heat. The heat from the hot body is used to warm a colder body. In approaching these heat exchange calculations we use the principle that for an isolated system, the heat energy lost (Q_{lost}) from the hot body will equal the heat gained (Q_{gain}) by the cold body.

Heat gained by cold object = Heat lost by hot object

Example 5.7

A 0.05 kg bar of unknown metal is heated to 200.0°C and then immersed into a container with 0.40 kg of water initially at 20.0°C. If the final equilibrium temperature of the system is 22.4°C, find the specific heat of the metal.

Solution:

Using the heat exchange method,

$$Q_w = -Q_s,$$

$$m_w c_w (T_f - T_c) = -m_s c_s (T_f - T_h)$$

$$0.4\ \text{kg} \times (4186\ \text{J/kg}) \times (22.4°\text{C} - 20.0°\text{C}) = -0.05\ \text{kg} \times c_s \times (22.4°\text{C} - 200.0°\text{C})$$

Solving this for c_s, we obtain,

$$c_s = 453\text{J/kg.°C}.$$

Electrical heating to determine the specific heat capacity of a solid

The electrical method can be used to find the specific heat capacity of a solid or a liquid. The diagram in Figure 5.17 The electrical method can be used to find the specific heat capacity of a solid or a liquid. The diagram in Figure 5.17 shows the apparatus used to find the specific heat capacity of a solid by electric method. In this case, the hot body losing the heat energy is the electrical heater. We assume only the solid object gains heat. We ignore any heat energy supplied to the thermometer and any heat lost to the surroundings, and assumed that the electrical heater is 100% efficient in this calculation.

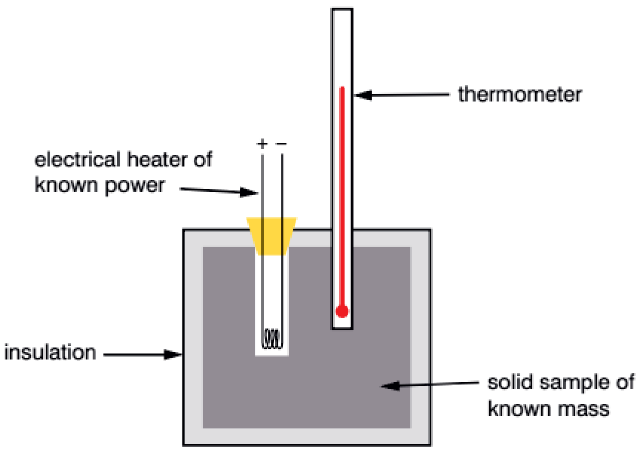


Figure 5.17: Finding the specific heat capacity of a known mass

Key term:

Calorimetry *is the measurement of the quantity of heat exchanged between substances.*

If we know the power rating of the heater and we know how long the heater is switched on for we can determine the quantity of heat energy supplied to the block as:

$$\mathbf{E = Pt}$$

where E is the energy supplied in the time t, and P is the power rating of the heater.

For a given mass, m_s, and change in temperature, ΔT, the specific heat capacity, c_s, can be determined from the principle of calorimetry as

$$Pt = m_s c_s \Delta T$$

Example 5.8

A 100 W electrical heater, running for 5 minutes, warmed a 0.50 kg block of an unknown metal. The initial temperature of the metal block was 20.0 °C and its final temperature was 86.6 °C. Determine the type of the metal.

Solution

$$t = 60\times 5 \text{ s} = 300 \text{ s}, \ \Delta T = T_f - T_i = 86.6°C - 20°C = 66.6°C$$

The total energy supplied to the metal block:

$$E = Pt = 100 \text{ W} \times 300 \text{ s} = 30000 \text{ J}$$

The heat gained by the block: $Q = m_s c_s \Delta T = 0.5\text{kg} \times c_s \times 66.6°C$

Since heat supplied equals heat gained, we have

$$0.50 \text{ kg} \times c_s \times 66.60 \text{ °C} = 30000.00 \text{ J}$$

$$c_s = \frac{30000.00 \text{ J}}{0.50\text{kg} \times 66.60°C} = 900.90 \text{J/kg.°C}$$

Comparing this value with the value given in Table 5.1, the metal block is an aluminum block.

I. Electrical heating to determine the specific heat capacity of a liquid

Electrical heating can also be used to determine the specific heat capacity of a liquid. An insulated container could be used for the liquid. Alternatively, a calorimeter could be used to hold the liquid.

In this case, the liquid is continuously stirred and we will take into account the heat energy supplied to the calorimeter as well as that supplied to the liquid. The diagram in Figure 5.18 shows the use of a calorimeter to determine the specific heat capacity of water.

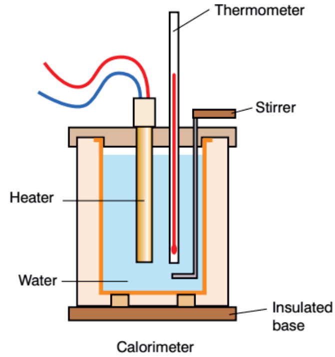


Figure 5.18: Determining the specific heat capacity of a liquid

Applying the heat exchange method for a given mass of liquid, m_l, specific heat capacity, c_l, using a calorimeter of mass, m_c, specific heat capacity, c_c, and change in temperature, ΔT,

$$Pt = m_l c_l \Delta T + m_c c_c \Delta T$$

$$c_l = \frac{Pt}{m_l \Delta T} - \frac{m_c c_c}{m_l}$$

Example 5.9

A copper calorimeter and stirrer of mass 350 g contains 227 g of a liquid. A 1000 W heater running for 1 minute and 15 seconds heated this combination of liquid and calorimeter from 20 °C to 88 °C. Calculate the specific heat capacity of this liquid given that the specific heat capacity of copper is 420 J/kg.K.

Solution:

$t = 75$ s, $P = 1000$W, $\Delta T = 88\ °C - 20\ °C = 68\ °C$, $c_c = 420$ J/kg.K, $m_l = 0.225$kg, $m_c = 0.35$ kg

Heat supplied: $Pt = 1000W \times 75\ s = 75000J$

Heat gained by the calorimeter and the liquid:

$$Q = m_l c_l \Delta T + m_c c_c \Delta T$$

$$Q = 0.225kg \times c_l \times 68\ °C + 0.350\ kg \times 420\ J/kg.°C \times 68\ °C$$

$$Q = (17\ kg.°C)c_l + 9996\ J$$

Using the heat exchange method,

$$Pt = m_l c_l \Delta T + m_c c_c \Delta T$$

$$75000\ J = (15.3\ kg.°C)c_l + 9996\ J$$

After some rearrangements, we obtain

$$c_l = \frac{75000\ J - 9996\ J}{15.3\ kg.°C} = 4{,}248.6 J/kg.°C$$

The type of the liquid is water.

Mixture method

This method can be adapted to measure the specific heat capacity of a solid or liquid. The diagram in Figure 5.19 shows the method used to determine the specific heat capacity of a solid. The solid, of known mass, m_s, is heated in a water bath to a temperature of T_h. This temperature will be the initial temperature of the solid. The solid is then quickly transferred to the cold water of known mass, m_w, in the calorimeter. Once in the calorimeter, the hot object (in this case a steel bolt) loses some heat energy to the colder water, and the colder calorimeter and stirrer. We stir the water and record the highest temperature on the thermometer, which will be the final temperature, T_f, of the mixture.

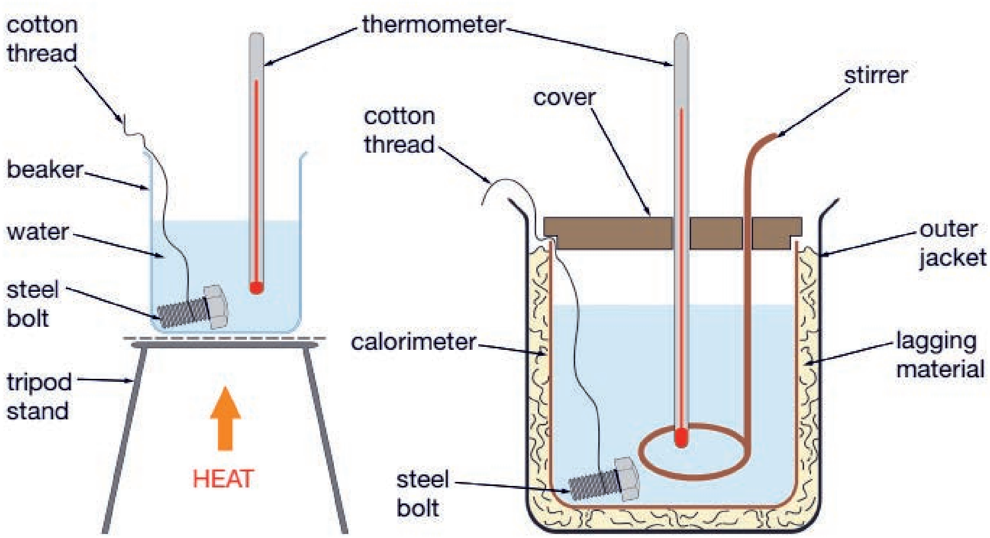


Figure 5.19: Determining the specific heat capacity of a solid (bolt in this case) using the method of mixtures

In this case, the heat lost by the hot solid object (in this case, the bolt) will be equal to the heat gained by the water and the calorimeter/stirrer.

$$-Q_s = Q_w + Q_c$$

where,

- The heat lost by the solid: $-Q_s = -m_s c_s (T_f - T_h) = m_s c_s (T_h - T_f)$
- The heat gained by the cold water: $Q_w = m_w c_w (T_f - T_c)$
- The heat gained by the calorimeter: $Q_c = m_c c_c (T_f - T_c)$

Substituting these expressions in the former equation, we obtain an expression, from which the specific heat capacity of the liquid can be derived:

$$m_s c_s (T_h - T_f) = m_w c_w (T_f - T_c) + m_c c_c (T_f - T_c)$$

Example 5.10:

In the above activity, suppose the temperature and mass of the bolt is 100 °C and 125 g, respectively. Mass of water is 200 g, and mass of copper calorimeter, copper stirrer is 100 g, and initial temperature of water and calorimeter is 20°C. If the equilibrium temperature is 25°C, what is the specific heat capacity of the bolt?

Solution:

Heat lost by the bolt:

$$-Q = m_b c_b (T_h - T_f) = 0.125\text{kg} \times c_b (100°\text{C} - 25°\text{C}) = (9.375\text{kg.°C}) c_b$$

Heat gained by the water and calorimeter + stirrer:

$$Q = (m_w c_w + m_c c_c)(T_f - T_c)$$

$$Q = (0.2\text{kg} \times 4200\text{J/kg.°C} + 0.1\text{kg} \times 385\text{J/kg.°C})(75°\text{C}) = 65{,}887.5\text{J}$$

Now we know that,

Heat lost by the bolt = Heat gained by the water and calorimeter + stirrer

$$(9.375\text{kg.°C}) c_b = 65{,}887.5\text{J}$$

$$c_b = \frac{65{,}887.5\text{J}}{9.375\text{kg.°C}} = 7028\text{J/kg.°C}$$

Activity 5.3

Discuss uses of specific heat in your daily life in groups and share to your class mates.

Uses of specific heat in our daily life are given below:

- Utensils for making tea or coffee, or cooking vegetables or rice, are made of low-specific heat materials. They have a polished bottom. As a result, they heat up faster. For example, copper, aluminum, and so on.
- High specific heat materials can be used as insulators. Wood, for example, has high specific heat. During the summer, wooden houses will keep the inside cooler. Builders can select appropriate building materials based on location and altitude. This enables the construction of warmer or cooler houses.
- Utensil handles at home are made of materials with high specific heat. They provide thermal insulation. Specific heat also influences insulating capability and conductivity.
- Probably the most relevant, real-life example of how a substance's specific heat capacity affects your everyday life is the specific heat of water with regard to the ocean as a heat reservoir. Essentially, the oceans of the Earth are enormous heat storage systems and the high specific heat capacity of the ocean's water is why we have stable (or relatively stable) temperature ranges within our climate systems.

5.6.2. Measuring specific latent heat

Measuring specific latent heat of ice or that of steam can be done by the two methods.

- Electrical method: Energy is supplied for a known time by an electrical heater of known power.
- Method of mixture: Mixing water and ice in a calorimeter.

Measurement of specific latent heat of ice using mixture method

The apparatus in Figure 5.20 can be used to determine the specific latent heat of fusion, L_f, of ice by mixture method. The copper calorimeter and stirrer is weighed before being half filled with water. The mass of the water present in the calorimeter is then determined before the water is heated to at least 10 °C above room temperature. Small quantities of ice are then added to the water, while stirring, until the temperature is below room temperature and all the ice has melted. The mass of the calorimeter, stirrer and water is then determined to find out the mass of ice added.

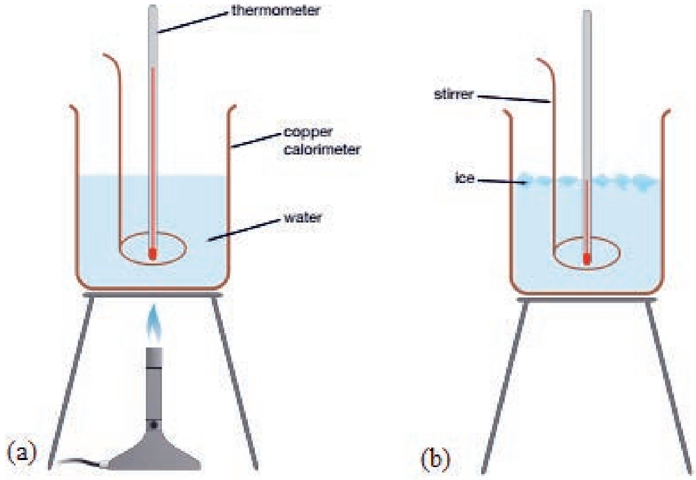


Figure 5.20: A simple experiment to determine the specific latent heat of fusion of ice.

We will use the principle here that the heat energy lost from the water and the calorimeter will go to warming and melting the ice and then warming the cold water produced when the ice melts.

Heat absorbed by ice and cold water = Heat liberated from calorimeter/stirrer and water

$$m_i L_i + m_i c_i \left(T_f - T_{0i}\right) = m_{hw} c_w \left(T_{0w} - T_f\right) + m_c c_c \left(T_{0w} - T_f\right)$$

Example 5.11:

The following experimental data are obtained using this method to determine the specific latent heat of fusion of ice.

- Mass of calorimeter, $m_c = 0.15$ kg,
- Mass of water, $m_w = 2.00$ kg.
- Mass of ice, m_i, added = 0.60 kg
- Initial temperature of ice, $T_{0i} = -10$ °C
- Initial temperature of water, $T_{0w} = 49$ °C
- Final temperature of water, $T_f = 20$ °C
- Specific heat capacity of water, $c_w = 4200$ J/kg.K
- Specific heat capacity of ice, $c_i = 2100$ J/kg.K
- Specific heat capacity of calorimeter, $c_c = 420$ J/kg.K

Solution

$$0°C - T_{0i} = -(-10°C) = 10°C, T_f - 0°C = 20°C,$$

$$T_{0w} - T_f = 49°C - 20°C = 29°C$$

The latent heat of fusion of ice L_f may be calculated from the following equation:

$$m_i L_i + m_i c_i (0°C - T_{0i}) + m_i c_w (T_f - 0°C) = m_{hw} c_w (T_{0w} - T_f) + m_c c_c (T_{0w} - T_f)$$

Substituting values in to this equation, we obtain

$$L_f = 346{,}045$$

Measurement of specific latent heat of steam

When steam, at 100 °C, is bubbled into water, it first condenses to water and then its temperature decreases as it loses heat to the surrounding water. If this occurs in an insulated container then the heat lost by the steam is equal to the heat gained by the water and the container.

The specific latent heat of vaporisation of water (L_v) can then be calculated using the formula:

$$m_s L_v + m_s c_w \Delta T_s = m_c c_c \Delta T_c + m_w c_w \Delta T_c$$

where m_s is mass of steam, m_w and m_c are masses of water and calorimeter, respectively; c_c and c_w are specific heat capacities of calorimeter and water, respectively; $\Delta T_s = T - T_s$ is fall in temp of condensed steam, and $\Delta T_c = T - T_c$ is rise in temp of calorimeter or rise in temp of water; provided that T is the final temperature and T_s (=100°C in this case) and T_c are initial temperatures of steam and cold water, respectively.

Group Discussion

Discuss uses of specific latent heat in your daily life in groups and share to your class mates.

Daily life is filled with examples of latent heat:

- Steaming Food
- Cooling drinks with cold water and ice
- Extinguishing fire by using boiling water
- Melting Ice on the road by using Salt
- Steam electric energy
- Transformation of water to cloud then to water/rain is the result Latent

heat.

Activity 5.4

1. In a small group, devise and carry out an investigation into the amount of heat necessary to convert a known quantity of ice to water at 0°. Show your plans to your teacher. How will you measure how much heat is supplied? What measurements will you need to take?

Review Questions 5.6

1. A 1.0 kg bar of unknown metal is heated to 400.0°C and then immersed into a container with 2.0 kg of water initially at 15.0°C. If the final equilibrium temperature of the system is 20.8°C, find the specific heat of the metal. What is the type of the metal? Neglect the container specific heat and mass.
2. A calorimeter of mass 250 g contains 200 g of water. A 1000 W heater running for 5 minutes, heated this combination of liquid and calorimeter from 20 °C to 80 °C. Calculate the specific heat capacity of this calorimeter
3. In an experiment to determine the specific heat of a metal, a 0.25 kg block of the metal at 200 °C is dropped in a copper calorimeter of mass 0.02 kg containing 150 cm^3 of water at 25 °C. The final temperature is 40 °C. What is the specific heat of the metal? If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?
4. A copper block of mass 20 kg is heated in a furnace to a temperature of 500 °C and then immediately placed on a large ice block. What is the maximum amount of ice that can melt?

UNIT SUMMARY

- Heat is energy in transit due to temperature differences.
- Two objects are said to be in thermal equilibrium if they are equal in temperature. Two or more objects, which are in thermal equilibrium, do not exchange heat energy.
- Internal energy of a system or a body with well defined boundaries is the total of the kinetic energy due to the motion of molecules and the potential energy associated with the vibrational motion and electric energy of atoms within molecules. It includes the energy in all the chemical bonds. Internal energy increases with rising temperature and with changes of state or phase from solid to liquid and liquid to gas.
- Heat and work are two different ways of transferring energy from one system to another. Heat is the transfer of thermal energy between systems, while work is the transfer of mechanical energy between two systems. Heat can be transformed into work and vice verse, but they aren't the same thing. Heat and work both contribute to the total internal energy of a system.
- Conduction is the process by which heat energy is transmitted through collisions between neighboring atoms or molecules.
- Convection (or convective heat transfer) is the transfer of heat from one place to another due to the movement of fluid.
- Radiation heat transfer occurs via electromagnetic waves. Unlike conduction and convection, radiation does not need a medium for transmission.
- Heat capacity is the amount of heat required to raise the temperature of an object by 1 $^{\circ}C$. It is mathematically defined as

$$C = \frac{Q}{\Delta T}$$

where Q is heat transferred to the object, ΔT is change in temperature of the object and C is heat capacity of the object. The SI unit of C is joule per kelvin (J/K); you know that 1 J/K=1J/ $^{\circ}C$).

- Specific heat capacity is the change in temperature of a unit mass of a substance by 1o C. The mathematical form of c is

$$c = \frac{C}{m}, \text{ or, } c = \frac{Q}{m\Delta T}$$

The SI unit of specific heat capacity is J/kg.°C, or J/kg.K.

- Thermal expansion is the tendency of matter to change in shape, volume, and area in response to a change in temperature.
- Thermal expansion of solids can be classified in to three: linear expansion, surface (area) expansion and volume expansion. We can determine only cubic thermal

expansion for liquids and gases as they do not have their own definite shapes.

- When a changed in temperature ΔT caused a change in length of ΔL on a rod of original length L_0 at an initial temperature T_0 and negligible width and thickness, the fractional change in length is given by,

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

where α is the linear coefficient of thermal expansion, has unit of $^0C^{-1}$.

- When the area of a plate changes from A_0 to A when the temperature changes from T_0 to T, we have the relation given by:

$$A = A_0\beta(T - T_0) = 2A_0\alpha(T - T_0$$

where $\beta = 2\alpha$ is the area coefficient of thermal expansion, having a unit of $^0C^{-1}$.

- Similarly, when the volume of an object changes from V_0 to V when the temperature changes from T_0 to T, we have:

$$V = V_0\gamma(T - T_0) = 2A_0\beta(T - T_0) = 3A_0\alpha(T - T_0)$$

where $\gamma = 2\beta = 3\alpha$ is the volume coefficient of thermal expansion, with a unit of $^0C^{-1}$.

- In measuring he coefficient of cubic expansion of a liquid in a container, we get an apparent value since the container itself experiences the change in volume. The real value of coefficient of volume expansion of the liquid is given by

$$\gamma_r = \gamma_a + \gamma_c$$

where γ_r is the real value, γ_a is the apparent value and γ_c is the coefficient of cubic expansion of the container.

- The energy absorbed or released by a substance during a change in its physical state (phase) that occurs without changing its temperature is called latent heat. Latent heat on by an object of mass, m, is given mathematically by

$$\mathbf{Q = mL}$$

where L is called the latent heat of the substance; i.e., $L = L_f$ for heat of fusion and $L = L_v$ for heat of vaporization. Its SI unit is joule per kilogram (J/kg).

- Latent heat depends on the nature of the phase change as well as on the properties of the substance.
- Latent heat of fusion is the amount of heat absorbed to convert a unit mass of a substance from solid to liquid - leaving the temperature of the system unaltered.
- Latent heat of vaporization is the amount of heat absorbed to change a unit mass of liquid to gas.
- Phase diagram is a graphical representation of the physical states of a substance

under different conditions of temperature and pressure.

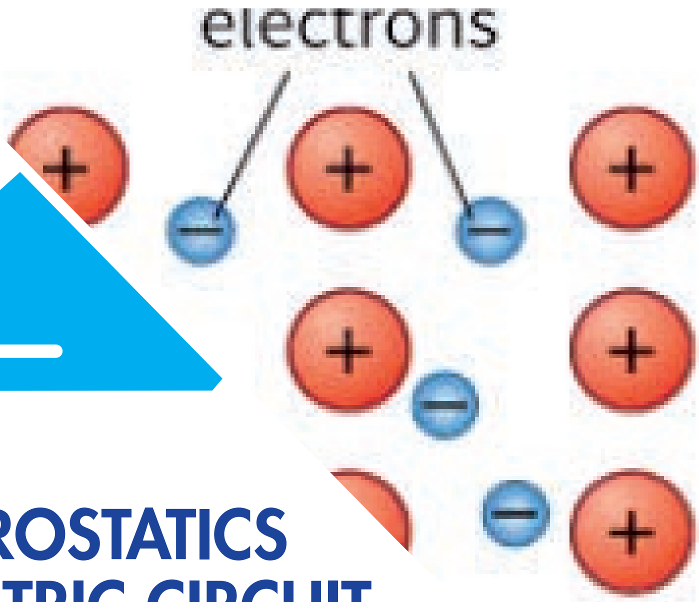
- The triple point represents the combination of pressure and temperature that facilitates all phases of matter at equilibrium.
- Critical Point is the point in temperature and pressure on a phase diagram where the liquid and gaseous phases of a substance merge together into a single phase.
- Calorimetry is the measurement of the quantity of heat exchanged between substances. The experimental approach of measuring heat capacities and the heat changes during chemical and physical processes is called calorimetry. A calorimeter is a device used to measure specific heat capacity and specific latent heat of solids or liquids.
- There are several methods we can use to determine the specific heat capacity of a substance, but all methods always involve heat exchange and the following should be known:
  the mass of the substance,
  the amount of heat energy supplied,
  its starting temperature, and
  the final temperature.
- The law of calorimetry states that for an isolated system, the heat energy lost from the hot body will equal the heat gained by the cold body.

Heat gained by cold object = Heat lost by hot object

END OF UNIT QUESTIONS 5

1. What is/are the condition/s for net heat flow from one place to another place? Is it necessary to have a heat-conducting material for heat transfer?
2. When does net heat flow stop? Do two objects in thermal equilibrium exchange heat?
3. What is the direction of the spontaneous flow of heat?
4. What is the relationship between temperature and heat? What about between temperature and internal energy? Is internal energy of a confined gas solely dependent on temperature?
5. If the temperature of a confined gas is increased, which form of energy is said to be increased in the gas?
6. Suppose heat is supplied to an inflated balloon, which is firmly tied at its neck. What are connections between the heat supplied, the thermodynamic work and the internal energy in the balloon?
7. Describe conduction, convection and radiation heat transfer mechanisms. Name the general mode of heat transfer in solids, in liquids and gases.
8. Which way is heat transfer believed to occur in a long, hollow cylinder kept at consistent but varied temperatures on its inner and outer surfaces?
9. How can you change the boiling point of water?
10. At 20.0 degree Celcius, the length of a sheet of steel is 100.0 cm and the width is 50.0 cm. If the coefficient of linear expansion of steel is 0.000011/C, determine the change in area and the final area at 100.0C.
11. The temperature of a metal bar rises by 10.0°C when it absorbs 9000.0 J of heat energy. If the mass of the bar is 1000.0 g, what is its specific heat capacity?
12. Is thermal energy the same as heat, internal energy or has another definition?
13. What important properties can be obtained in the phase diagram?
14. What does the critical point of water at (217.7atm, 374.0°C) mean?

UNIT 6



ELECTROSTATICS AND ELECTRIC CIRCUIT

Introduction

In this unit we will study the two branches of electricity, namely electrostatics (static electricity) and current (dynamic) electricity. Under electrostatics we study the properties of electric charges at rest and under current electricity we study the properties of charges in motion. The main difference between static and current electricity is that with static electricity, the charges are stationary and build on the surface of an insulator, while in current electricity electron flowing through a conductor form what we call an electric current.

Learning outcomes: Students will be able to:

- *calculate the electric field resulting from a point charge.*
- *determine the magnitude and direction of the electric force among any point charges.*
- *acquire knowledge and understanding in electrostatic phenomenon*
- *demonstrate an understanding of the components and functions of electrical circuits that are commonly found at home and in the workplace*
- *construct, analyze, simple electrical circuits, using schematic diagrams, working with electrical tools and components, and examining small everyday electrical devices and appliances*
- *define a capacitor and explain some of its applications in life.*
- *apply Kirchhoff's rules to solve circuit problems.*

6.1 Coulomb's Law

By the end of this section, you will be able to:

- *state Coulomb's law.*
- *calculate the magnitude and direction of electric force between any two charges.*
- *solve problems involving Coulomb's law.*
- *explain Coulomb's law using the idea of vectors.*
- *explain the meaning of a coulomb.*

? Brainstorming Questions

1. What are the compositions of an atom?
2. What are the two types of charges and what are their origin?
3. How can a body be charged positively and negatively?
4. What does the law of electrostatics say?

Properties of Electric Charges

Charge

Before we go into discussing Coulomb's law, we need to revise what you have learnt in the previous grades about charge and its properties. Charge is the fundamental property of matter associated with electrons and protons which are found in any atom. There are only two types of charges: negative and positive charges. By convention, negative charges are carried by electrons and positive charges are carried by protons.

When the number of electrons and protons in an object are equal that object is said to be electrically neutral. However, if this electrical balance is disturbed, then the object is said to be electrically charged. A negatively charged object contains more electros than protons, but a positively charged body contains more protons than electrons.

Properties of charges

Property I: There are two kinds of electric charges: Positive and Negative.

Property II: Two similarly charged bodies repel each other and two oppositely charged bodies attract each other.

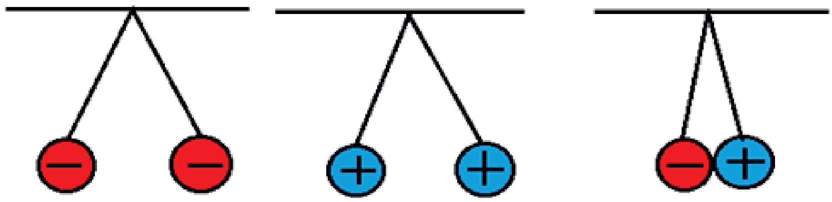


Figure 6.1: Like Charges repel and unlike charges attract each other.

Property III: Electric charge is always conserved, i.e., it cannot be created or destroyed. In any charging process, the number of electrons gained by one object is equal to the number of electrons lost by the other.

Property IV: Electric charge is quantized. Charge comes in discrete packets, it means that charge (q) always occurs as some integral multiple of the charge of an electron, *e*. It can be written as: $q = \pm ne\,(where\ n = 1, 2, 3, ...)$

The smallest unit of charge that is carried by one electron or proton is called the elementary charge, e. The magnitude of the charge of an electron is:

$$1e = -1.6\times10^{-19}C$$

The charge carried by a proton is also the same as that of the charge carried by an electron except it is positive.

$$1\,p = +1.6\times10^{-19}C$$

Example 6.1

What number of protons needed to make a charge of +1.0 C?

Solution:

$$n = \frac{q}{e} = \frac{1.0C}{1.6\times10^{-19}C} = 6.25\times10^{18}\ protons$$

Similarly, the number of electrons needed to make a charge of -1.0 C is 6.25×10^{18} *electrons.*

Activity 6.1:

Discuss in pair about the following questions. Present what you have agreed in your group to the class.

1. What would happen if electrons were positively charged and protons negatively charged?
2. Is the Earth charged or neutral? Discuss why is so.
3. Is there a force of attraction or repulsion between a charged and a neutral body?
4. Can positive charges move and create electric current electrostatically?

Exercise 6.1

(a) How many electrons are needed to form a charge of −2.0 nC?

(b) How many electrons must be removed from a neutral object to leave a net charge of +0.50 μC?

Electrostatic Force

A force of attraction or repulsion between two charged bodies, is known as the **electrostatic force**. Electrostatic force is either attractive or repulsive, unlike the gravitational force which is only attractive. If the objects have opposite charges they attract each other, while if their charges are similar, they repel each other.

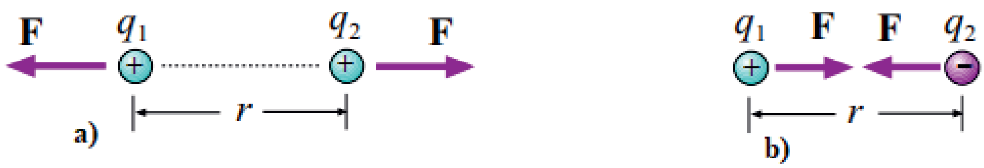


Figure 6.2: Like charges repel and Unlike charges attract each other.

In 1785, the French physicist Charles Coulomb, proposed a law that describes the force that one charged particle exerts on another. This law is remarkably similar in form to Newton's law of gravitation. Coulomb's law states that any two-point charges exert an electrical force on each other that is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. where q_1 is the charge on the one particle, q_2 is the charge on the second, and r is the distance between the two.

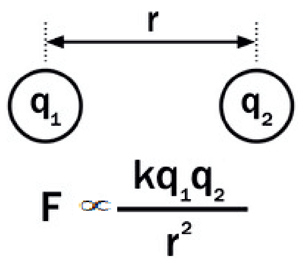


Figure 6.3: Coulomb's Law

Then, the magnitude of the electrostatic force between two point-charges is given by:

$$F = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

The proportionality constant k is called the **electrostatic, or Coulomb constant.** It can also be written in terms of the permittivity of free space ε_0 whose value is, $\varepsilon_0 = 8.85\times10^{-12} C^2/N.m^2$. With this, the value of k becomes:

$$k = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\times3.14\times8.85\times10^{-12} C^2/N.m^2} = 9.0\times10^9 N.m^2/C^2$$

Example 6.2:

Two tiny spherical water drops with identical charges of –, have a center-to-center separation of 1.0cm. a) What is the electrostatic force acting between them? b) How many excess electrons are on each drop giving it its charge imbalance?

Solution:

Step I: Conceptualize the problem: There are two identical negative charges of water drops. We are required to find the electrostatic force from Coulombs law (a) and the number of excess electrons that gives this charge imbalance by the charge is quantized property of electric charges.

Step II: Identify the given and required quantities:

What you have is: $q_1 = q_2 = -1.0\times10^{-10} C$, $r = 10cm = 0.1m$,

what you need to determine is: a) F =? b) n = ?

Step III: Apply the appropriate equation that best fits for the solution.

$$F = k\frac{|q_1||q_2|}{r^2} = 9\times10^9 N.m^2/C^2 \frac{|-1.0\times10^{-10}C||-1.0\times10^{-10}C|}{(0.1m)^2} = 9.0\times10^{-17} N$$

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1.0\times10^{-10}C}{1.6\times10^{-19}C} = 6.25\times10^8$$

That means 6.25×10^{8} electrons give this charge imbalance so that the two charged water drops exert 9.0×10^{-17} N of repulsive force to each other.

Exercise 6.2:

1. What are the similarities and differences between Newton's law of universal gravitation and Coulomb's law?
2. Which force is greater between an electron and a proton, the gravitational or the electrostatic? By how much is one larger than the other?

Activity 6.2:

1. Take a balloon or plastic comb and rub against your hair. Then take it near a stream of water falling from a tap. Discuss and try to explain what you observe.

Figure 6.4: A charged body (balloon) attracts a neutral body (stream of water)

Activity 6.3

A positively charged light metal ball is suspended between two oppositely charged metal plates on an insulating thread as shown below. Describe the motion that the ball undergoes.

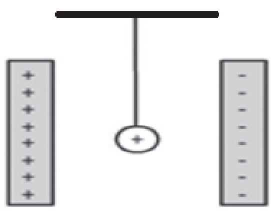


Figure 6.5: Coulomb's Law in Vector Form

Force on a Charge due to Multiple Electric Charges

Electrostatic force is a vector quantity. Therefore, if there are more than one force acting on a charged particle, then the resultant force acting on the particle will be the vector sum of all these forces. This is known as the **superposition principle**.

Consider a system of n stationary charges $q_1, q_2, q_3 \ldots q_n$ in vacuum. The force on q_1 due to $q_2, q_3 \ldots q_n$ is obtained by taking the vector sum of the individual forces.

$$\overrightarrow{F_{net}} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \overrightarrow{F_{14}} \ldots\ldots \overrightarrow{F_{1n}}$$

Where, F_{12} is the force that charge q_2 exert on q_1, F_{13} is the force that q_3 exert on q_1, … F_{1n} is the force that q_n exert on q_1.

Example 6.3:

Three point charges are placed on the x-axis: $q_1 = +6\mu C$ at $x_1 = 5.0cm$, $q_2 = -5\mu C$ at $x_2 = -3.0cm$, and $q_3 = +2\mu C$ at the origin ($x = 0$). What is the magnitude and direction of the total electrostatic force acting on q_3?

Solution:

The first step should be determining the magnitude of the force each charge exerts on q_3, using Coulomb's law:

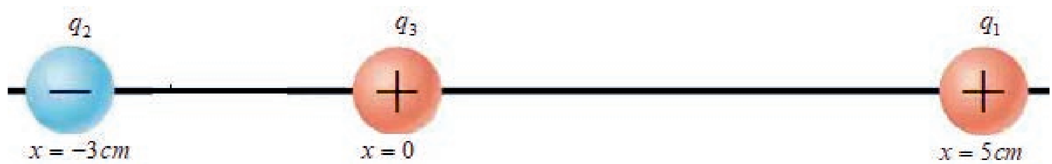
The force charge q_1 exerts on charge q_3 is given by:

$$F_{13} = k\frac{q_1 q_3}{r_{13}^2} = 9.0\times10^9\left(\frac{6\times10^{-6}\times2\times10^{-6}}{(5\times10^{-2})^2}\right) = 43.2N$$

Similarly, the magnitude of the force charge q_2 exerts on charge q_3, F_{23} is given by

$$F_{23} = k\frac{q_2 q_3}{r_{23}^2} = 9.0\times10^9\left(\frac{5\times10^{-6}\times2\times10^{-6}}{(3\times10^{-2})^2}\right) = 100.0N$$

After the magnitude of each force is determined the next step will be determining the directions of the forces exerted on q_3 due to q1 and q2 by considering the sign of the pair of these charges.



Charge q_1 (positive) repels charge q_3 (positive) towards left: $\overrightarrow{F_{13}} = 43.2N - \hat{i}$

Charge q_2 (negative) attracts charge q_3 (positive) towards left: $\vec{F}_{23} = 100.0N - \hat{i}$

Therefore, since these forces act in the same direction (both to the left), the resultant force, F_R acting on charge q_3, will be their sum.

$$\vec{F}_R = \vec{F}_{13} + \vec{F}_{23} = 143.2N\hat{i} + (-100.0N\hat{i}) =- 143.2N\hat{i}$$

Thus, the magnitude of the resultant force acting on charge q_3 is $143.2N$, and the force is directed along the negative x-axis.

Exercise 6.3:

Referring to the question on the above example 6.3, determine the magnitude and direction of the total electrostatic force acting (a) on q_1, and (b) on q_2.

Review questions 6.1

1. State Coulomb's law in your own words.
2. Write Coulomb's law mathematically both in a scalar and vector form.
3. What is the meaning of a Coulomb?
4. Two charges +3μC and +12μC are fixed 1 m apart, with the second one to the right. Find the magnitude and direction of the net force on a –2 μC charge when placed halfway between the two.
5. Two small, identical conducting spheres repel each other with a force of 0.050 N when they are 0.25 m apart. After a conducting wire is connected between the spheres and then removed, they repel each other with a force of 0.060 N. What is the original charge on each sphere?
6. In the picture below, X is a small negatively charged sphere with a mass of 1.0kg. It is suspended from the roof by an insulating rope which makes an angle of 60 degrees with the roof. Y is a small positively charged sphere which has the same magnitude of charge as X. Y is fixed to the wall by means of an insulating bracket. Assuming the system is in equilibrium, what is the magnitude of the charge on X?

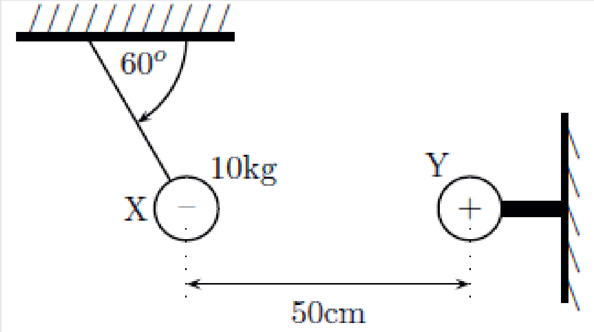


Figure 6.6: Question 6

6.2 Electric Fields

By the end of this section, you will be able to:

- *define the terms, electric field, and electric flux.*
- *sketch electric field lines.*
- *solve problems involving electric field.*
- *map an electric field lines pattern using electric lines of force.*
- *calculate the magnitude and direction of electric field due to a point charge and two point charges.*

? Brainstorming

1. What is a gravitational field?
2. What causes gravitational field?
3. Discuss the concepts and recall the equations of (a) gravitational field lines, and (b) gravitational field strength.

An **electric field** is a region in space where one charge experiences a force from another charge. Let us consider a point charge Q placed in vacuum, at point O. If we place another point charge q at a point P, where OP = r, then the charge Q will exert a force on q as per Coulomb's law. We say that the charge Q produces an electric field in its surrounding.

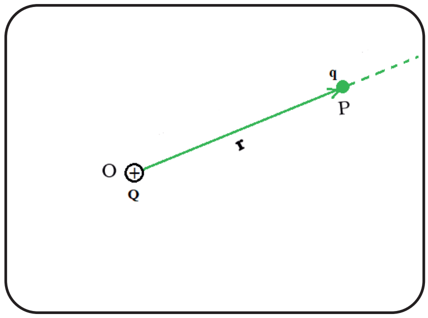


Figure 6.7: Electric field of a charge Q

Electric field lines

The electric field produced by a charged particle can be represented by lines of force called **electric field lines**. It is drawn in the direction that a positive charge would move if placed at a point in the field. Electric field lines are helpful to visualize the vector nature of an electric field. The Figures 6.8 (a) to (f) shows the electric field lines for different arrangement of charges.

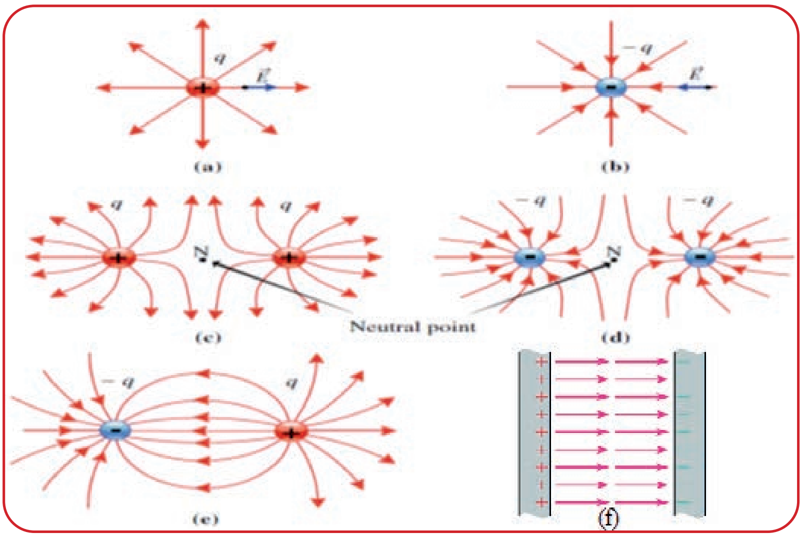


Figure 6.8: Electric field lines of (a) a positive point charge, (b) a negative point charge, (c) two equal positive charges, (d) two equal negative charges, (e) an electric dipole, and (f) a uniform field of oppositely charged parallel plates.

Properties of electric Field lines

Electric field lines have the following properties

1. Electric field lines do not cross each other.
2. Electric field lines begin on positive charges and radiate away from them toward negative charges, where they terminate.
3. Electric field lines are always perpendicular to the surface of the charged body.
4. The closer the lines, the stronger the electric field, and the farther apart they are the weaker is the field.
5. Equally spaced electric field lines indicate the field is uniform. The electric field between two parallel plates, shown in Figure 6.8, is a common example of a uniform electric field.

Activity 6.4:

1. Form a group containing 3 or 4 members. Discuss in your group on the mechanism to investigate the nature of electric field lines between two similar and opposite charges.
2. What would happen when (a) a conductor and (b) an insulator is placed in an electric field?

The Electric field strength

The magnitude and direction of the electric field is expressed by a quantity called electric field strength or electric field intensity ($\vec{E}$). Electric field strength, is the force acting per unit positive charge for a stationary point charge. In equation form:

$$\vec{E} = \frac{\vec{F}}{q}$$

Electric Field of Point Charge: The electric field of a point charge can be obtained from Coulomb's law. From Coulomb's law, the equation for the force between two electric charges is:

$$F = k\frac{qQ}{r^2}$$

The electric field strength is then the force per unit of charge.

$$E = \frac{F}{q} = k\frac{Q}{r^2}$$

where E is the electric field strength due to a point charge Q, and r is the distance from charge Q to the point of interest. From the above equation of electric field strength, we see that the electric field is depend only on the source charge Q and the distance r; it is independent of the test charge q.

The SI unit for the electric field strength is newton per coulomb (N/C). The direction of the electric field will be in the direction of the force the source charge exerted on a positive test charge. Thus, electric field lines will direct away from a positive source charge and towards a negative source charge.

Example 6.4:

Calculate the strength and direction of the electric field E due to a point charge of 2.0 nC (nano-Coulombs) at a distance of 5.0 mm from the charge.

Solution:

Step I: Conceptualize the problem: the strength of the field is its magnitude at a distance of 5mm = 5x10^{-3}m. We are therefore required to find the magnitude of the electric field.

Step II: Use appropriate equation that best fits for the required quantity.

$$\mathbf{E} = k\frac{Q}{r^2} = 9.0\times10^9\,Nm^2/C^2\,\frac{2.0\times10^{-9}C}{(5.0\times10^{-3})^2} = 7.2\times10^5\,N/m^2$$

Since the source charge is positive in sign, the field is directed away from Q.

Exercise 6.4:

What electric force does the electric field found in the above example 6.4. would exert on a point charge of –0.25 μC?

Electric field for multiple point charges

The electric field from multiple point charges can be obtained by taking the vector sum of the electric fields of the individual charges. Consider n number of charges whose electric fields have magnitudes of $|\vec{E_1}|, |\vec{E_2}| .... |\vec{E_n}|$ at a given point. The net electric field is found by taking the vector sum of each electric field:

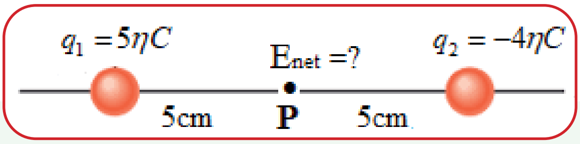
$$\vec{E_{net}} = \vec{E_1} + \vec{E_2} + \vec{E_3} + ...\vec{E_n}$$

Note that the direction of $\vec{E_1}$ is the direction of the force q_1 exerted on the positive test charge placed at that point, and the direction of $\vec{E_2}$ is the direction of the force q_2 exerted on the positive test charge placed at that point, and so on.

Example 6.5

Two-point charges $q_1 = 5nC$ and $q_2 = -4nC$ are separated by a distance of 10 cm. What is the magnitude of the electric field in between the charges?

Solution:



The magnitude of the electric field at point P due to q_1 is:

$$E_1 = k\frac{q_1}{r_1^2} = 9\times10^9\,N.m^2/C^2\left(\frac{5\times10^{-9}C}{(0.05m)^2}\right) = 1.8\times10^4\,N/m$$

Since q_1 is positive it exerts a repulsive force to the right on the positive test charge placed at point P.

Thus, $\vec{E}_1 = 1.8\times10^4\,N/m\,\hat{i}$

Similarly, the magnitude of the electric field at point P due to q_2 is:

$$E_2 = k\frac{q_2}{r_2^2} = 9\times10^9\,N.m^2/C^2\left(\frac{4\times10^{-9}C}{(0.05m)^2}\right) = 1.44\times10^4\,N/m$$

Since q_2 is negatively charged it exerts an attractive force to the right on the positive test charge placed at point P.

Thus, $\vec{E}_2 = 1.44\times10^4\,N/m\,\hat{i}$

Finally,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 1.8\times10^4\,N/m\,\hat{i} + 1.44\times10^4\,N/m\,\hat{i} = 3.24\times10^4\,N/m\,\hat{i}$$

Exercise 6.5:

Repeat example 6.5, if (a) both q_1 and q_2 are positively charged, and (b) they are both negatively charged.

Electric flux

One of the properties of electric field lines is that, the number of field lines crossing a unit area, placed normal to the field at a point, is a measure of the strength of electric field at that point. This means that the number of field lines per unit area, is proportional to the magnitude of the electric field, or $E \sim N/A$. We can rewrite this proportion as the number of field lines is proportional to the *product* of E and A, $N \sim EA$.

We can now define electric flux. Electric flux is a measure of the number of electric field lines that crosses a given area. The product of the electric field and the surface area, A, perpendicular to the field is called the **electric flux** and represented by the symbol ϕ.

$$\phi = EA$$

If the surface under consideration is not perpendicular to the field, as in Figure 6.9 (b), the expression for the electric flux became,

$$\phi = EA\cos\theta$$

E cosθ is the projection of area normal to E. In a vector form, the above equation can be written as the dot product of the electric field vector and area vector.

$$\phi = \vec{E} \cdot \vec{A}$$

The **area vector** of a flat surface of area *A* has the following magnitude and direction:

- Magnitude is equal to area of the surface (*A*)
- Direction along normal or perpendicular to the surface.

This means, if the electric field vectors are tangent to the surface at all points, that is if they don't penetrate the surface, the electric flux through the surface is zero. Electric flux ϕ is a scalar quantity, whose SI unit is N.m2/C.

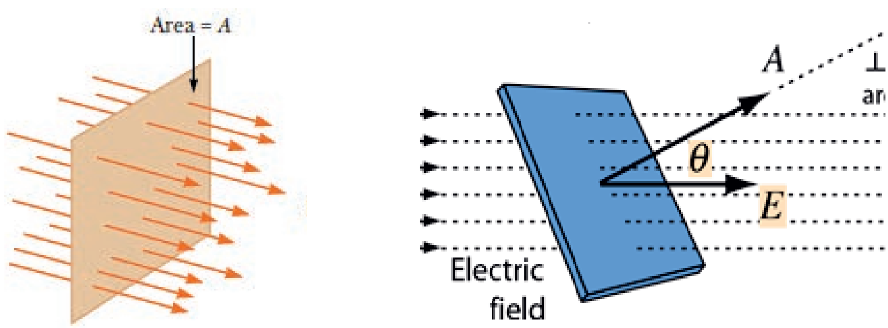


Figure 6.9: Field lines of a uniform electric field penetrating a plane of area A perpendicular to the field. The electric flux ϕ through this area is equal to EA.

Example 6.6:

A 2cm × 2cm square lies in the xy-plane. Find the electric flux through the square for the electric field vectors, $\vec{E} = (50\hat{k} + 20\hat{j})N/C$.

Solution:
In this case direction of the area vector, which is perpendicular to the surface, is in the z-axis. Because the square lies in the xy-plane.

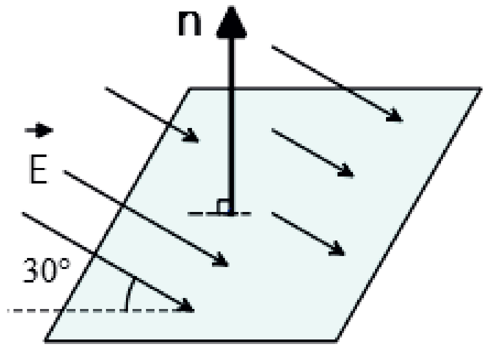
$$\vec{A} = (2cm \times 2cm)\hat{k} = 4cm^2\hat{k},$$

$$\phi = \vec{E}\cdot\vec{A} = \vec{A} = 4\times10^{-4}m^2\hat{k}\cdot(50\hat{k}+20\hat{j})N/C$$

$$= 4\times10^{-4}\times50 + 0\times20N.m^2/C = 0.02N.m^2/C$$

Exercise 6.6:

Find the electric flux through the surface with sides of 15cm×15cm positioned in a uniform electric field of E = 150N/C as shown in the figure below.



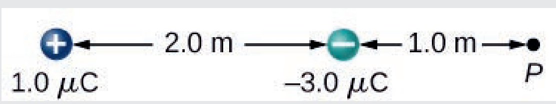
Review Questions 6.2

1. Define the following terms, (a)electric field, (b) electric field strength, (c) electric field lines, and (f) electric flux.
2. Map electric field lines for different arrangement of charges.
3. Compare and contrast the following gravitational quantities with their counterpart electrical quantities:
   A. gravitational field and electric field.
   B. Gravitational field strength and electric field strength.
4. For the following figures find the possible region where the electric field could be zero.

A. 

B. 

5. How field lines indicate the magnitude of the field they represent?
6. What is the net electric field strength at point P in the system shown below.



7. A particle of charge 2 x 10-8C experiences an upward force of magnitude 4 x 10-6 N when it is placed in a particular point in an electric field. (a) What is the electric field at that point? (b) If a charge q = -1.0 x 10-8 C is placed there, what is the force on it?

6.3 Electric Potential

By the end of this section, you will be able to:

- *Define the terms electric potential, equipotential surface.*
- *Solve problems involving electric potential.*
- *Explain the meaning of a volt, potential difference, and emf.*

? Brainstorming Questions

- Discuss the concepts and recall the equations of (a) gravitational potential, and (b) gravitational potential energy.
- What is the relationship between the following pair of quantities?
  a. electric potential and electrical potential energy?
  b. electrical potential energy, and electric work?
  c. Electric potential and potential difference?
  d. Potential difference and electric field?
  e. Electrical potential and gravitational potential?

Electric Potential Energy

The concept of electric potential energy is similar to that of gravitational potential energy. In raising a ball with mass m to any height, you are giving it gravitational potential energy. Electric potential energy is similar but with charges instead of masses. Instead of raising a ball in the gravitational field of the Earth, you move a charge in the electric field of another charge.

In order to move one positive charge closer to another positive charge, (charge q closer to charge Q in Figure 6.10) work has to be done by an external agent. This is simply because there is a force of repulsion between the charges. In the process of doing work, energy is transferred from you to the charge that you are pushing. As a result, the charges gain electrical potential energy.

We define electrical potential energy of a test charge q in terms of the work done on the charge q. Work done by external force in bringing a unit positive charge from point A to B, in Figure 6.10 is the electrical potential energy of q.

$$W = q(V_B - V_A) = q\left(k\frac{Q}{r_B} - k\frac{Q}{r_A}\right)$$

If we take point A to be at infinity, i.e., $r_A \approx infinity$

$$W = k\frac{qQ}{r}$$

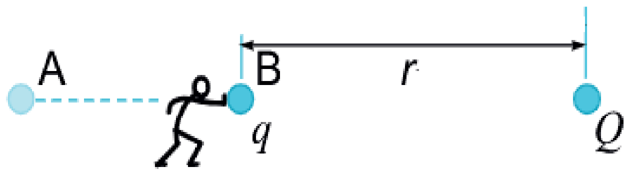


Figure 6.10: Work must be done to push one positive charge towards another.

Thus, the potential energy of a test charge 'q' in the vicinity of a source charge 'Q' will be:

$$U = k\frac{qQ}{r}$$

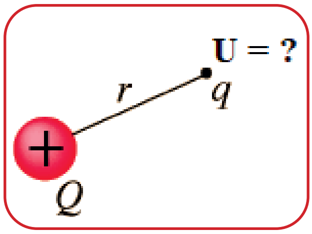


Figure 6.11: Electric potential energy

Electric Potential

Electric potential is another concept which helps us to visualize the behavior of electric fields. In unit 4, gravitational potential is defined from the idea of potential energy. The potential at a point is the potential energy of unit mass at that point. We will approach the idea of electrical potential in the same way. Electric potential (denoted by V) is equal to the work done per positive charge in carrying it from infinity to that point in an electric field. The work to be done is equal to the change in the potential energy of the test charge in the field of the source charge.

$$V = \frac{W_{\infty \to r}}{q} = \frac{\Delta U}{q}$$

The potential due to a single point charge

In electrostatics the zero of potential is at infinity, at a larger distance from the source charge, because the electric field and force approach zero there. In the same way as we consider gravitational potential energy at the ground to be zero, electric potential is defined as the work done per unit positive charge to move a positive test charge from infinity to its current position within an electric field.

Mathematically:

$$V = \frac{W}{q} = \frac{\left(k\frac{Q\cancel{q}}{r}\right)}{\cancel{q}} = k\frac{Q}{r}$$

Thus, the equation for electric potential at a distance r from a source charge Q is given by:

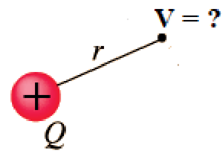


Figure 6.12: Electric Potential

$$V = k\frac{Q}{r}$$

Electrical potential is a scalar quantity. Its SI unit is Volt (V) in the honor of Alessandro Volta. $1V = \frac{1J}{1C}$.

Electrical Potential Difference

Consider a positive test charge +q placed at A in the electric field of a source charge Q. The test charge moves towards B under the influence of the electric field of the source charge. In the process the test charge loses electrical potential energy and gains kinetic energy. Thus, at A, the test charge has more potential energy than at B, A is said to have a higher electrical potential than B.

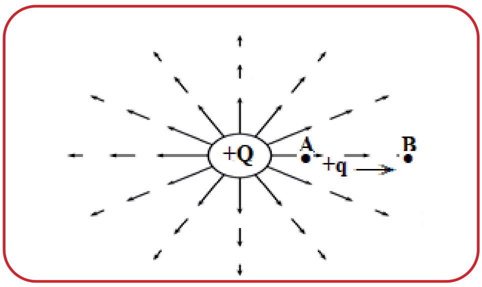


Figure 6.13: Potential Difference

The potential difference between two points (A and B in this case) in an electric field is defined as the work required to move a unit positive test charge from the point of lower potential to that of higher potential.

If an amount of work W is required to move a charge Q from one point to another, then the potential difference between the two points is given by,

$$\Delta V = \frac{W}{q}$$

Activity 6.5:

A difference in height is needed for water to flow, see the figure below. What is then needed for current to flow? Discuss in pairs.

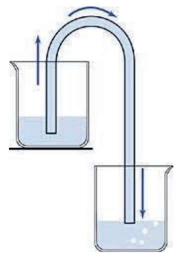


Figure 6.14: Gravitational potential difference

Example 6.7

Determine the electric potential produced by a charge of 1μC at a distance of 1mm from it.

Solution

$$V = \frac{kq}{r} = \frac{9\times10^{9}\,Nm^2/C^2 \times 1\times10^{-6}\,C}{1\times10^{-3}\,m}$$
$$= 9\times10^{6}\,J/C = 9\times10^{6}\,V$$
$$= 9MV$$

Potential difference in a uniform electric field

Figure 6.15, below shows a uniform electric field produced between two equally and oppositely charged parallel plates.

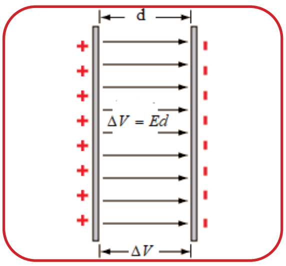


Figure 6.15: Potential difference in a uniform electric field

To measure the potential difference between two points in the field, we equate the work done in moving a charge from one point to the other with the gain in electrical potential energy.

$$W = \Delta PE$$
$$Fd = q\Delta V$$
$$qEd = q\Delta V$$
$$\Delta V = Ed$$

Thus, the electrical potential difference between the two plates is expressed as, the electric field strength times the distance between the plates.

From the previous section, we already know the units for electric field is newtons per coulomb. We now got the unit of electric field to be volt per meter. Thus, the following relation among units is valid:

$$1N/C = 1V/m$$

Potential due to a system of point charges

Consider a system of charges $q_1, q_2, \ldots, q_n$ with position vectors $r_1, r_2, \ldots, r_n$ relative to some origin (Figure 6.16). The potential at P due to the charge q_1 is $V_1 = k\frac{Q_1}{r_1}$. Similarly, the potential at P due to q_2 is $V_2 = k\frac{Q_2}{r_2}$, and the potential at P due to q_3 are given by $V_3 = k\frac{Q_3}{r_3}$.

where r_1 is the distance between q_1 and point P, r_2 is the distance between q_2 and point P, and so on. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges.

The total potential at P is:

$$V = V_1 + V_2 + \ldots + V_n$$

$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + \ldots k\frac{Q_n}{r_n}$$

$$V_p = k\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots + \frac{q_n}{r_n}\right]$$

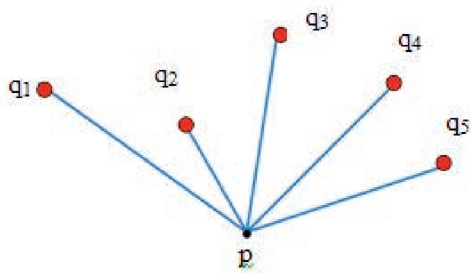


Figure 6.16: Potential of multiple charges

$$V_p = k\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad (for\ n\ point\ charges)$$

Note that the sum in this equation is an algebraic sum. It is not a vector sum. It means the signs of the charges are used in calculating the potential. As a result, the potential of a positive charge is positive and the potential of a negative charge is negative.

Example 6.8

Two charged particles are placed on the x-axis of a coordinate system. The first ($q_1 = 2\mu C$) is at the origin. The second ($q_2 = -5\mu C$) is at $x = 1.0\,m$. Determine the potential at $x = 0.2\,m$.

Solution:

$$V = V_1 + V_2 = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2}$$

$$V = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right) = 9\times10^9\left(\frac{2\times10^{-6}}{0.2} + \frac{-5\times10^{-6}}{0.8}\right)$$

$$V = 3.375\times10^4 V$$

Exercise 6.7:

Referring to example 6.8, (a) determine the potential at $x = 1.5m$. (b) At what point in between these two charges the electric potential is zero?

Example 6.9

Calculate the change in electrical potential energy in moving a from 5 cm to 0.1 cm from a charged sphere with a charge of .

Solution:

$$W = q\Delta V = q\left(V_B - V_A\right) = q\left(\frac{kQ}{r_B} - \frac{kQ}{r_A}\right) = kqQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$W = 9\times10^9\times10^{-6}\times10^{-9}\left(\frac{1}{0.01} - \frac{1}{0.05}\right)$$

$$W = 7.2\times10^{-4} J = 72\,mJ$$

Equipotential line or surface

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Equipotential lines are lines connecting points of the same electric potential. When these points lie on a two- or three-dimensional surface, such as the surface of a metallic sphere, they are called equipotential surfaces. If a charge moves along the equipotential line then, there is no change in its potential and hence, no electrical work is done.

Characteristics of equipotential lines/surfaces:

1. Two equipotential surfaces never intersect each other.
2. The equipotential lines of a uniform field are parallel to each other and the equipotential lines of a point charge are concentric sphere (see Figure 6.17)

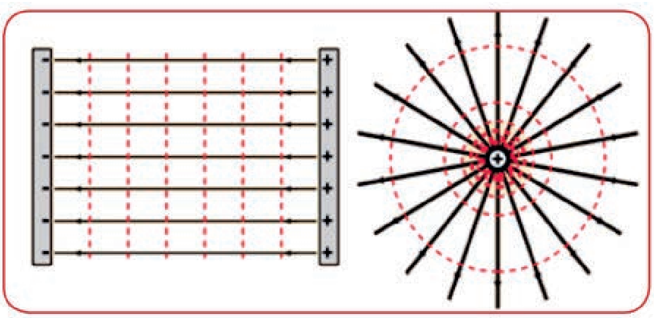


Figure 6.17: Equipotential lines of (a) a uniform field, (b) a point charge.

3. Since the electric field (E) is stronger near the charge, so equipotential surfaces of point charge are crowded near the charge as shown in the Figure 6.17.
4. No work is required to move a charge along an equipotential line because there is no change in potential.
5. Equipotential lines are always perpendicular to the electric field lines.

$$W = \vec{F} \cdot \vec{S} = Fs\cos\theta = qEs\cos\theta$$

Since the work done is zero,

$$qEs\cos\theta = 0$$

$$\Rightarrow \theta = \cos^{-1}(0) = 90^0$$

Key term

Equipotential lines are lines connecting points of the same potential

Review Questions 6.3

1. Write the definition of the following terms in your own words. (a) electric potential, (b) potential difference, (c) volt, and (d) equipotential line/surface.
2. Write two units of electric field strength.
3. If the potential is constant in a certain region, what is the nature of the electric field at that region?
4. What would happen to its electrical potential energy (a) when an electron is released in a uniform electric field, and (b) when a proton is released in a uniform electric field?
5. How are electric field lines and equipotential lines drawn relative to each other?
6. What is the strength of the electric field between two parallel conducting plates separated by 1.0 cm and having a potential difference between them of 1.5V?
7. Two-point charges are located on the corners of a rectangle with a height oh 0.05 m and a width of 0.15 m. The first charge ($q_1 = -5\mu C$) is located at the upper left-hand corner, while the second charge ($q_2 = +2\mu C$) is at the lower right-hand corner. (a) Determine the electric potential at the upper right-hand corner of the rectangle. (b) What is the potential difference for a point at the right-hand corner of the rectangle relative to the lower left-hand corner?
8. Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.
9. What are the signs and magnitudes of a point charge that produces a potential of -2.0 V at a distance of 1.0 mm?
10. Two point charges 10pC and -2pC are separated by a distance of 1.0 . Find the potential mid-way between them.

6.4 Electric Current, Resistance and ohm's law

By the end of this section, you will be able to:

- *define the terms electric current, current density, resistance, conductivity, resistivity and drift velocity.*
- *explain the effect on the current (brightness) and potential difference of connecting light bulbs in series.*
- *explain the effect on the current (brightness) and potential difference of connecting light bulbs in parallel.*
- *draw electric circuits consisting of three resistors connected in series and parallel.*
- *determine equivalent resistance of resistors connected in series and parallel.*
- *describe the principles of potentiometer and wheatstone bridge.*
- *describe how to convert galvanometer to a voltmeter and ammeter.*
- *explain the meaning of an ohm, resistance.*
- *explain the si units of electric current, current density, resistance, resistivity, conductivity.*
- *state and apply Kirchhoff's laws.*
- *solve problems involving network of resistors.*

? Brainstorming

- Write down what you understand, from previous grades, by the terms current, potential difference, emf and resistance.
- Set up a simple circuit to measure the current in a lamp and the potential difference across it? Sketch the circuit and swap it with a classmate to check.
- What drives the current in a conductor?
- How the flow of electrons are produced through a conductor?
- Why does current flows through a conductor but not through an insulator?

Electric Current, Resistance and Ohm's law

Electric current

Electric current is a flow of charge. In metals, current is due to a flow of electrons. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on the free or conduction charges of the conductor, causing current. Even though it is electrons' motion that produces current, for historic reason, what we call electric current is the conventional flow of protons. Hence, it flows from the '+' terminal of a battery or power supply through the circuit to the '–' terminal.

Electric current is defined as the rate of flow of charges through some cross-sectional area. if is the net charge that flows through a cross-sectional area A in time (t),then the current I is given by the equation:

$$I = \frac{\triangle q}{\triangle t}$$

The SI unit of electric current is ampere, denoted by A. One ampere is a flow of one **coulomb** (1 C) of charge in 1 s. Small currents are measured in milliampere (mA), and microampere (μ A). 1mA = 10^{-3}A and 1μ A=10^{-6}A.

Activity 6.6:

Discuss in group what is needed to store water in a tanker and distributed to the needy. Compare it with how current is distributed to resistors using the source of voltage, see the Figure below.

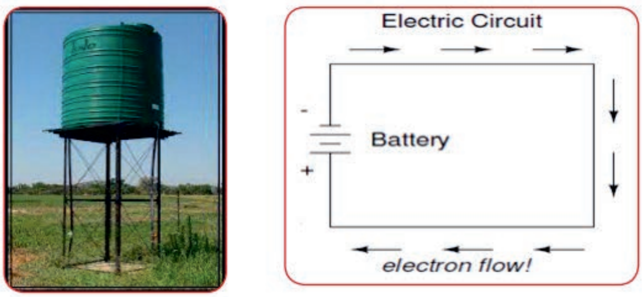


Figure 6.18: Analogy of water current (a) and electric current (b)

Electric current and current density

Current density is a quantity which is used to describe the density of an electric current. It is denoted by the symbol J, and defined as the magnitude of the electric current per cross-sectional area.

$$J = \frac{I}{A}$$

The SI unit of current density is amperes per square meter. **Current density** is a vector quantity, which means it has both magnitude and direction.

Current density is also assumed to be proportional to the electric field, (E), that produces it, and the conductivity (σ) of the conductor through which it flows. The conductivity of a conductor is a measure of its ability to conduct electricity, whereas resistivity is the reverse of conductivity. With this, the relationship between electric field E and the current density J became:

$$J = \sigma E = \frac{E}{\rho}$$

Where σ = conductivity of the conductor, and ρ resistivity. The SI unit of ρ is Ohm-meter (Ω.m), where 1 Ω.m = 1 V/A, and the SI unit of ρ is Siemens per meter (S/m), or Ώ$^{-1}$.

Example 6.10

A current of 6 mA, flows through a copper wire of correctional are 12 mm². What is the current density?

Solution:

$$I = 6\ mA = 6\times10^{-3}A, \quad A = 12mm^2 = 12\times10^{-6}m^2$$

$$J = \frac{I}{A} = \frac{6\times10^{-3}A}{12\times10^{-6}m^2} = 500A/m^2$$

Exercise 6.9

Since current density is a vector quantity it has both magnitude and direction. What is then its direction?

Drift velocity

A conductor contains a large number of loosely bound electrons which we call free electrons or conduction electrons. The remaining material of which the conductor is made keep on vibrating about their mean positions. The average amplitude of vibration depends on the temperature. While the free electrons are moving through the conductor, they collide with these materials/ions. As a result, the speed and direction of the electron changes randomly at each such collision, causing the electron to move in a zig-zag path as shown in the Figure 6.19, below.

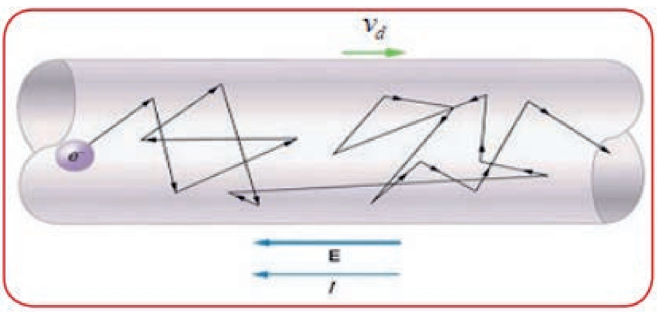
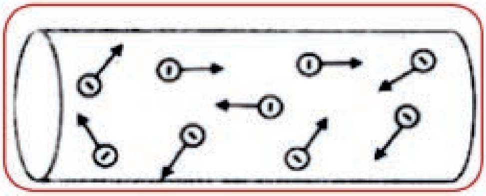


Figure 6.19: Motion of electrons through a conductor

The potential difference provided by the source creates an electric field inside the conductor. This field exerts a force on each electron in the direction opposite to the field. In every collision the electrons lose their kinetic energy and start to drift. The average velocity by which the free electrons move through a conductor is called their **drift velocity**.



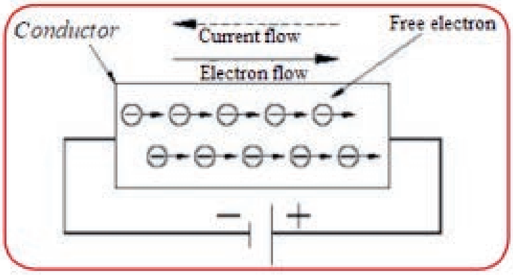


Figure 6.20: When a conductor is not connected to source, the free electrons will make a random motion (a), but when a pd is applied they will make a directional motion (b).

In a time interval Δt, an electron moves a distance d=vΔt along the wire. If the number of electrons per unit volume of wire is n then the number of electrons in the shaded cylinder of Figure 6.15 is $nAv\Delta t$. Therefore, the total charge on this volume of electrons is $nAv\Delta te$, where e is the charge of one electron.

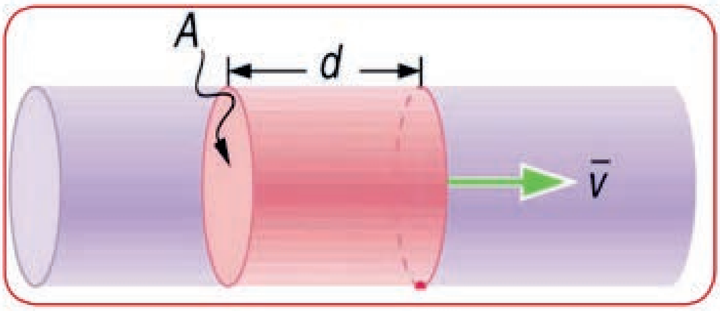


Figure 6.21:

The current in the circuit will then be:

$$I = \frac{Q}{\Delta t} = nAve.$$

$$\Rightarrow J = \frac{I}{A} = nv_d e$$

Exercise 6.9

1. Calculate the mean drift velocity of the electrons in a copper wire of cross-sectional area $5.0\text{x}10^{-6}m^2$ carrying a current of 1.0 A. There are $8.5\ \text{x}10^{-6}m^2$ free electrons in one m^3 of copper.
2. What are the directions of the following quantities in an electric circuit? (a) electrons, (b) electric current, (c) current density, (d) electric field, and (d) drift velocity.

Activity 6.7:

The drift speed of electros in a copper wire is very slow, a fraction of a millimeter per second, but as a generator is switched on in a power plant, far away from our house, a bulb gives off light almost instantly. How could this happen? Discuss in group and present your group's opinion to the class.

Resistance and Ohm's Law

Resistance

An electric wire is made of metal. Inside a metal, there are negatively charged electrons that are free to move about. We call these **conduction** or **free** electrons, because they are the particles that allow a metal to conduct an electric current. The atoms of a metal bind tightly together; they usually form a regular array, as shown in Figure 6.21. In a typical metal, such as copper or silver, one or more electrons from each atom breaks free to become conduction electrons. Since there are equal numbers of free electrons (negative) and ions (positive), the metal has no overall charge – it is neutral.

When electrons are moving through a conductor, they collide with each other and with the fixed positive ions of the material. This electric property that impedes current is called **resistance**. Resistance is therefore defined as a measure of the opposition to current flow in an electrical circuit.

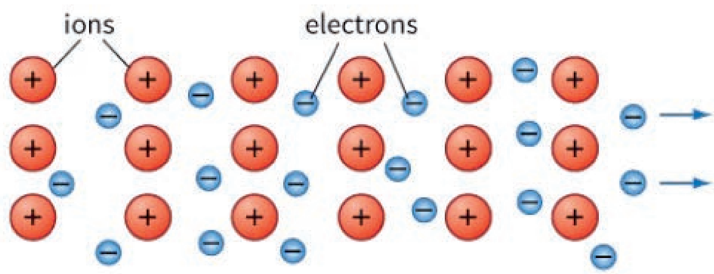


Figure 6.22: In a metal, conduction electrons are free to move among the fixed positive ions.

Ohm's Law

The German physicist Georg Simon Ohm (1787–1854) demonstrate experimentally that the current in a metal wire is directly proportional to the potential difference between its ends, provided that physical conditions, such as temperature, remain constant. This means that, for a metal at constant temperature, the graph of I against V will be a straight line through the origin.

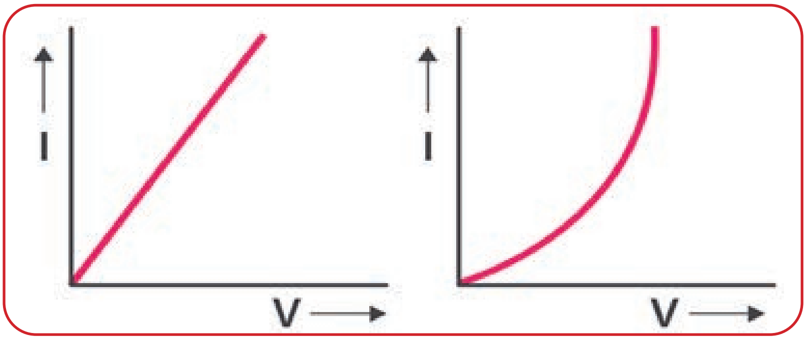


Figure 6.23: Current vs. voltage for an ohmic and non-ohmic conductors

Whenever one quantity is proportional to another quantity, there must always be some constant that relates the two quantities. In this case the constant that relates voltage and current according to Ohm's law, is the resistance, R. Thus, for a conductor (resistor) which passes current *I* when the pd across it is *V*, its resistance *R* is given by the equation

$$R = \frac{V}{I}$$

The SI unit of resistance is Ohm denoted by Ω, a Greek letter read as Omega.

$$1\Omega = 1V / A.$$

Large resistances are measured in Kilo Ohm (KΩ) and Mega ohm (MΩ).

$$1K\Omega = 103\,\Omega \quad \text{and} \quad 1M\Omega = 106\,\Omega.$$

Note that this equation is valid for wires and resistors at constant temperature, which are called ohmic. For non ohmic conductors, such as a filament lamp, the current is not directly proportional to the potential difference. Filaments are materials used in lamps that are composed of metals that glow when a certain amount of current flows through them. The resistance of a filament depends on its temperature. A filament bulb gives a curve with a decreasing gradient because its resistance increases with temperature, Figure 6.23 (b).

Example 6.11

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Solution

Rearranging Ohm's law for R and substituting known values gives

$$R = \frac{V}{I} = \frac{12V}{2.5A} = 4.8\Omega .$$

Exercise 6.10

The current through a component is 2.0 mA when connected to a source which provides 12 V. Assuming its resistance is unchanged, what pd is required across the component when the current is 2.0μA?

Activity 6.8:

1. The potential difference (PD) across a resistor means that there is a change in potential across the resistor. Is there any change in current as it passes through a resistor? Explain
2. How is the potential difference (PD) drop in a resistor similar to the pressure differencein a fluid flowing through a pipe.

Resistors

An object of conducting material, having a resistance of desired value, is called a **resistor**. In an electric circuit, a resistor is represented by symbol shown in Figure 6.24.

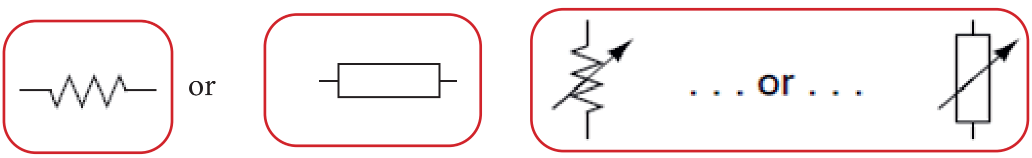


Figure 6.24: Symbol of (a) resistor and (b) variable resistor

Combination of Resistors

Usually, we find circuits where more than two components are connected together. There are two basic ways in which to connect more than two resistors. They are series and parallel.

Resistors in Series.

In series connection components are connected end-to-end in a line to form a single path for electrons to flow. The defining characteristic of a series circuit is that there is only one path for electrons to flow.

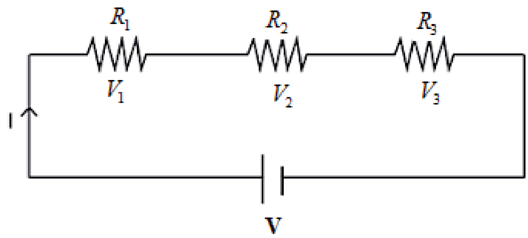


Figure 6.25: Three resistors (R_1, R_2, and R_3), connected in series.

In series circuit the amount of current is the same through any component in the circuit.

$$I = I_1 = I_2 = I_3 ...$$

Using Ohm's law ($V = IR$), the potential difference across each resistor is given by:

$$V_1 = I_1R_1, \ \ V_2 = I_2R_2, \ and\, V_3 = I_3R_3$$

The potential difference V across all three resistors is given by

$$V = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$$

The total resistance R of the three resistors is then given by

$$R = R_1 + R_2 + R_3$$

Resistors in parallel

Figure 6.26 below shows a circuit containing three resistors of resistances $R_1, R_2, and\ R_3$ connected in parallel. In parallel connection, there are more than one path for electrons to flow. The defining characteristic of a parallel circuit is that all components are connected between the same set of electrically common points (P and N). In a purely parallel circuit, the potential difference across any resistor is the same,

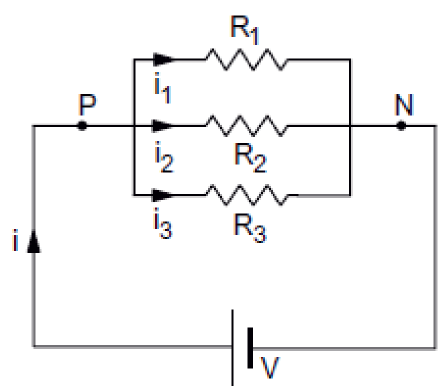


Figure 6:26 Resister connected in parallel

$$V = V_1 = V_2 = V_3 .$$

To find the equivalent resistance of the combination, let us apply a potential difference V between the points P and N. The current 'I' is divided at the junction P. Suppose a current I_1 goes through R_1, I_2 through R_2 and I_3 through R_3. These combine at N to give a total current. Therefore, $I = I_1 + I_2 + I_3$.

So,

$$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V}{R_3}$$

Since the pd across each resistor is the same, $V = V_1 = V_2 = V_3$. This gives

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

V cancels throughout to give

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For only two resistors connected in parallel, the equivalent resistance can be determined by the equation.

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

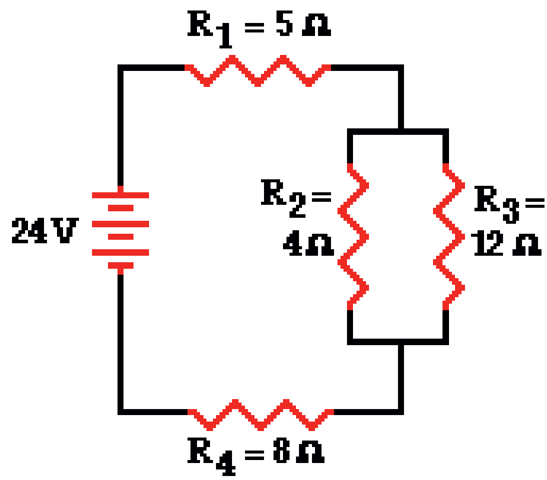
Activity 6.9:

Discuss in group and give a mathematical reason for the following fact.

1. For resistors connected in series, the total resistance is larger than the resistance of the resistor with the largest resistance, but for resistors connected in parallel, the total resistance is smaller than the resistance of the resistor with the smallest resistance.

Example 6.12

For the circuit shown in the figure below, determine (a) the equivalent resistance of the circuit, (b), the current through each resistor, and the potential difference across each resistor.



We start by combining resistors R_2 *and* R_3 in parallel.

$$R_{23} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{4 \times 12}{4+12} = 3\Omega$$

Then, all the three are connected in series, Therefore,

$$R_{eq} = R_1 + R_{23} + R_4 = 5\Omega + 3\ \Omega + 8\Omega = 16\Omega$$

The current from the battery can then be determined by using Ohm's law.

$$I = \frac{V}{R} = \frac{24V}{16\Omega} = 1.5\,A$$

Since R_1, R_{23} *and* R_4 are in series with the source, they will have the current of the source. Thus,

$$I_T = I_1 = I_{23} = I_4 = 1.5\,A$$

Using Ohm's law:

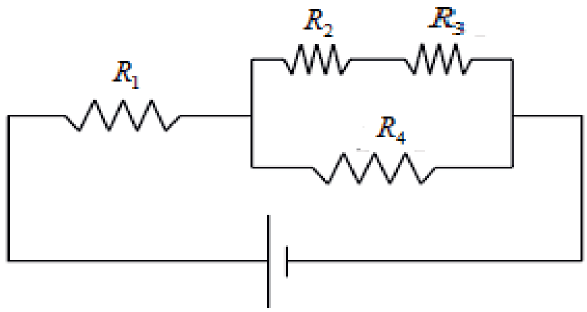
$$V_1 = I_1 R_1 = 1.5\,A \times 5\Omega = 7.5V$$
$$V_{23} = I_{23} R_{23} = 1.5\,A \times 3\Omega = 4.5V$$

$$\Rightarrow I_2 = \frac{V_{23}}{R_2} = \frac{4.5V}{4.0\Omega} = 1.125\,A \quad \text{and} \quad \Rightarrow I_3 = \frac{V_{23}}{R_3} = \frac{4.5V}{12\Omega} = 0.375\,A$$

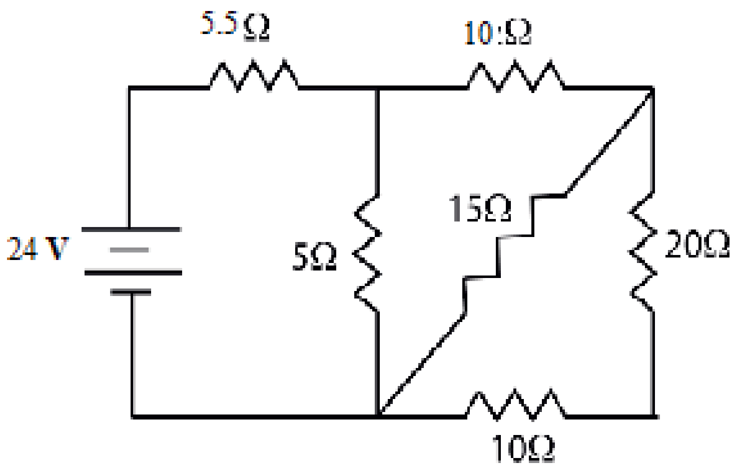
$$V_4 = I_4 R_4 = 1.5\,A \times 8\Omega = 12V$$

Exercise 6.11

1. For the circuit shown below, list resistors that are connected (a) in series, and (b) in parallel.



2. What is the equivalent resistance of the circuit shown in the figure? (b) What current flows through the 5Ω resistor?



Activity 6.10:

Use the links below to perform virtual experiment on DC Circuits. You may need your teacher's guidance on how to use it.

https://phet.colorado.edu/en/simulations/circuit-construction-kit-dc
http://phet.colorado.edu/sims/html/circuit-construction-kit-dc-virtual-lab/latest/circuit-construction-kit-dc-virtual-lab_en.html

Title: Verification of the laws of combination of resistance

Objectives:

1. To study the characteristics of resistances connected in series.
   A. Compare the measured value with the calculated value or resistors connected in series.
   B. Verify that the same current flows through all resistors that are connected in series.
   C. Verify that voltage is added for resistors connected in series.
2. To study the characteristics resistances connected in parallel.
   A. Compare the measured value with the calculated value for resistors connected in parallel.
   B. Verify that the same potential difference is found is across resistors that are connected in parallel.
   C. Verify that current is added for resistors connected in parallel.

Theory:

Materials Required:

- Digital multimeters (2)
- DC power supplies, 0-15 V (2)
- Resistors and connecting wires (3)
- Circuit board (1)

Procedure:

Series Circuit

I. Record the resistances of three resistors (R_1, R_2 and R_3) determined by measurement

II. Construct a series circuit with these resistors.

III. Measure the equivalent resistance R_{eq}, of the circuit by connecting the terminals of the multimeter across the beginning of R_1 and end of R_3. Then compare your measured value with that determined by the equation: $R_{eq} = R_1 + R_2 + R_3$

IV. Connect power supply. Using the multimeter measure the potential differences across all the resistances and the current flowing in the circuit. Then apply ohms law (R= V/I). Finally, compare your result with what you have found in number iv.

V. Measure the potential differences across each of the resistors. Add the potential differences. Do they add up to the voltage across the power supply?

Calculation: In Series circuit

I. Experimental value of Req = ………….. ohm

II. Theoretical value of Req = ………….. ohm

III. Difference (if any) = ………….. ohm

Note the following.

- Have your lab instructor approve the circuit before plugging in the power supply.
- Whenever you use the multimeter, do not forget to adjust the scale of the multimeter to the quantity being measured.

Parallel Circuit

I. Connect the above three resistors in parallel with each other on the circuit board. Have your instructor check the circuit before plugging in the power supply. After approval, plug in the power supply.

II. Record this voltage. Now measure the potential difference across the resistor R1 and record this voltage. Do the same for the R2 and the R3 resistors. Are the values the same?

III. Measure current vs. potential as before. Graph current vs. voltage on the same plot as the series graph and determine the total equivalent resistance of the circuit.

IV. Disconnect the power supply from the circuit and measure the total resistance of the circuit with the multimeter as before.

Calculations in parallel circuit

(a) Experimental value of R_{eq} = ………….. ohm

(b) Theoretical value of R_{eq} =………….. ohm

(c) Difference (if any) = …………..ohm.

Measuring Instruments

Measurement of Current

An ammeter is a device used to measure current flowing in a circuit. To measure the current flow in a circuit the ammeter must be connected in series with the resistor, so that the same current passes through both the resistor and the ammeter. The internal resistance of an ammeter should be as small as possible, so that little energy is dissipated in it.

Figure 6.27: Ammeters and How to connect them.

Exercise 6.12:

In the circuit, shown below, an ammeter has a resistance of 0.5 Ω . Determine the current in the circuit, (a) before an ammeter is introduced, and (b) after it is introduced. (c) Based on your result of question (b), discuss the effect of inserting the ammeter.

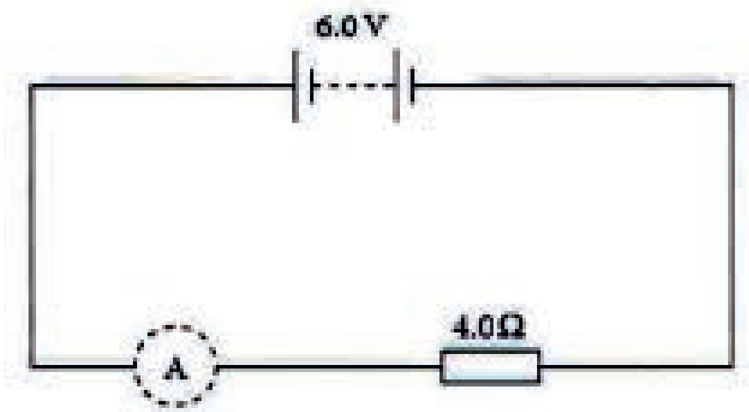


Figure 6.38: Investigating the effect of ammeters

Measurement of Voltage

Voltmeter is a device used to measure the potential difference between two points in the circuit. For this reason, they are connected in parallel. It could be voltage across the terminal of a battery or across a resistor or any other circuit element. Voltmeters should have a very high resistance to take as little current as possible. The ideal resistance of a voltmeter would be infinite.

Figure 6.29: Voltage measuring devices

Exercise 6.13

A voltmeter shown in the figure below which can measure up to 6V has a resistance of 50 kΩ. Determine the pd across the 20 kΩ in the circuit, (a) when the voltmeter is not in place, and (b) when the voltmeter is in place. (c) Based on your result of question (b), discuss the effect of inserting a voltmeter.

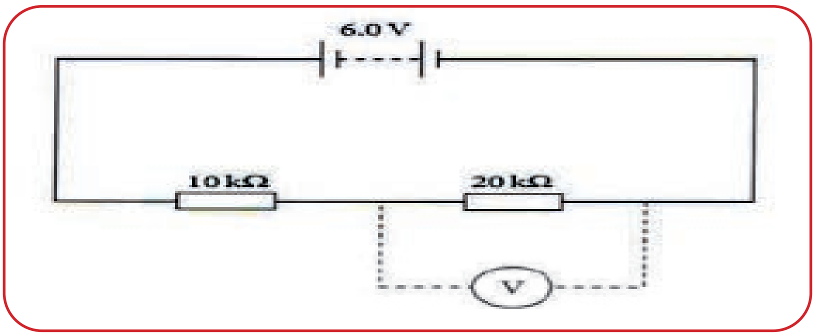


Figure 6.30: Investigating the effect of a voltmeter

Conversion of galvanometer to ammeter and voltmeter

A galvanometer is an instrument used for measuring small electric current in electric circuit. It works by deflecting a pointer in response to an electric current flowing through a coil in a constant magnetic field. A galvanometer has a coil pivoted (or suspended) in a magnetic field. When an electric current passes through the coil, it deflects. The deflection is proportional to the current passed.

There are different types of galvanometers. The most common type, shown in Figure 6.31, is the moving coil galvanometer in which it consists of a light coil of wire suspended from a metallic ribbon between the poles of a permanent magnet.

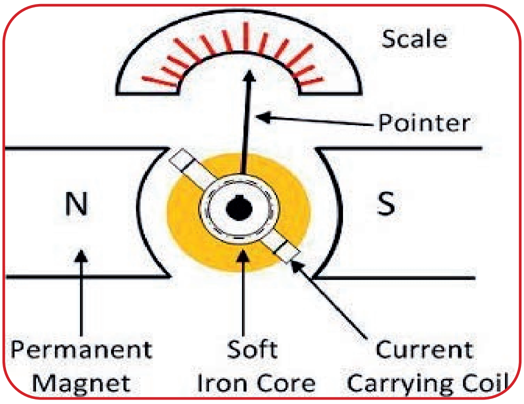


Figure 6.31: The moving-coil galvanometer

converting a galvanometer into an ammeter?

Since a galvanometer is a very sensitive instrument it can't measure heavy currents. In order to convert a galvanometer into an ammeter, a very low resistance known as "shunt" resistance is connected in parallel to it, as shown in Figure 6.32. The shunt allows most of the current passes through it and the remaining small current through the galvanometer.

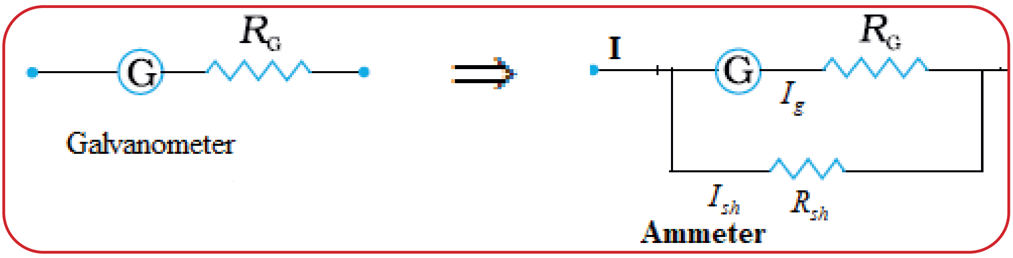


Figure 6.32: Converting galvanometer to an ammeter

Let R_g be the resistance of the galvanometer and I_g be the current for full scale deflection in the galvanometer. The value of the shunt resistance required to convert the galvanometer into an ammeter is,

$$V_g = V_{sh}$$

$$(I - I_g) R_{sh} = I_g R_g$$

$$R_{sh} = \left(\frac{I_g}{I - I_g}\right) R_g$$

Example 6.14

A galvanometer of full-scale deflection 5 mA is to be converted into a 10 A ammeter. If its coil has a resistance of 50 Ω, what value shunt must be fitted?

Solution:

$$R_{sh} = \left(\frac{I_g}{I - I_g}\right) R_g$$

$$R_{sh} = \left(\frac{5\times10^{-3}}{10-5\times10^{-3}}\right)50 = \left(\frac{0.005}{10-0.005}\right)50 = \left(\frac{0.005}{9.995}\right)50 = 0.025\,\Omega$$

Converting a galvanometer into voltmeter

A galvanometer can be converted into voltmeter by connecting with it with a very high resistance called multiplier in series with it as shown in Figure 6.33.

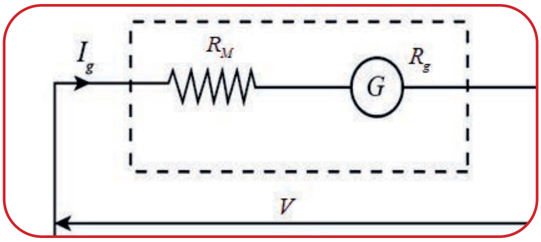


Figure 6.33: Converting a galvanometer into voltmeter

Let R_g be the resistance of a galvanometer and R_M the resistance of the multiplier connected in series with it. Let V volt be the potential difference to be measured by the voltmeter and I_g, be the current. Potential difference across the two resistors in the circuit is the sum of potential difference across galvanometer and potential difference across the multiplier.

$$V = I_g R_M + I_g R_g = I_g (R_M + R_g)$$

$$\therefore R_M + R_g = \frac{V}{I_g}$$

$$R_M = \frac{V}{I_g} - R_g$$

This is the required value of multiplier resistance which must be connected in series to the galvanometer to convert it into a voltmeter.

Example 6.14

A galvanometer has a resistance of 40 Ω and is of 3 mA full scale deflection. How would you modify it to a 0 – 10 V voltmeter.

Solution:

$$V = I_g (R_g + R_{sh})$$

$$R_{sh} = \frac{V}{I_g} - R_g = \frac{10}{5\times10^{-3}} - 40$$

$$R_{sh} = 1960\ \Omega$$

Exercise 6.14

Explain how you will convert a galvanometer into (a) an ammeter, and (b) a voltmeter.

The Potential divider

A potential divider consists of two or more resistors in series with each other and with a source of fixed potential difference. The potential difference of the source is divided between the components in the circuit, as they are in series with each other, Figure 6.34.

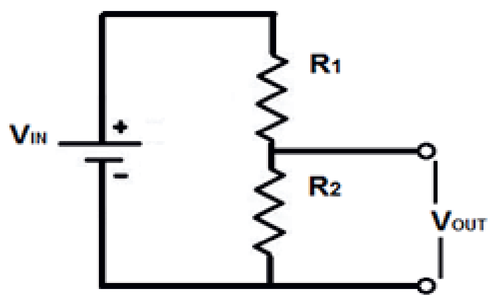


Figure 6.34: Potential divider circuit

For the potential divider circuit in Figure 6.34

The total resistance of the combination is: $R_T = R_1 + R_2$

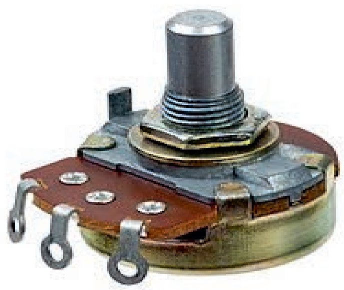
The current through the resistors is: $I = \frac{V}{R_T} = \frac{V}{R_1 + R_2}$

The potential drop across resistor R_1: $V_1 = IR_1 = \left(\frac{R_1}{R_1 + R_2}\right)V$

The potential drop across resistor R_2: $V_2 = IR_2 = \left(\frac{R_2}{R_1 + R_2}\right)V$

Potentiometer

A potentiometer is a three-terminal resistor with a sliding or rotating contact that forms an adjustable voltage divider. It works by varying the position of a sliding contact across a uniform resistance wire. In a potentiometer, the entire input voltage is applied across the whole length of the resistor, and the output voltage is the voltage drop between the fixed and sliding contact as shown in Figure 6.35. Thus, a potentiometer is essentially a type of potential divider circuit.



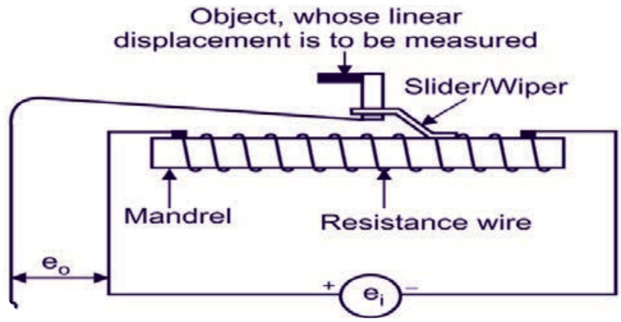


Figure 6.35: Potentiometer

The Potentiometer can also be used for the following purposes.

- to measure the emf of a given cell,
- to measure the internal resistance of a cell,
- to compare the emf of different cells, and
- as a variable resistor.

Figure 6.36 shows the two symbols of a potentiometer in an electric circuit.

Figure 6.36: Symbols of a potentimeter

Potentiometer to measure the emf of a cell

A potentiometer consists of a piece of resistance wire, usually 1 m in length, stretched horizontally between two points. In Figure 6.37, the ends of the wire are labelled A and B. A driver cell is connected across the length of wire. Suppose this cell has an emf of 2.0V. We can then say that point A is at a voltage of 2.0V, B is at 0V and the midpoint of the wire is at 1.0 V. In other words, the voltage decreases steadily along the length of the wire.

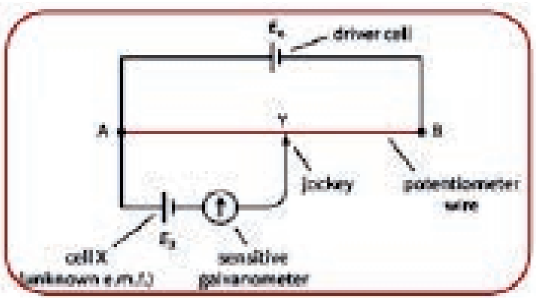


Figure 6:37 A potentiometer connected measure the e.m.f. of cell X,

Now, suppose we wish to measure the emf. E_X of cell X (this must have a value less than that of the driver cell). The positive terminal of cell X is connected to point A. (Note that both cells have their positive terminals connected to A.) A lead from the negative terminal is connected to a sensitive galvanometer, and a lead from the other terminal of the galvanometer ends with a metal jockey. This is a simple connecting device with a very sharp edge that allows very precise positioning on the wire.

If the jockey is touched onto the wire close to point A, the galvanometer needle will deflect in one direction. If the jockey is touched close to B, the galvanometer needle will deflect in the opposite direction.

Clearly, there must be some point Y along the wire that, when touched by the jockey, gives zero deflection. In finding this position, the jockey must be touched gently and briefly onto the wire; the deflection of the galvanometer shows whether the jockey is too far to the left or right. It is important not to slide the jockey along the potentiometer wire as this may scrape its surface, making it non-uniform so that the voltage does not vary uniformly along its length.

When the jockey is positioned at Y, the galvanometer gives zero deflection, showing that there is no current through it. This can only happen if the potential difference across the length of wire AY is equal to the emf. of cell X. We can say that the potentiometer is balanced. If the balance point was exactly halfway along the wire, we would be able to say that the emf. of X was half that of the driver cell. This technique – finding a point where there is a reading of zero – is known as a null method.

To calculate the unknown emf. E_X we measure the length AY. Then we have:

$$E_X = \frac{AY}{AB} \times E_0$$

where E_0 is the emf. of the driver cell.

Potentiometer to compare the emf of two cells

Suppose we have two cells whose emfs E_X and E_Y we want to compare. Each is connected in turn to the potentiometer, giving balance points at C and D, see Figure 6.38. (In the diagram, you can see immediately that E_Y must be greater than E_X because D is further to the right than C.)

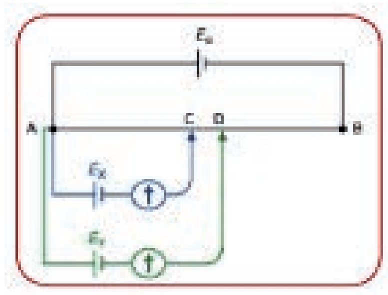


Figure 6.38: Comparing two emfs using a potentiometer.

The ratio of the emfs of the two cells will be equal to the ratio of the two lengths AC and AD:

$$\frac{E_X}{E_Y} = \frac{AC}{AD}$$

If one of the cells used has an accurately known emf, the other can be calculated with the same degree of accuracy.

Measurement of Resistance

Resistance can be measured directly by using ohmmeter. Experimentally resistance is measured using the following two methods.

1. The ammeter-voltmeter method.

In this method the current is measured using an ammeter and the voltage using the voltmeter, see Figure 6.39. Then, using Ohm's law ($R = \frac{V}{I}$), resistance will be determined. The variable resistor is used to adjust the current and voltage as necessary.

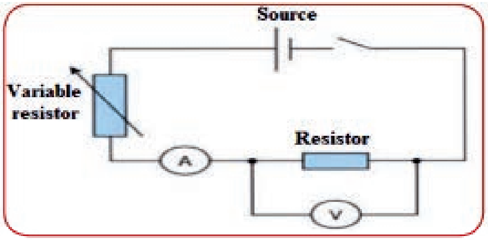


Figure 6.39: Circuit to measure resistance

2. Using Wheatstone bridge

Another more precise method of measuring resistance is using a Wheatstone bridge. A Wheatstone bridge is an arrangement of four resistances which can be used to measure one of them in terms of the rest. The arrangement is shown in Figure 6.40.

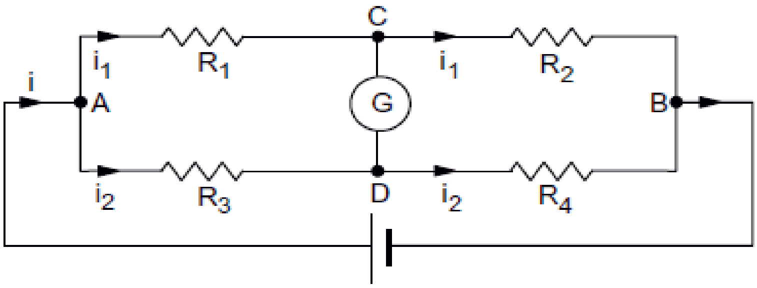


Figure 6.40: A Wheatstone bridge

Four resistors with resistances R_1, R_2, R_3 *and* R_4 are connected to form a loop. There are four joints A, B, C and D. A battery is connected between two opposite joints A and B and a galvanometer is connected between the other two opposite joints C and D.

The current i from the battery is divided at A in two parts. A part i_1 goes through R_1 and the rest i_2 goes through R_3. When no current is flowing through, the bridge, it is said to be balanced. In this case, the current in R_2 is the same as the current in R_1 and the current in R_4 is the same as that in R_3. As there is no current through the galvanometer, the potential difference across its terminals is zero.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

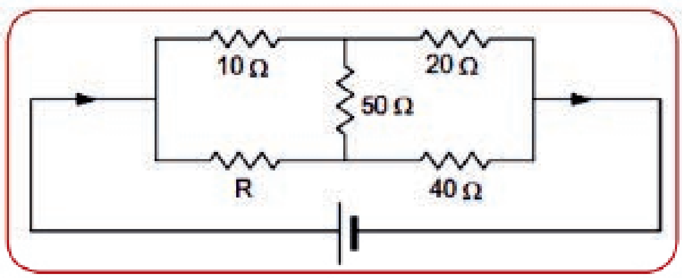
This is the condition for which a Wheatstone bridge is balanced. To measure the resistance of a resistor, it is connected as one of the four resistors in the bridge. Given the other three resistors, the value of the fourth unknown resistor will be easily determined.

Activity 6.11:

In a small group, research some applications of a Wheatstone bridge. Present your findings to the rest of the class in a form of your choice.

Example 6.15:

Find the value of R in the figure below so that there is no current in the 50 Ω resistor.



Solution:

$$R_1 = 4\Omega,\ R_2 = R, R_3 = 20\Omega, R_4 = 40\Omega$$

$$R_2 = \frac{R_1R_4}{R_3} = \frac{10\times 40}{20} = 20\Omega$$

Activity 6.12:

Experiment, WHEATSTONE BRIDGE

OBJECTIVE

The objective of this experiment is to determine the unknown resistance using Wheatstone bridge.

APPARATUS

Galvanometer, various medium resistors, multimeter, power supply (DC)

THEORY:

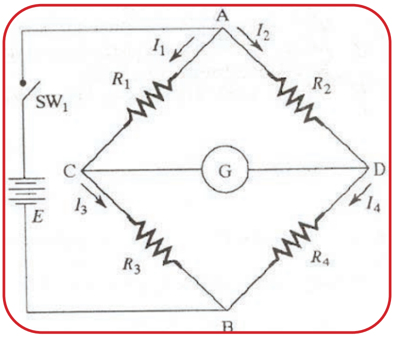


Figure 6.41: Wheatstone bridge

- Setup the circuit that is shown in the figure.
- Do the required measures to complete the table with the results of your experiments.
- Determine the resistances of any unknown resistors using a multimeter.
- Compare those values to the value obtained using the balancing equation.

Parameter	Rx
Experimental	
Theoretical	
% of error	

$$\%\ of\ error = \frac{Theoretical - Experimental}{Experimental} x100\%$$

DATA ANALYSIS

Compared to the direct measurement of resistance using a multimeter, what level of accuracy may be expected when applying a Wheatstone bridge?

Comparing the Brightness of Bulbs

Activity 6.13:

What are you referring when you buy a light bulb from electric shops? Discuss in pairs.

We usually see the wattage to buy a bulb that give us a brighter light. Watts are not a measure a brightness, they're a measure of energy consumption per unit time. It measures, how much electricity a bulb uses. The amount of light emitted by a bulb (brightness) is measured in **Lumens.** So, when buying light bulbs see their Lumens instead of their wattage. The higher the lumens, the more light the bulb emits.

Of course, we can use power (watts) to compare the brightness of bulbs. The higher the wattage rating, the more energy consumed, and brighter the bulb will be. From the equation for electric power, $P = I^2R$, we see that brightness depends on current and resistance.

Key term

The amount of light emitted by a bulb (brightness) is measured in Lumens.

Example 6.16

Compare the brightness of two lamps arranged in parallel with the brightness of similar lamps arranged in series (Both arrangements having one cell)?

Solution:

In parallel, both the bulbs will have the same voltage across them as that of the battery. However, in series connection, the voltage gets divided between the two bulbs. So, the bulbs which are connected in parallel will glow brighter.

Example 6.17

Compare the brightness of the following the three lamps A, B, and C in the figure below.

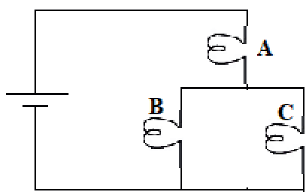


Figure 6.42:

Solution:

Of all the three bulbs, Bulb A is brightest, because it gets the full current from the battery. At the junction, the current then splits, resulting to the current in bulbs B and C to be half of that in A (assuming the bulbs to be identical). Thus, Bulbs B and C will have the same brightness but less than that of A.

Exercise 6.15:

What would happen to the brightness of the bulbs in example 6.17, if another bulb D is connected in parallel to that of bulbs B and C, See Figure 6.43 below?

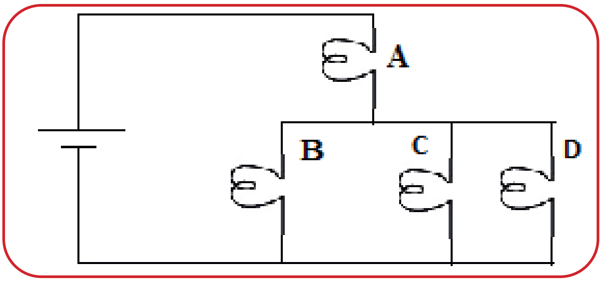


Figure 6.43:

Activity 6.14:

Look at the data in the table below. The Lumens, and wattage of Incandescent, compact fluorescent light (CFL) and Light Emitting Diodes (LED) bulbs are given. Discuss in group the advantages and disadvantages of using CFL od LED lamps over the incandescent lamps.

Table 6.2: Comparing Incandescent, CFL and LED lamps.

Lumens	 Incandescent Watts	 CFL Watts	 LEDs Watts
450	40W	8 – 12W	6 – 7W
800	60W	13 – 18W	7 – 10W
1100	75W	18 – 22W	12 – 13W
1600	100W	23 – 30W	14 – 20W
2600	150W	30 – 55W	25 – 28W

Kirchhoff's Rules

The following two circuit rules, which are called Kirchhoff's rules, can be used to analyze any dc circuit, regardless of how many loops and cells are in the circuit.

Kirchhoff's First Rule (Junction rule)

Kirchhoff's first rule states that, at any junction in a circuit, the total current entering the junction is equal to the total current leaving the junction. This means, the sum of all the currents directed towards a junction in a circuit is equal to the sum of all the currents directed away from that junction.

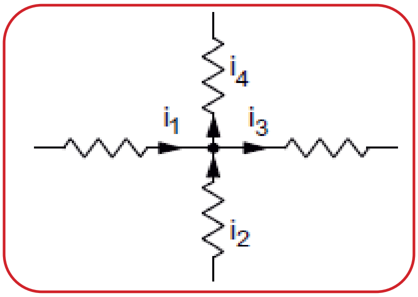


Figure 6.44: Kirchhoff's junction rule

Thus, in Figure 6.44, the currents I_1 *and* I_2 enters into the junction, while the currents I_3 *and* I_4 leave the junction.

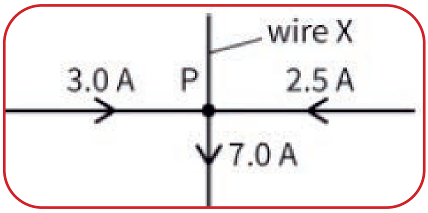
Thus,

$$I_1 + I_2 = I_3 + I_4$$

Kirchhoff's junction rule is based on the law of conservation of charge. The fact that no point in a circuit keeps on accumulating charge or keeps on supplying charge, the net charge coming towards the point should be equal to that going away from it in the same time.

Exercise 6.16:

The figure below is part of a circuit. Calculate the current in the wire X. State the direction of this current (towards P or away from P).



Kirchhoff's Second Rule (Loop Rule)

Kirchhoff's loop rule is stated as:

For any complete loop of a circuit, the sum of the emfs round the loop is equal to the sum of the potential drops around the loop'. It follows from the law of conservation of energy.

When using this rule, one starts from a point on the loop and goes along the loop, either clockwise or anticlockwise, to reach the same point again. In using Kirchhoff's loop rule consider the following points.

1. Assume all voltage sources and resistances are given. (If not label them

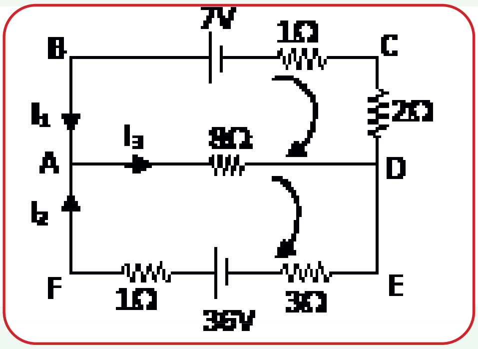
   V_1, V_2, ...R_1, R_2 *etc*
2. Label each branch with a branch current. (I_1, I_2, I_3 *etc*)
3. Apply junction rule at each junction.
4. Applying the loop rule for each of the independent loops of the circuit.
5. Solve the equations by substitutions.

For Kirchhoff's loop rule, the following convention is used:

1. The e.m.f of the cells is taken as positive when the direction of motion is from negative terminal to the positive terminal, but negative when traversing from positive to negative terminal.
2. The potential difference across a resistor (IR) is taken as positive if we move in the opposite direction to the current, and negative when moving in the direction of the current.

Example 6.17:

Find the currents I_1, I_2 and I_3 in the circuit shown below.



Solution:

Applying Kirchhoff's first rule to junction A: $I_1 + I_2 = I_3$ (1)

Applying Kirchhoff's second rule for loop ABCDA (clockwise)

$$-7 + I_1(1) + I_1(2) + I_3(8) = 0$$

$$3I_1 + 8I_3 = 7 \qquad (2)$$

Applying Kirchhoff's second rule for loop ADEFA (clockwise)

$$-I_3(8) - I_2(3) + 36 - I_2(1) = 0$$

$$2I_3 + I_2 = 9 \qquad (3)$$

Substituting $I_1 = I_3 - I_2$ into equation 2

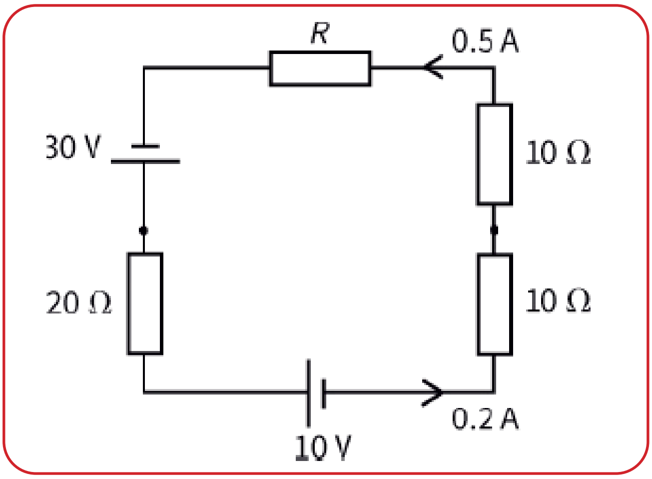
$$3(I_3 - I_2) + 8I_3 = 7$$

$$11I_3 - 3I_2 = 7 \qquad (4)$$

Solving equation (3) and (4) simultaneously yields $I_3 = 2A$
Substituting for I_3 into equation (3) yields $I_2 = 5A$
Substituting for I_2 and I_3 into equation (1) yields $I_1 = -3A$
The negative sign tell us that I_1 is opposite in direction to that shown in Figure 6.3:

Exercise 6.17:

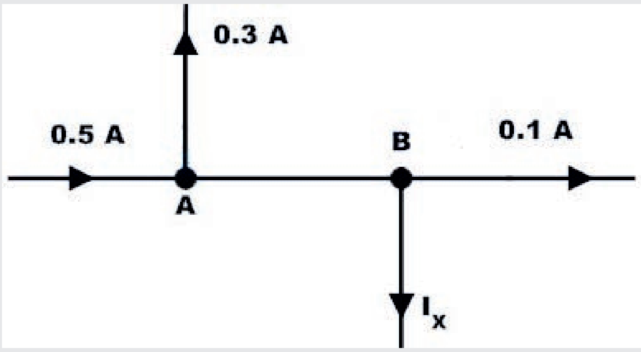
By using Kirchhoff's second law determine the resistance R of the resistor shown in the circuit below.



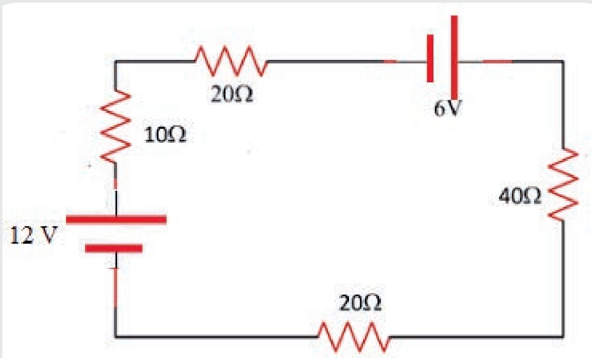
Review Questions 6.4

1. Write the definition of the following terms using your own words: electric current, current density, resistance, conductivity, resistivity and drift velocity.
2. Write the SI units of the following quantities. Electric current, current density, resistance, resistivity, and conductivity.
3. What is the meaning of one ohm?
4. Explain how a potentiometer works.
5. What is the use of a Wheatstone bridge? Discuss how it works.
6. Describe how to convert galvanometer to a voltmeter and ammeter.
7. State Kirchhoff's first and second laws.
8. A cell of emf 1.5 V and internal resistance 0.5 Ω is connected to a 2.5 Ω resistor. Calculate: (a) the current, (b) the terminal pd, (c) the power delivered to the 2.5 Ω resistor, and (d) the power wasted in the cell.
9. The pd across the terminals of a cell was 1.1 V when the current from the cell was 0.20 A, and 1.3 V when the current was 0.10 A. Calculate: (a) the internal resistance of the cell, (b) the cell's emf.

10. The figure shown below is part of an electric circuit. Determine the value of the unknow current I_x.

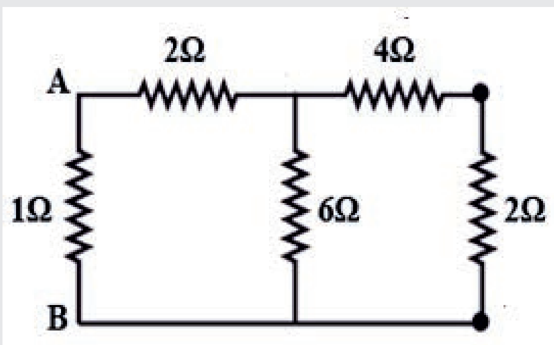


11. Determine the electric current that flows in the circuit below.



12. A galvanometer with a resistance of 36Ω is converted into an ammeter using a shunt resistance of 4Ω. What fraction of the total current passes through the galvanometer?

13. Determine equivalent resistance between points A and B in circuit shown below.



6.6 Capacitors and Capacitance

By the end of this section, you will be able to:

- *Define the terms capacitors, capacitances, dielectric.*
- *Explain the effect of inserting dielectric in the gap between the plates of a capacitor.*
- *Solve problems related to capacitance of parallel plate capacitor.*
- *Calculate the equivalent capacitance of capacitors connected in series and parallel.*

Brainstorming

1. What is the function of a capacitor?
2. How is a capacitor is charged?
3. Mention devices that we use in daily life which uses a capacitor for storing energy.

Capacitor and Capacitance

A combination of two conductors placed close to each other but not touching form a capacitor. In charging a capacitor one of the plates is connected to the positive terminal and the other to the negative terminal of a DC source. The electric field created forces electrons to flow from the positive plate to the negative plate until the potential difference between the plates of the capacitor becomes equal to the emf of the battery.

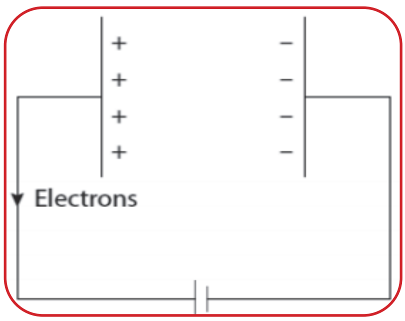


Figure 6.46: A capacitor, with the positive plate carrying a positive charge and the negative plate an equal amount of negative charges.

Finally, positive charges will be accumulated on the positive plate and negative charges on the negative plate, see Figure 6.47.

The following are circuit symbols for capacitors.

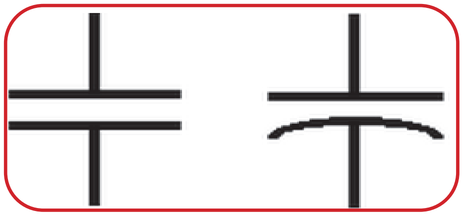


Figure 6.47: Circuit symbol of a capacitor

For a given capacitor, the charge Q stored on the capacitor is proportional to the potential difference V between the plates, $Q \propto V$. Thus,

$$Q = CV \quad \Rightarrow C = \frac{Q}{V}$$

The proportionality constant C is called the *capacitance* of the capacitor. It is independent of Q and V but depends on geometrical factors. The SI unit of capacitance is farad (F). Small values of capacitances are measured in micro, nano and pico farads. 1 μF = 10^{-6} F, 1 nF =10^{-9} F, and 1 pF =10^{-12} F.

Key terms

- *A capacitor is a device used to store electric charge, and hence electric energy.*
- *A capacitor is a device used to store electric charge, and hence electric energy.*

Parallel-plate Capacitor

A parallel-plate capacitor consists of two large plane plates placed parallel to each other with a small separation between them (Figure 6.48). Suppose, the area of each of the facing surfaces is A and the separation between the two plates is d. Also, assume that the space between the plates is filled with vacuum.

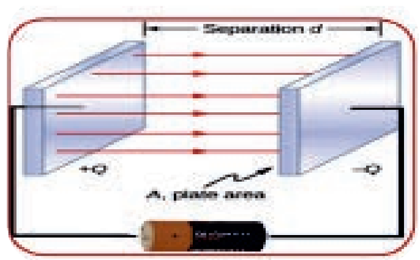


Figure 6.48: Capacitor

If a charge Q is stored on the positive plate, a charge –Q will be stored on the negative plate. The charges will appear on the facing surfaces. The charge density on each of these surfaces has a magnitude

$$\sigma = \frac{Q}{A}$$

The electric field between the plates is given by the equation

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

The potential difference between each plate is given by:

$$V = Ed$$

Therefore

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

Substituting this into our defining equation for capacitance we get:

$$C = \frac{Q}{V} = \frac{Q}{Qd / A\varepsilon_0} = \varepsilon_0 \frac{A}{d}$$

This equation gives the capacitance of two parallel plates in a vacuum. If a dielectric material of dielectric constant k is inserted between the plates, the equation for capacitance became:

$$C = k\varepsilon_0 \frac{A}{d}$$

Hence, the capacitance of a parallel plate capacitor depends on the size, distance of separation of the plates, and the medium between them.

Example 6.19

Consider a parallel plate capacitor with plates of area $200\ cm^2$ and separated by a sheet of mica thickness 2mm and dielectric constant 6. If the capacitor is connected to 12V source, determine the charge stored on the plates of the capacitor.

Solution:

$$C = k\varepsilon_0 \frac{A}{d} = 6 \times 8.85 \times 10^{-12} C^2 / N.m^2 \times \frac{2 \times 10^{-2} m^2}{2 \times 10^{-3} m}$$

$$= 5.31 \times 10^{-10} F$$

$$Q = CV = 5.31 \times 10^{-10} F \times 12V = 6.37 \times 10^{-9} F = 6.37 nF$$

Key term

A dielectric is an insulator such as a glass, paraffin, wood or pastic that is inserted in the gap between the plates of a capacitor.

Exercise 6.18

The dimensions of a parallel plate capacitor are 5cm by 10m, they are separated by a distance of 2mm, and connected to a 24V dc source. If an insulator of dielectric constant 5 is inserted between the plates, determine the amount of charge stored on the plates.

Activity 6.15:

Discuss in group about the following questions.

If you were asked to design a capacitor with large capacitance, how are you going to design?

Dielectrics

A dielectric is an insulating or nonconducting materials such as glass, paraffine, wood or plastic, that is inserted in the gap between the plates of a capacitor. When a dielectric is inserted in the electric field between the plates, there will be some charge distribution in it. The positive charges within the dielectric are displaced lightly towards the negative plate, and the negative charges are displaced slightly towards the positive plate.

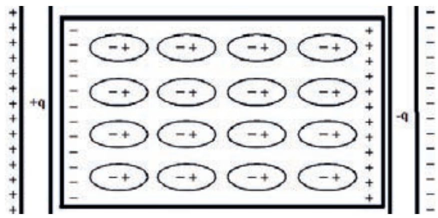


Figure 6.49: Polarization in a dielectric inside an electric field of a capacitor

This slight separation of charge is known as polarization. Due to polarization the potential difference and hence the electric field between the plates will decrease. Since, the dielectric is not disconnected from the source, the potential difference between the plates finally will be that of the source. As a result, the capacitor can hold more charge, and thus, its capacitance increased, $C = kC_0$.

A dielectric constant, k, also called relative permittivity or specific inductive capacity, measures the ability of a capacitor to store charge. It is the property of the dielectric. It is equal to the ratio of the capacitance of a capacitor filled with the given material to the capacitance of an identical capacitor in a vacuum without the dielectric material.

$$k = \frac{Capacitance\ with\ dielectric}{Capacitance\ without\ dielectric} = \frac{C}{C_0}$$

The dielectric constant is a number without dimensions. The value of the dielectric constant for vacuum is one and for any other material is it is greater than one. Inserting a dielectric always increases the capacitance of the capacitor.

Example 6.20

If a dielectric is inserted in the gap between the plates of an isolated capacitor, what are the effects on (a) the capacitance of a capacitor, (b) the potential difference between the plates of a capacitor, (c) amount of the charge stored on the plates, and (d) the electric field in the region between the plates of a capacitor.

Solution:

- When inserting a dielectric in a capacitor its capacitance always increases.
- Due to polarization the potential difference between the plates decreases.
- Since the capacitor is isolated (disconnected from the source) the amount of charge stored on the plates remains the same.
- From the equation $E = \Delta V / d$, the electric field in the region between the plates decreases.

Activity 6.16:

1. Discuss in group the effect of inserting a dielectric on a capacitor which is not disconnected from the source on (a) the capacitance of a capacitor, (b) the potential difference between the plates of a capacitor, (c) the charge stored on the plates, and (d) the electric field in the region between the plates.
2. What about if a dielectric is withdrawn from a capacitor?

Dielectric strength

For any insulator, there is a maximum electric field that can be maintained without ionizing the molecules. This means that there is a maximum allowable voltage that that can be placed across the conductors. This maximum voltage depends on the dielectric in the capacitor. The corresponding maximum field E_b is called the **dielectric strength** of the material. It is the maximum electric field that the material can withstand without undergoing electrical breakdown and becoming electrically conductive (i.e. without failure of its insulating properties). The table below shows the dielectric strength of some materials.

Table 6.3: Dielectric constant and dielectric strength of some materials

Material	Dielectric Constant	Dielectric strength (V/m)
Air	1.0	3×10^{6}
Glass	4.8 - 10	30×10^{6}
Mica	4.8 - 10	200×10^{6}
Paraffin paper	2.0	40×10^{6}
Rubber	3.2	21×10^{6}
Water (distilled)	78.5	3×10^{6}

Activity 6.22: Project

Search an internet or read books on how to make a capacitor, and construct it using locally available materials. Demonstrate your capacitor to your class by showing a practical work, like charging your cell phone.

Combination of Capacitors

Capacitors are manufactured in a certain standard capacitance and working voltages. But these values may not be compatible with the ones that we effectively need for a particular application. In such cases we obtain the desired values by combining capacitors in different ways. Two or more capacitors may be connected either in series or in parallel.

Capacitors connected in parallel

Consider three capacitors connected in parallel as shown in Figure 6.50.

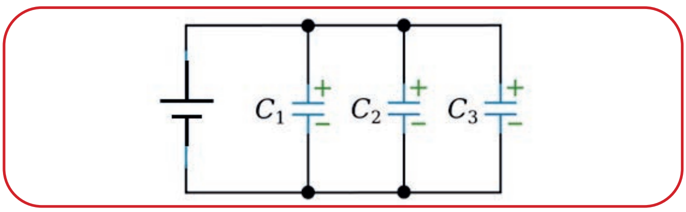


Figure 6.50: Three capacitors C_1, C_2 and C_3 connected in parallel.

The same potential difference is provided for each capacitor.

a

$$V = V_1 = V_2 = V_3$$

Charges Q_1 is stored on capacitor C_1, and charge Q_2 on capacitor C_2 and so on. The total charge will therefore be:

$$Q = Q_1 + Q_2 + Q_3$$

Since potential difference across the capacitors is the same, V. Then,

$$Q_1 = C_1V\, ,\ Q_2 = C_2V \ \ and\ \ Q_3 = C_3V\ .$$

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)V\ .$$

From which, $\dfrac{Q}{V} = C_1 + C_2 + C_3$.

But we know that $C_{eq} = \dfrac{Q}{V}$, Which yields

$$C_{eq} = C_1 + C_2 + C_3.$$

For any number of capacitors connected in parallel then

$$C_{eq} = C_1 + C_2 + C_3 ... + C_n$$

Capacitors Connected in Series

Consider three capacitors of capacitances $C_1, C_2, and\ C_3$ *connected* in series to a potential difference of V as shown in Figure 6.51.

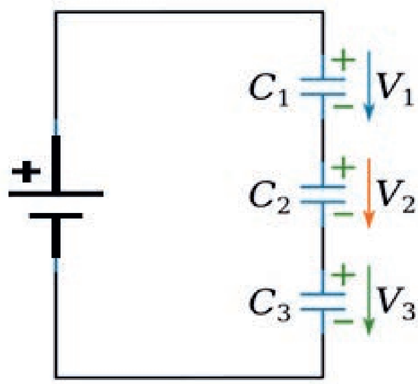


Figure 6.51: Three capacitors C1, C2 and C_3 *connected in series*

Since the capacitors are connected in series and are charged by induction, they will all carry the same amount of charge. Thus, the magnitude of charge on all plates is the same. That is,

$$Q_{total} = Q_1 = Q_2 = Q_3$$

Therefore, if V_1 is the voltage across C_1, V_2 is the voltage across C_2, and V_3 is the voltage across C_3, then

$$V_1 = \frac{Q}{C_1},\quad V_2 = \frac{Q}{C_2} \; and \; V_3 = \frac{Q}{C_3}$$
$$V = V_1 + V_2 + V_3$$
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

From which,

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The equivalent capacitance of the capacitors is given by

$$C_{eq} = \frac{Q}{V} \quad \Rightarrow \frac{1}{C_{eq}} = \frac{V}{Q}$$

Combining these equations, we get

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

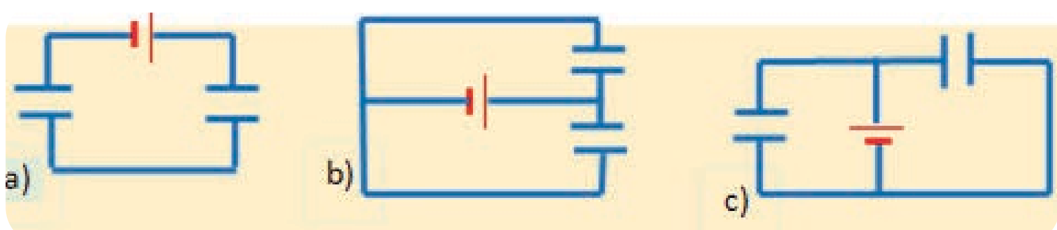
For any number of capacitors in series therefore,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \frac{1}{C_n}$$

The above equation indicates that the equivalent capacitance of a series combination of capacitors is always less than any individual capacitance in the combination.

Exercise 6.19:

For each circuit in the following figure, are the capacitors connected in series, in parallel, or in neither mode? What is your reason to say about their mode of connection?



Equivalent capacitance for multiple capacitor circuits

In most electronic circuits, capacitors are connected in series, in parallel or in combinations of series and parallel as well. Consider a capacitor network shown in Figure 6.54.

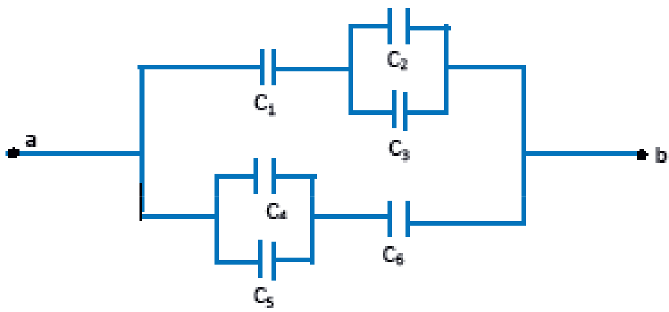


Figure 6.54: A capacitor network.

We can see in the upper portion of the circuit, C_1 is connected in series to the combinations of C_2 and C_3. But, C_2 and C_3 are connected in parallel. Again, in the lower portion of the circuit, C_6 is connected in series to the combinations of C_4 and C_5. Again, C_4 and C_5 are connected in parallel to each other. Therefore, the circuit can be reduced as follows.

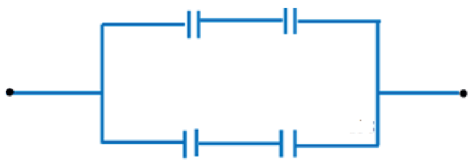


Figure 6.55: Reduced equivalent circuit of the network shown in Figure 6.54.

$C_{23} = C_2 + C_3$ and $C_{45} = C_4 + C_5$ (Since they were connected in parallel)

Now, refer to the capacitors in upper portion of the circuit in Figure 6.55. C_1 and C_{23} are connected in series to each other. Similarly, the capacitors in the lower portion of the circuit C_{45} and C_6 are connected in series. So, the circuit can further be reduced as in Figure 6.56.

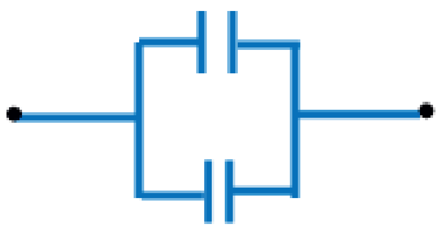


Figure 6.56: The equivalent capacitance of the network in Figure 615

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C_1 C_2 + C_1 C_3}{C_1 + C_2 + C_3}$$

and

$$C_{456} = \frac{C_{45} C_6}{C_{45} + C_6} = \frac{C_4 C_6 + C_5 C_6}{C_4 + C_5 + C_6}. \quad \text{(Since they are connected in series)}$$

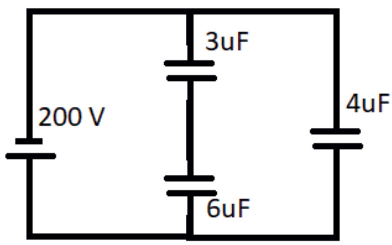
The equivalent capacitance of the six capacitors between a and b is therefore the parallel combination of these two capacitors is

$$C_{eq} = C_{123} + C_{456} ..$$

When numerical values for each capacitors are given, it is advisable to substitute the numerical values in each step.

Exercise 6.20:

For the network of capacitors shown in the figure below, determine (a) the equivalent capacitance, (b) the charge stored on each capacitor, and (c) the voltage across each capacitor.



Activity 6.18

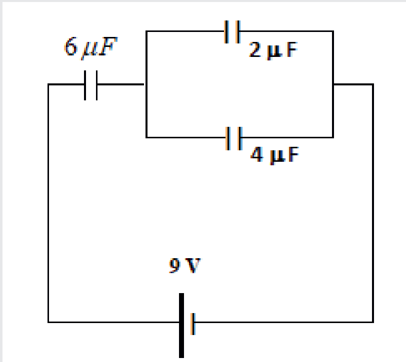
Mention the different applications of a capacitor.

Review Questions 6.5

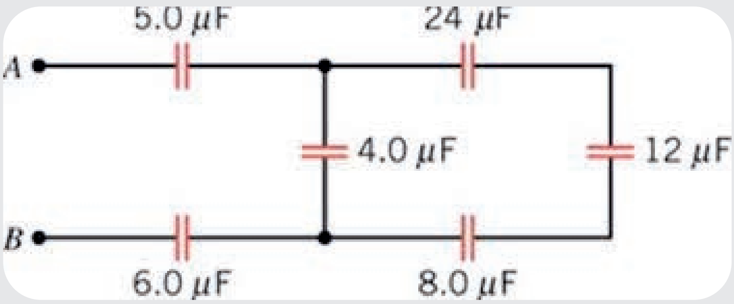
1. Write the definition of the following terms using your own words. capacitors, capacitances, dielectric.
2. Mention the factors that affect the capacitance of a capacitor.
3. Explain the effect of inserting dielectric in the gap between the plates of a capacitor on the following quantities. Its capacitance, (b) the amount of charge stored on the plates, (c) the potential difference between the plates, and (d) the electric field in the region between the plates.
4. A pair of parallel flat metal plates are placed a distance of 10 mm apart. The

plates are circular, with a radius of 10 cm. How much charge must be placed on each plate to produce an electric field of 500 V/m between them?

5. For the network of capacitors shown in the figure below, determine (a) the equivalent capacitance, (b) the charge stored on each capacitor, and (v) the voltage across each capacitor.



6. Calculate the equivalent capacitance between points A and B for the circuit shown in the figure below.



6.7 Electric Circuits in Our Surroundings

By the end of this section, you will be able to:

- *apply the knowledge about electric circuits to explain household electric installation.*

? Brainstorming

Make a list of electrical appliances in your home.
Group them into three, (a) electric power powered, (b) battery powered, and (c)

heat powered.

Household Electric Installations

In our country Ethiopia, the voltages of the mains are 220 V – 240 V. Heating appliances include electric kettle, immersion heater, electric irons, hair driers and electric stoves. All heating appliances contain a heating element that converts electrical energy into heat energy.

Activity 6.19:

Have you visited the new national Science Museum located Addis Ababa? If so, present a report about the smart house you visited. If not arrange a means to visit this fantastic museum.

Color Code of Resistors

Resistors and capacitors that we discussed so far are frequently used in many electronic devices and they are said to be the major circuit components for many electronic systems. The value of the resistance of resistors in electronic devices is labeled by a color code. There are colored rings in any resistor. According to the international standard organization ISO, the orders of the rings are arranged and each color is represented by a number.

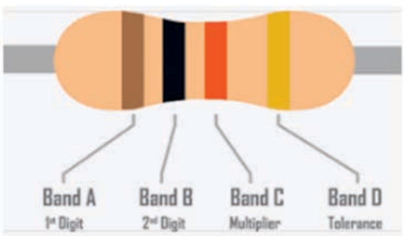


Figure 6.56: Color code of resistors

The first band represents the first digit number, the second band the second digit number and the third band represents multiplier to the base 10. The fourth band is the tolerance (the permissible percentage error ± of the value.

The trick in reading color code is noticing that the first three digits (numerical values) in a four band resistor are closer in equal gap, whereas the fourth band (the tolerance) on the other end is in a bit wider gap! In a five band resistor, the first four digits are closer in equal gap and the fifth band (the tolerance) is a bit further

For example in the illustrated resistor, the numerical value of the resistor can be calculated by referring the color code chart and replace the corresponding number for each color. The first three closer bands are: brown, black and red, which represent first digit number, second digit number and the multiplier respectively. Referring the color chart, the first band is brown =1 the second band is black =0 , the third band is red=2. Therefore, the numerical value of this resistor is

$$10 \text{ X } 10^2 = 1000 \text{ Ohms} = 1K\Omega.$$

The color of the fourth band is gold, the tolerance value of gold in the chart is ± 5%. This resistor has a value of $1K\Omega \pm 5\%$.

The following chart depicts the ISO standard for each color in the resistor.

Colour	1st Band	2nd Band	3rd Band	Multiplier	Tolerance
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	± 0.5% (D)
Blue	6	6	6	1MΩ	± 0.25% (C)
Violet	7	7	7	10MΩ	± 0.10% (B)
Gray	8	8	8		± 0.05%
White	9	9	9		
Gold				0.1Ω	± 5% (J)
Silver				0.01Ω	± 10% (K)

Activity 6.20:

Given the following resistor as the color of the first three bands in order are: green, blue and yellow, the last band (the tolerance) is silver. What is the numerical value of this resistor?

Activity 6.21: Project

Go to a nearby radio, television or computer repair house them. Ask them for a used electronic components like resistors or capacitors. Read their values by using the color code and present as a demonstration to your class

UNIT SUMMARY

- Electric charges are quantized and are integral multiples of a fundamental charge e.
- There are two types of charges: positive and negative. Unlike charges attract each other and like charges repel each other.
- The electrostatic force between two charges is directly proportional to the product of the two charges and inversely proportional to the square of the separation between them. This law is called Coulomb's law.
- The electrostatic force is positive if the force is repulsive and negative if the force is attractive.
- Electric field lines originate from the positive charge and terminate into the negative charge.
- Electric field lines do not cross to each other and are perpendicular to the surface of the charge source.
- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_s = R_1 + R_2 + R_3 + \ldots$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The total resistance of an electrical circuit with resistors connected in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

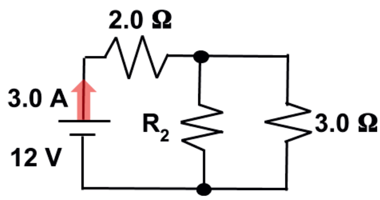
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is reached.
- Total capacitance in series $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$
- Total capacitance in parallel $C_{eq} = C_1 + C_2 + C_3 + \ldots$

- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.
- Capacitors are devices that store charge and electrostatic energy.
- A capacitor consists of two isolated conductors (the plates) with equal and opposite charges +q and −q. Its capacitance C is defined from , where V is the potential difference between the plates.
- The SI unit of capacitance is the farad. (1 farad =1coulomb per volt)
- Other practically applicable units of capacitance are: micro Farad, nano Farad and pico Farad.
- When two or more capacitors are connected in series, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitance of the capacitors.
- When two or more capacitors are connected in parallel, the equivalent capacitance is the sum of the capacitance of each capacitor.

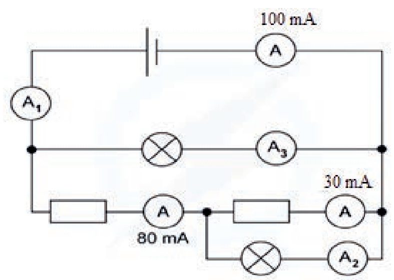
END OF UNIT QUESTIONS

1. If two charges separated by a distance d, have an electrostatic force F, What will be the magnitude of the new electrostatic force if their separation is doubled to 2d.
2. What is the source of an electric field in a certain region?
3. Which one of the following cannot be the unit of electric field
   a) Volt/meter
   b) newton/ coulomb
   c) newton.coulomb
4. When resistors are connected in series which property of the resistor is practically changed?
   a) The length
   b) the thickness
   c) the resistivity
5. When resistors are connected in parallel, which property of the material is changed?
   a) The length
   b) the thickness
   c) the resistivity
6. Two charges $Q_1 = -5\mu C$ and $Q_2 = +2\mu C$ are separated by a distance of 4cm. Calculate the electrostatic force between these charges. Is the electrostatic force an attractive or repulsive force? Why?

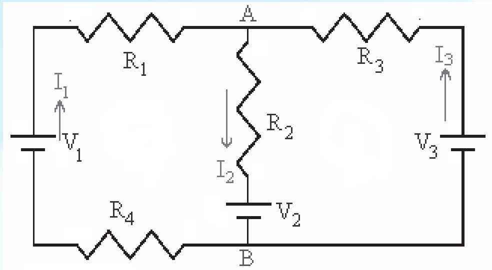
7. In the above question, what is the electric field produced due to Q_2 at a point where Q_1 is placed? What is the direction of the electric field? Similarly, determine the electric field produced by Q_1 at a point where Q_2 is placed. What is the direction of the electric field?
8. What are the largest and smallest resistances you can obtain by connecting a 36.0 Ω, a 50.0 Ω, and a 700 Ω resistor together?
9. What is the resistance of ten 75 Ω resistors (a) when connected in series? (b) when connected in parallel?
10. When do you get a brighter light, connecting bulbs in series or in parallel?
11. Why heating coils like an electric kettle is made of materials with large resistivity?
12. Why and how the resistivity of conductors depends on its temperature?
13. Why temperature dependence of the resistivity differs for conductors, semiconductors and superconductors?
14. What is the difference between emf and terminal voltage?
15. What is internal resistance?
16. How do you connect resistors so as to get larger/smaller equivalent resistance?
17. How do you connect resistors so as to get larger current from the circuit?
18. A battery of unknown emf ε and internal resistance r is connected in series with an ammeter and a resistance box. The current was 2.0 A when R = 4.0 Ω and 1.5 A when R = 6.0 Ω. Calculate ε and r.
19. A 12 V battery causes a current of 3.0 A to flow into the circuit shown in the figure below. Find (a) the value of the unknow resistor R2, and (b) the potentiial difference across the 3Ω resistor.



20. Determine the current that flows through the ammeters, $A_1, A_2,$ *and* A_3 in the circuit below.

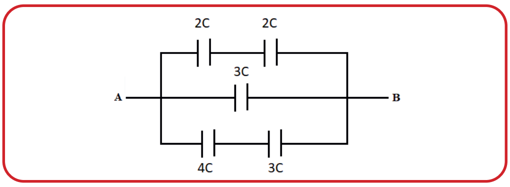


21. In the circuit shown below, If V1 = 1V, V2 = 2V, V3 = 3V, and let each resistor has a 1 Ω resistance. Determine the value of the currents $I_{1,} I_{2,}$ *and* I_3.



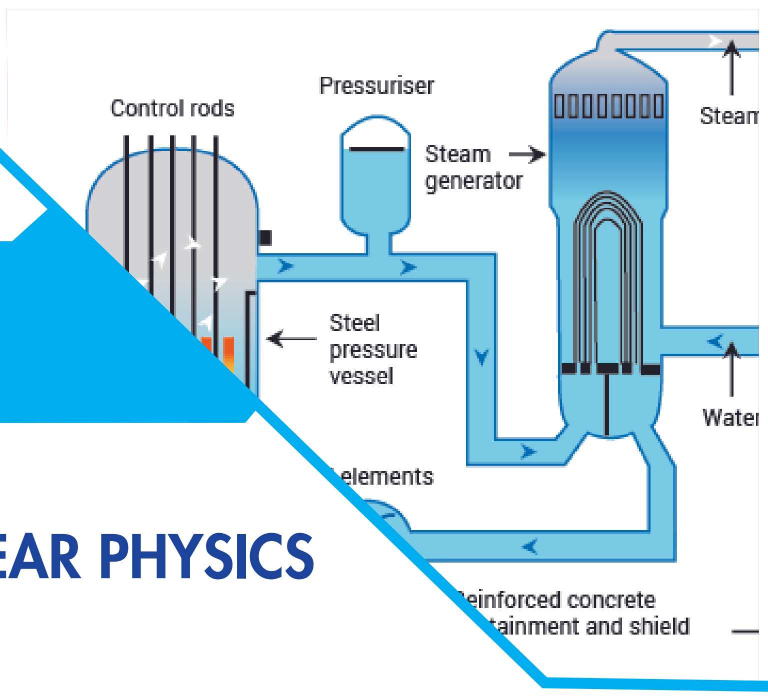
22. If you want to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

23. Find the total capacitance of capacitance between points A and B, for the network shown in the figure below?.



UNIT 7

NUCLEAR PHYSICS



Introduction

Nuclear physics is the field of physics that studies atomic nuclei and their constituents and interactions that hold them together. Nuclear interactions include radioactive decay, the break-up of a nucleus (fission), and the merging of nuclei (fusion).

The main driving strong nuclear force produced by the nucleus of an atom acts within a very short distance - just a few femtometres (10^{-15} metres) across. The strong nuclear force is the force that is responsible for the binding of protons and neutrons into atomic nuclei. The strong binding energy will help to produce strong radiation upon different reaction processes and radiation decay.

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A nuclide may be stable or unstable. Unstable nuclides emit radiations spontaneously. These radiations have different forms which are known as α, β and γ radiation with charges similar to proton, electron, and neutron in the atomic system.

Through research, nuclear physicists are leading us on a journey of discovery into the nucleus of the atom - the very heart of matter. The goal is a roadmap of matter that will help unlock the secrets of how the universe is put together. Significant efforts have been made from the early 19^{th} century on the observation of different subatomic systems by different scientists. Nowadays, the research reactor (RR) and the power reactor (PR) are widely in use globally. Research reactors are small nuclear reactors that are primarily used to produce neutrons, unlike nuclear power reactors, which are larger and used to generate electricity. The two reactors are based on the fission processes. However, the fusion is still in progress in laboratories and computer work. In all these processes, it is important to aware that the fuel elements for nuclear reactions are heavy unstable nuclei such as U, Th, K, Pt and so on.

Nuclear radiations have very powerful ionization effects; some of which is fatal to human beings if proper precautions are not in use. It is mandatory to use the nuclear safety rules of the International Atomic Energy Agency (IAEA) for the proper use of nuclear instruments and materials. There should be a separate inspection bodies for both the RR and PR once any country has these reactors in use.

The purpose of this unit is studying the different properties of nuclei so far you have seen above, uses and quantitative measurements of nuclear energy; and precautions to be taken to take care of ionization radiations.

At the end of the unit, students will be able to:

- *appreciate the basic concept of the atomic nucleus.*
- *describe qualitatively the basic nuclear processes (concepts of radioactivity, and nuclear reaction).*
- *discuss the types of nuclear forces.*
- *explain application of radioactivity and nuclear reaction to society.*
- *describe the application of radioactivity and nuclear reaction to explain the energy source of stars.*
- *define terms like nuclear fission, nuclear fusion, and isotope.*
- *gain the basic knowledge of nuclear medicine.*
- *demonstrate high level of awareness to protect themselves and their community from radiation hazard.*

7.1 The nucleus

At the end of this topic, you should be able to:

- *identify particles inside the nucleus*
- *discus the historical origins of the particles of the nucleus*
- *define and discuss isotope of an atom*
- *describe the relationships between nuclear mass and binding energy of isotopes;*
- *calculate binding energy of the nucleus of different isotopes;*
- *identify the nuclear forces, (strong nuclear force, weak nuclear force)*

? Brainstorming:

Discuss in small groups and tell to the whole class about the following.

1. Discus about the volume and the mass of the nucleus compared to that of the atom; about constituting particles of the nucleus and their charges.
2. Who discovered the nucleus? How?
3. What force keeps protons and neutrons at the center of the atom? What force keeps the electrons and the nucleus together?

About the Atom

Atoms are the basic building blocks of matter. An atom consists of a positively charged central nucleus that is surrounded by one or more negatively charged electrons. The nucleus contains one or more relatively heavy particles known as protons (positively charged) and neutrons (electrically neutral), which are collectively called nucleons. Table 7.1 summarizes charges and masses of the three subatomic particles.

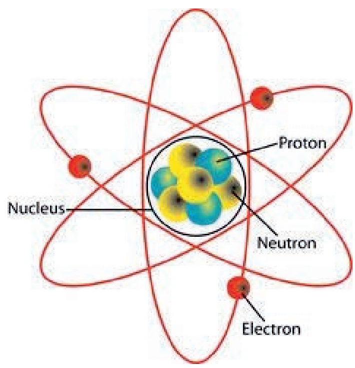


Figure 7-1: Structure of an atom- Every atom has these particles except the smallest isotope of hydrogen which has no any neutron.

Table 7-1: Mass and charge of subatomic particles

Subatomic particles	Charge (in C)	Mass (in kg)	Mass (in u)
Electron	-1.602×10^{-19}	9.1094×10^{-31}	0.00054858 u
Proton	$+1.602\times10^{-19}$	1.6726×10^{-27}	1.0072766 u
Neutron	0	1.6749×10^{-27}	1.0086654 u

Note: Where 1 amu (atomic mass unit) = 1.660×10^{-27} kg.

Since the mass of an electron is comparatively very small, only the nucleus contributes to the mass of an atom, which typically contains more than 99.9% of the *mass* of the atom. Thus, the sum of the number of protons and neutrons in the nucleus is expressed as mass number (A) and the number of protons is its atomic number (Z) for a neutral element. A neutral element in the periodic table of chemical symbol X is uniquely designated by:

$$^{A}_{Z}X$$

Some examples of designation of elements are oxygen $^{16}_{8}O$, helium $^{4}_{2}He$ and carbon $^{12}_{6}C$. Although the nucleus contains much of the mass of the atom, it is very dense and occupies only less than 0.01% of the volume of the atom. The atom and the nucleus are assumed spherical and the atomic radius is estimated by:

$$R = R_0A^{1/3} = \left(1.2\times10^{-15}\,m\right)A^{1/3}$$

where A is the mass number of the atom; R_0 is the radius of the nucleus which is roughly equal to $R_0 = 1.2fm$ and 1 femtometer (1 fm) = $10^{-15}m$.

Isotopes

An element may have atoms with different number of neutrons. Atoms with the same number of protons but different numbers of neutrons are called isotopes.

They share almost the same chemical properties, but differ in mass and therefore in physical properties.

Key terms:

- *Atomic nucleus consists of electrically positively charged particles called protons and electrically neutral particles called neutrons. These are held together by the strongest known fundamental force called the strong nuclear force.*
- *Isotopes are members of a family of an element that all have the same number of protons but different numbers of neutrons.*

Some examples of isotopes of elements:

Carbon isotopes: $^{12}_{6}C$, $^{13}_{6}C$ and $^{14}_{6}C$.

Hydrogen isotopes: $^{1}_{1}H$(protium), $^{2}_{1}H$(deuterium) and $^{3}_{1}H$(tritium).

Uranium isotopes: $^{232}_{90}U$, $^{233}_{90}U$, $^{234}_{90}U$, $^{235}_{90}U$, $^{236}_{90}U$ and $^{238}_{90}U$.

Example 7.1

- Calculate the radii of the smallest isotope of hydrogen $\left({}_1^1H\right)$ and uranium-238.
- Compare the volume of U-238 atom with the volume of its nucleus.

Solution:

The radius of ${}_1^1H$ is

$$R_{H-1} = \left(1.2\times10^{-15}\,m\right)A^{1/3} = \left(1.2\times10^{-15}\,m\right)(1)^{1/3}$$
$$R_{H-1} = 1.2\times10^{-15}\,m = 1.2fm$$

The radius of U-238 becomes

$$R_{U-238} = \left(1.2\times10^{-15}\,m\right)A^{1/3} = \left(1.2\times10^{-15}\,m\right)(238)^{1/3}$$

$$R_{U-238} = 7.437\times10^{-15}\,m = 7.437fm$$

The volume of U-238 atom is,

$$V_{atom} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(7.437fm\right)^3 = 12{,}819.01\ fm^3$$

The volume of the nucleus of U-238 atom,

$$V_{nuc} = \frac{4}{3}\pi R_0^3 = \frac{4}{3}\pi\left(1.2\,fm\right)^3 = 7.24\ fm^3$$

Taking the ratio of the first to the second one, we obtain

$$V_{nuc} = \left(\frac{7.24\ fm^3}{12819.01\,fm^3}\right)V_{atom} = 5.65\times10^{-4}V_{atom}$$

The volume of the nucleus is 0.0565% of the volume of the atom.

7.1.1 Historical origins of the nucleus and its constituting particles

Activity 7.1:

- Collect pictures and diagrams showing particles of the nucleus, important physicists who contributed to the discovery of the nucleus and its constituent particles; illustrations of experiments related to the nucleus.
- Discus in groups why electrons could not be part of the nucleus.

Discovery of the nucleus

In 1909, two of Lord Rutherford's students, Geiger and Marsden, aimed positively charged particles (stream of helium nuclei-$^{4}_{2}He^{+2}$) at an extremely thin piece of gold foil (100 nm thickness). In order to study the deflection caused by the α-particles, they placed a fluorescent zinc sulfide screen around the thin gold foil. They used the apparatus similar to that shown in Figure 7.2.

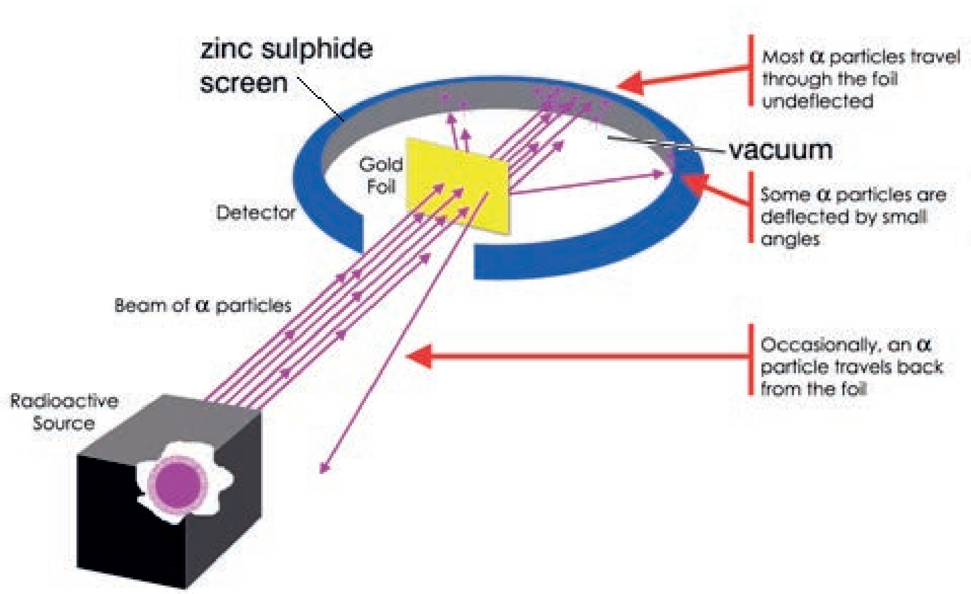


Figure 7-2: Rutherford's scattering experiment apparatus

Rutherford repeated the experiment many times, and come up with the same result. His evidences and conclusions are summarized as follows.

Evidences	Conclusions
Most of the -particles passed through the gold sheet with no or small deflection (0°-10°).	Most of the space in an atom is empty.
Some of the particles were deflected by very small angles (10° - 90°)	The positive charge in an atom is not uniformly distributed, but concentrated in a very small volume.
Very few of the -particles were deflected back towards the source (90°- 180°), only very few being at 180°.	Most of the mass of the atom, and all the positive charge, is concentrated in a tiny central space, what Rutherford called the nucleus.

Based on his observations, Rutherford proposed a new atomic model in 1911, called the solar system model as all the positive charge and most of the mass is concentrated in a tiny central nucleus. Most of the atom is empty space, and electrons orbit at the edge. He also claimed that the electrons surrounding the nucleus revolve around it with very high speed in circular paths. He named these circular paths as orbits (See Figure 7.3).

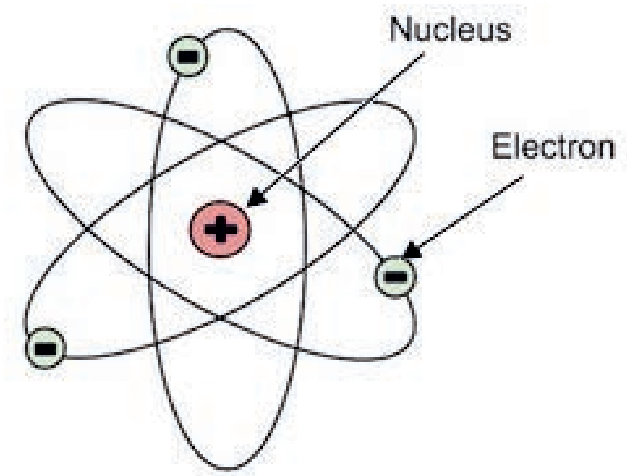


Figure 7-3: Rutherford's atomic model

What are the drawbacks of the Rutherford atomic model? Rutherford's atomic model failed to explain the stability of electrons in a circular path. He stated that electrons revolve around the nucleus in a circular path, but electrons in motion would undergo acceleration and cause energy radiation. Eventually, electrons should lose energy and fall into the nucleus.

Discovery of the proton

Rutherford thought that a hydrogen nucleus must be the fundamental building block of all nuclei, and also possibly a new fundamental particle. Rutherford postulated the hydrogen nucleus to be a new particle in 1920, which he called proton. Rutherford named it the proton, from the Greek word "protos," meaning "first."

Discovery of the neutron

The British physicist James Chadwick discovered the neutron in 1932. He conducted experiments very similar to those of Rutherford. In Chadwick's case, he fired his atomic bullets at the element beryllium, not gold. The resulting collisions produced a new kind of particle that was just as massive as the proton, but it penetrated through several inches of lead.

If you can take a particle and penetrate several inches of lead, it implies, first of all, that the particle is moving very fast; and second, that the particle is electrically neutral, it has neither a positive nor a negative charge. So Chadwick found that the neutron is indeed electrically neutral, and he also found that its mass is slightly greater than that of the proton. Finally, he determined the mass of the neutron using the law of conservation of energy and momentum as well.

7.1.2 What keeps the nucleus together?

You know that the positively charged protons and electrically neutral neutrons are packed together in the nucleus. Then, what is holding the cluster of nucleons together instead of repelling each other due to the electrostatic force and fly apart? This question takes you to consider the two fundamental forces of nature: the strong and weak nuclear forces.

The strong and the weak nuclear forces

The strong nuclear force is a very short-range attractive force that acts between nucleons: protons and neutrons. The strong nuclear force is strong enough to withstand the electrical repulsion up within a distance of slightly more than the radius of a nucleon, 10^{-15} m. Both nucleons are affected by the nuclear force almost identically.

Look at the difference!

At the range of 10^{-15} m (slightly more than the radius of a nucleon), the strong force is approximately 100 times as strong as electromagnetism, 10^6 times as strong as the weak interaction, and 10^{38} times as strong as gravitation.

The weak nuclear force acts inside of individual nucleons, which means that it is even shorter ranged than the strong nuclear force. The weak nuclear force can split the electrically neutral neutron into a proton and an electron. In this process, sub-atomic particles are released near the speed of light. These fast particles allow nuclear fusion reaction- combination of two or more nuclides to form a single nuclide- by then releasing enormous energy.

Key terms:

- *The Strong Nuclear Force is an attractive force between protons and neutrons that keeps the nucleus together.*
- *The Weak Nuclear Force is responsible for the radioactive decay of certain nuclei.*
- *Nuclear binding energy is the energy required to separate an atomic nucleus completely into its constituent protons and neutrons, or, equivalently, the energy that would be liberated by combining individual protons and neutrons into a single nucleus.*

Nuclear binding energy

Nuclear binding energy is the energy that holds nucleons together. Another equivalent definition of binding energy is the minimum energy that is required to disassemble the nucleus of an atom into its constituent nucleons. Now the question is how can we determine the binding energy of a nuclide?

The mass of an atomic nucleus is less than the sum of the individual masses of the free constituent protons and neutrons. This difference in mass (Δ_m) is known as the mass defect. Once the mass defect is known, the nuclear binding energy (BE) can be calculated using Einstein's special theory of relativity, which states that mass and energy are equivalent. The remarkable equivalence between matter and energy is given in one of the most famous equations given by $E = mc^2$; where m is the mass converted in to an amount of energy E and $c = 3\times10^8$ m/s is the speed of light. From this theory, we can deduce that the mass defect is an amount of mass released in the form of energy during the formation of the nucleus. This energy is equal to the binding energy, which is then given by

$$BE = \Delta mc^2$$

Masses of subatomic particles are measured in "atomic mass unit (amu or u)";

$$1 \text{ amu (or 1 u)} = 1.660 \times 10^{-27} \text{ kg}$$

Nuclear energy is measured in "atomic energy unit (aeu)"; or "megaelectron volt (MeV)";

$$1\text{aeu} = c^2 \times 1\text{ amu} = 1.44\times10^{-10}\text{J}$$

$$1\text{MeV} = 1.602\times10^{-13}\text{ J, and } 1\text{aeu} = 931.1\text{MeV}$$

The mass defect of an atom can be calculated by:

$$\text{Äm} = Zm_p + (A - Z)m_n - M$$

where Zm_p is the total mass of the protons; $(A - Z)m_n$ is the total mass of the neutrons, and M is the mass of the nucleus.

Using this into the former equation, we obtain:

$$BE = \Delta mc^2 = [Zm_p + (A - Z)m_n - M]c^2$$

Therefore, when mass is given in amu, the nuclear binding energy of a nucleus in MeV becomes,

$$BE = [Zm_p + (A\text{-}Z)m_n - M]\times 931.1\text{MeV}$$

Masses of subatomic particles

$$m_p = 1.0072766u,\quad m_n = 1.0086654\ u, \text{ and } m_e = \frac{m_p}{1836} = 0.00054858\ u$$

Binding energy per nucleon: Binding energy per nucleon (BEN) is the average energy required to remove an individual nucleon from a nucleus. It is one of the most important experimental quantities, which is defined by

$$BEN = \frac{BE}{A}$$

where BE is the binding energy and A is the atomic number of the atom.

Example 7.2

You know that an atom of oxygen has the form ${}^{16}_{8}O$.

a) What is the mass defect in oxygen nucleus?

b) What is the binding energy per nucleon in Oxygen?

Solution

$Z = 8$, $A = 16$, $m_p = 1.00728\ u$, $m_n = 1.00867\ u$, $m_{nuc} = 16u$

a) The mass defect is

$$\Delta m = Zm_p + (A - Z)m_n - m_{nuc}$$

$$\Delta m = 8 \times 1.00728\ u + (16 - 8) \times 1.00867\ u - 16u$$

$$\Delta m = 0.1276\ u$$

b) The mass defect in amu is 0.1276. The binding energy in Oxygen is, then:

$$BE = [Zm_p + (A-Z)m_n - M] \times 931.1MeV = \Delta m \times 931.1MeV$$

$$BE = 0.1276 \times 931.1MeV = 118.808MeV$$

The binding energy per nucleon in Oxygen:

$$BEN = \frac{BE}{A} = \frac{118.808\ MeV}{16} \approx 7.4\ MeV$$

Activity 7.2

- Select few isotopes of some elements in the periodic table.
- Let individual students of your group do the calculations for the binding energy of different isotopes and the rest members of the group come up with a set of biding energy.
- Contribute your result in a whole class discussion about the least and highest binding energies.
- Give explanation about how the binding energy varies from isotope to isotope and from element to element

Nuclear stability

Nuclear stability is determined by the binding energy per nucleon. The net binding energy of a nucleus is that of nuclear attraction, minus the disruptive energy of the electrostatic force.

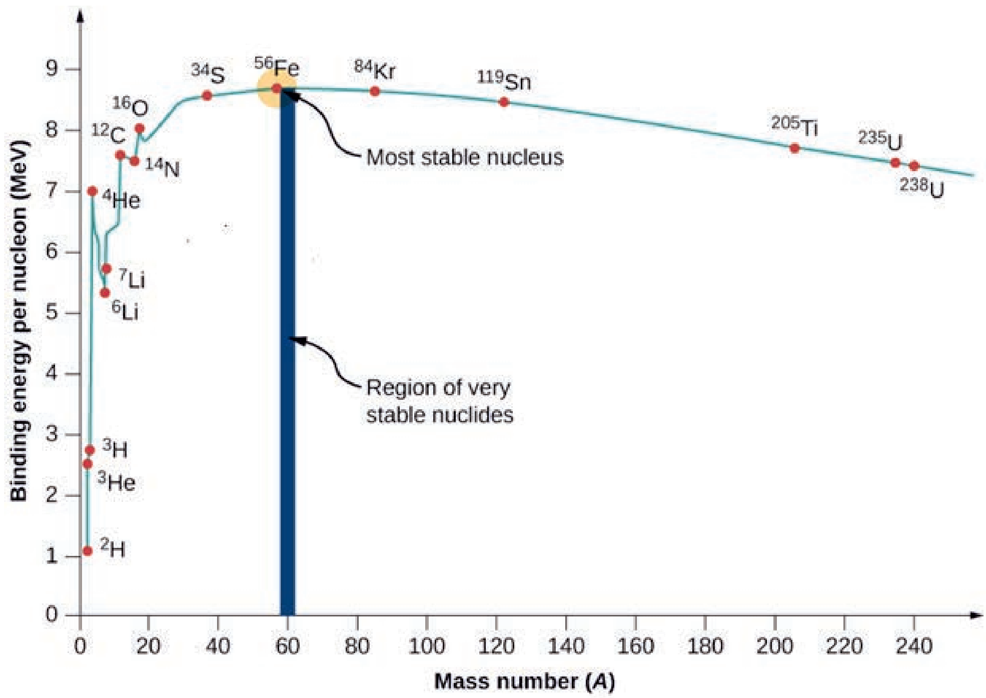


Figure 7.4: A plot of the binding energy per nucleon as a function of mass number A.

Any system will always try and move to a state of lower energy (or more stable state). As nuclei get heavier than helium, their net binding energy per nucleon grows more and more slowly and reaches its peak at iron (Fe- 56), which is 8.8 MeV/nucleon, , as shown in Figure 7.4. As nucleons are added, the total binding energy increases but so does the total disruptive energy of the electrostatic forces and, once nuclei are heavier than that of iron, the increase in disruptive energy has more effect than the increase in binding energy. For example, the binding energy per nucleon of uranium, which is at the end of the periodic Table, decreases to about 7.6 MeV.

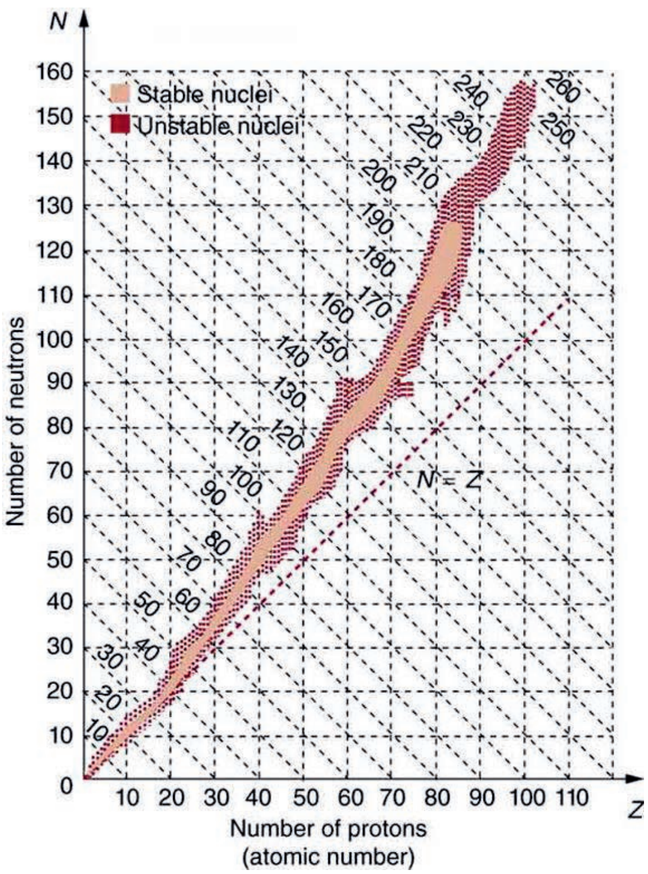


Figure 7.5: Graph of neutron number vs atomic number

Review Questions 7.1

1. Two nuclei have mass numbers in the ratio 27:125. What is the ratio of their nuclear radii?
2. Consider the atoms ^{16}O, ^{18}O, ^{18}F, ^{18}Ne and ^{20}Ne.
   a. Which atoms are isotopes of the same element?
   b. Which ones have the same number of neutrons?
3. X and Y are two elements with the same mass number but different atomic numbers 28 and 30, respectively. If X has a mass number of 58, what is the number of neutrons in Y? Are X and Y isotopes of the same element?
4. In his experiment of gold foil, when Rutherford bombarded the alpha particles on a very thin sheet of gold foil, only very few of the projectiles were observed deflecting back.
   a. Which force was responsible for the deflection?
   b. What does it imply that most of the alpha particles were passed undeflected?
5. What were failures in Rutherford's planetary model of the atom?
6. Describe how neutrons were discovered.
7. What are differences between strong nuclear force and weak nuclear force?
8. What are the major differences between the electrostatic and nuclear forces?
9. Calculate the binding energies per nucleon of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV. Use the following data: m(Fe-56) = 55.934939u, m(Bi-209) = 208.980388u, mp= m(H-1)=1.007900 u, m (neutron)=1.008665 u.
10. Explain factors affecting nuclear stability of isotopes.

7.2 Radioactivity

By the end of this section you should be to:

- *define the terms radio-activity, and distinguish between the three types of emissions, alpha radiation, beta- radiation and gamma- radiation*
- *recognize and discuss dangers of ionizing radiation;*
- *discuss radioactive dating and radiation detectors;*
- *identify radioactive sources in the school, home, and workplace and recommend protective measures to the school community.*

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) accidentally found that uranium-rich mineral called pitchblende emits invisible, penetrating radiations that can darken a photographic plate enclosed in an opaque envelope. Later it was found that there are some unstable isotopes that are emitting particles to be stable.

Larger nuclides tend to have more neutrons than protons to reduce the disruptive energy and stay stable. To reduce the disruptive energy by increasing number of neutrons and reducing number of protons, the weak nuclear force converts proton to neutron; or at some stage the only way for the nucleus to reach a lower energy state will be to emit particles. All these processes that lead unstable atoms to decay by particle emission are what we call radioactivity. unstable isotope is said to be a radioisotope, and the energy that is released is called radiation. After an atom expels energy from the nucleus, the composition of the nucleus changes, and we are left with a different element that is more stable.

Do you know...?

Pierre and Marie were awarded the Noble Prize in Physics in 1903 for their work on radioactivity. Marie Curie became the first woman to be awarded the Nobel Prize and the first person to obtain two Nobel prizes when she won the prize for the discovery of Polonium and Radium in 1911. She later discovered two new radioactive elements: Radium and Polonium which took her several years since these elements are difficult to extract and extremely rare. Unfortunately, the Merry Curie died young. She died of aplastic anemia, almost certainly a result of radiation exposure.

Key term:

Radioactivity is the phenomenon of the spontaneous disintegration of unstable atomic nuclei to atomic nuclei to form more energetically stable atomic nuclei. Radioactive decay is a highly exoergic, statistically random, process that occurs with a small amount of mass being converted to energy.

Types of nuclear radiation

There are three types of nuclear radiations: alpha (α), beta (β) and gamma (γ) radiation. Each of these comes about through a different process in the decaying nucleus, each one is composed of different particles and each one has different properties.

Alpha Particles Radiation

Alpha particles are subatomic fragments consisting of two neutrons and two protons, or helium nuclei (He-4). Alpha radiation occurs when the nucleus of an atom becomes unstable (the ratio of neutrons to protons is too low) and alpha particles are emitted to restore balance. Alpha decay occurs in elements with high atomic numbers, such as uranium, radium, and thorium. The nuclei of these elements are rich in neutrons, which makes alpha particle emission possible.

When a radioactive atom decays by α-emission, it leaves a daughter nucleus of atomic number two less than the parent atom of atomic mass number four less than that of the parent atom. Thus,

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

where X and Y are chemical symbols of the parent and the daughter nuclei, respectively.

For example, the abundant isotope of uranium, U-238, decays by alpha emission to give a thorium atom which can be written in a form:

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He $$

Some sources of α radiation are americium-241, plutonium-236, uranium-238, thorium-232, radium-226, radon-222 and polonium-210.

Beta-particle Radiation

Unlike alpha radioactivity, beta radioactivity requires the weak nuclear force. There are two beta decay types; beta minus (-) and beta plus (+). Beta minus particles are energetic electrons emitted from a radioactive nucleus. Beta minus particle emission occurs when the ratio of neutrons to protons in the nucleus is too high. An excess neutron transforms into a proton and an electron. The proton stays in the nucleus and the electron is ejected energetically. This process decreases the number of neutrons by one and increases the number of protons by one. Thus,

$$ {}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + {}^{0}_{-1}\beta $$

As we see in this reaction equation, any decay by beta-emission accompanies emission of an antineutrino, $\overline{v}$. Take a look at the decay of the uranium daughter product thorium-234 into protactinium-234:

$$ {}^{A}_{Z}Th \rightarrow + {}^{234}_{91}Pa + {}^{0}_{-1}\beta $$

Some beta negative emitters are tritium, cobalt-60, strontium-90, technetium-99, iodine-129, iodine-131, and cesium-137. There is also positron $\left({}^{0}_{1}e^{+}\right)$ or beta plus emission. Like the beta particle, a positron is immediately rejected from the nucleus upon its formation. For example, potassium-38 decays by positron emission, becoming argon-38. Positron emission decreases the atomic number by one, but the mass number remains the same. Thus,

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta$$

Gamma-particle Radiation

Gamma rays are high energy, high frequency, electromagnetic radiations. Gamma radiation usually accompanies alpha or beta decay. They have no charge and no mass so they rarely interact with particles in their path, so they have the least ionizing power of the three radiations. They are never completely absorbed, although their energy can be significantly reduced by several centimeters of lead, or several meters of concrete. If the energy is reduced to a safe level, gamma rays are often said to have been absorbed.

Gamma emitting radioisotopes are the most widely used radiation sources. The three radionuclide's by far useful are: cobalt-60, cesium-137, and technetium-99 m.

Activity 7.3

Discharge flame and radioactivity:

- Hold a candle flame near a charged electroscope and it discharges leaves of the electroscope. The reason is that the Flame produces ionization of nearby air.
- Repeat demonstration with opposite charge on electroscope. Holding weak radioactive source near electroscope may also cause discharge.

Ionization and penetration powers of nuclear radiations

When alpha, beta and gamma radiations, they produce ions and molecular fragments by knocking electrons from them. The greater mass presents the greater the ionizing power. Thus, alpha-radiation has the highest and gamma radiation has the least ionization powers.

On the other hand, gamma-radiation has the highest and alpha- radiation has the least penetration powers. Alpha radiation can be stopped by paper and skin, beta radiation by aluminum sheet and gamma radiation can be blocked by 2 inches thick lead. Figure 7.6, compares penetrating powers of these radiations.

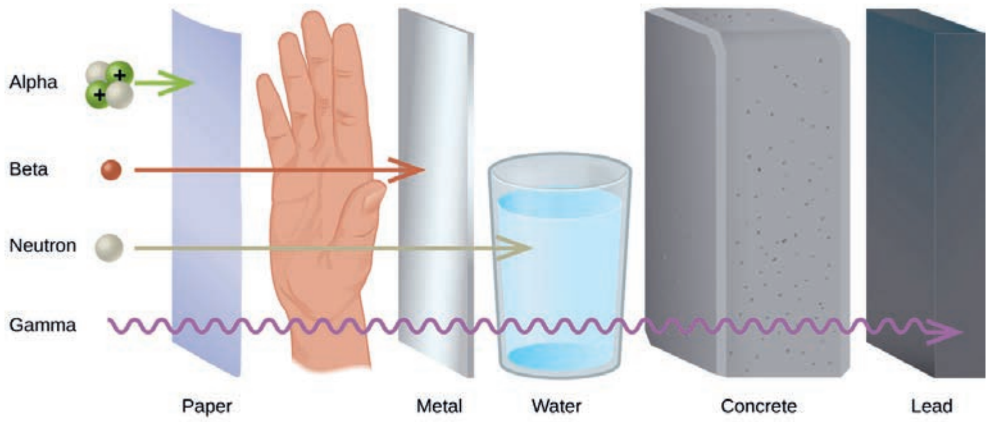


Figure 7.6: Penetration of the alpha, beta, and gamma radiations. Neutron is added to show impact of charge on penetration power.

Dangers of ionizing radiation

When radiation passes through cellular tissue, it ionizes water molecules which change into free radicals. These radicals are highly reactive and can interact with the important genetic material in the cell, the DNA. In addition, the DNA may also be ionized directly. Damage caused by these interactions may be fully repaired, in which case, the cell remains viable. However, if the damage is not successfully repaired and the DNA is not restored completely, the cell may either die or mutate.

Because the radiations ionize to different extents, the hazard level is also different for each one. The extent of the potential damage depends on several factors, including:

- the type of radiation
- the sensitivity of the affected tissues and organs
- the manner and length of time exposed
- the radioactive isotopes involved
- Characteristics of the exposed person (such as age, gender and underlying condition).

The depth of radiation damage is summarized below.

Radiation type	Effect inside body	Effect outside body
Alpha	Highly ionizing – very dangerous radiation poisoning and cancer possible.	Absorbed by surface layer of dead skin cells – no danger
Beta	Moderate ionization and danger should be minimized	Moderate so exposure ionization and danger, close exposure should be minimized
Gamma	Minimal ionization, cancer danger from long-term exposure	minimal ionization –cancer danger from long-term exposure

Effective Dose

The risk of developing adverse health effects depends on the radiation dose. The higher the dose the higher is the risk of adverse effects. Absorbed dose describes the amount of energy deposited per unit mass in an object or person. The units for absorbed dose are gray (Gy, sievert (Sv) and rad; where 1Gy = 1J/kg and 1rad = 0.01 Gy = 0.01 J/kg.

- 1 Gy is the deposit of a joule of radiation energy per kilogram of matter or tissue.
- 1 Sv = 1 joule/kilogram – a biological effect. The sievert represents the equivalent biological effect of the deposit of a joule of radiation energy in a kilogram of human tissue.

Radiation doses above 3 Gy (300 rad) can be fatal and doses above 6 Gy (600 rad) are almost certain to be fatal, with death occurring within several months (in shorter times at higher doses). For gamma rays and electrons, above 1 Gy, radiation causes a complex of symptoms, including nausea and blood changes, known as radiation sickness. For doses below 1 Sv (100 rem), there is little likelihood of radiation sickness, and the main danger is an increased cancer risk.

Safety precautions when using radioactive sources

Radioactive sources which may be used in schools are usually very weak. They can only be used in the presence of an authorized teacher. They are kept in a sealed container except when they are being used in an experiment or demonstration. They are immediately returned to the container when the experiment or demonstration is finished.

When using the radioactive source it should be:

1. handled with tongs or forceps, never with bare hands.
2. kept at arm's length, pointing away from the body.
3. always kept as far as possible from the eyes.

Hands must be washed after the experiment and definitely before eating.

Radiation Detectors

A radiation detector is a device that measures the ionization of radiations (i.e., creating electrons and positively charged ions), such as beta radiation, gamma radiation, and alpha radiation with the matter. There are different types of radiation detectors, most widely ones of which are discussed below.

Scintillators: A scintillator is a general term for substances that emit fluorescence when exposed to radiations of high energy- it is a type of phosphor. When a radiation collides with this substance, it absorbs its energy and internal electrons move from the ground state (stable state) to the excited state. When this electron returns to the original stable state, it releases its energy in the form of light emission (visible light or ultraviolet light), and this phenomenon called scintillation.

The incident radiation can be measured quantitatively by photo-electrically converting/ amplifying the emitted fluorescence with a photomultiplier tube (PMT) or the like Figure 7.7 shows a type of a scintillation detector. Scintillation detectors represent the best means for detecting gamma or x-radiation and are the second-most common detector type after G-M tubes. They have the ability to distinguish between alpha, beta, and gamma radiation, and can be configured to produce correspondingly different sounds through a meter.

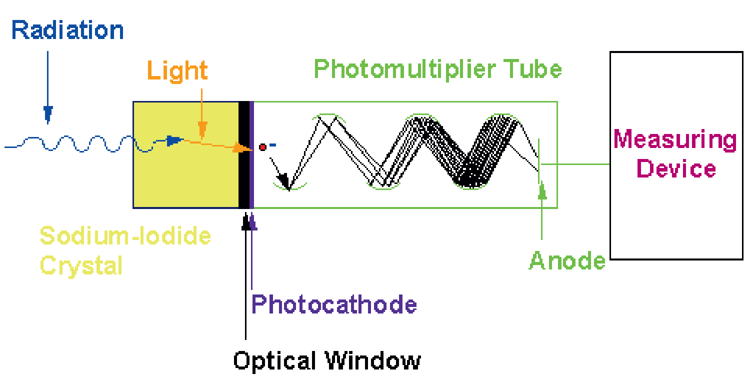


Figure 7.7: Scintilator

Geiger counter: A Geiger counter, also known as the Geiger-Muller tube, is used to quickly detect and measure radiation. It exploits the natural process of ionization to detect and measure radiation. When exposed to radioactive radiations, the stable gas within the chamber ionizes. This generates an electrical current that the counter records over a period of 60 seconds.

When ionization occurs and the current is produced, a speaker clicks and a reading is given — often in millisieverts (mSv). The central wire in between a gas-filled tube at high voltage is used to collect the ionization produced by incident radiation. Geiger counters can detect alpha, beta, and gamma radiation. However, they cannot differentiate which one is beta, or gamma or alpha radiation.

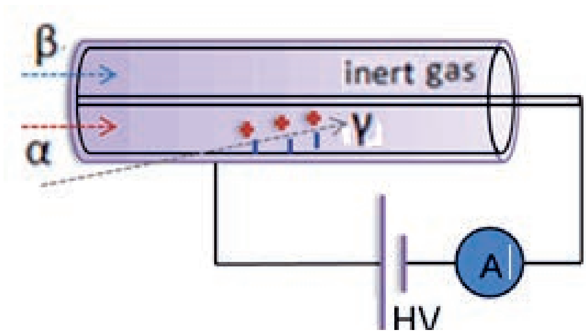
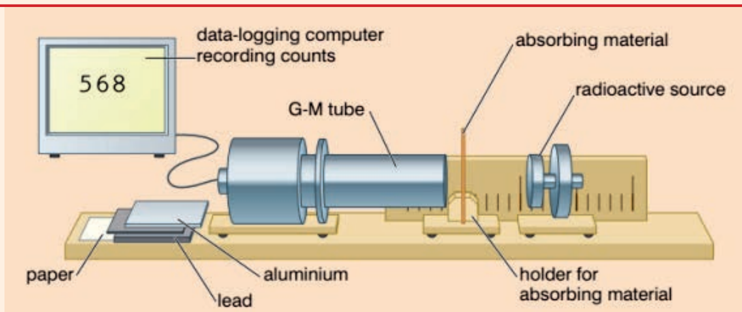


Figure 7.9: Geiger counter

Activity 7.4: Demonstrations

Work in a small group. Before you begin, make sure that you know the safety precautions that you must follow when using radioactive materials. Set up the equipment as shown in the figure below in cooperation with your teacher.



The Geiger-Muller tube can detect all the three types of radiations. Place absorber sheets which progressively increase in density between the source and the detector, and record the average count rate in each case. You can also carry out the experiment using computer software to remove all risk of exposure to radiation hazards.

Note that:

- Some counters can separate betas from gammas by absorbing the betas in a metallic cover.
- Few have windows thin enough to detect alphas.
- If you can detect betas and gammas, use aluminum sheets and lead sheets to show that betas are stopped by aluminum but lead is required to stop gammas.

If there is no any detector in your school, it might be possible to contact a local hospital regarding the use of nuclear medicine for analysis or treatment. If a tour cannot be arranged, perhaps your teacher may invite a specialist and He/she can speak to the class.

Use either a prism or diffraction grating spectrometer to show the difference between continuous or line spectra. A simple absorption spectrum can be shown by crushing leaves in alcohol. The chlorophyll solution will absorb at both ends of the spectrum. β rays spectrum is continuous among α and γ rays spectrum.

Activity 7.5

Research the common detectors for α-particle, β-particle and γ-rays

In a small group: Research the various forms of detectors for nuclear radiation. Prepare a summary of your research to present to the rest of your class. Choose an appropriate format for your presentation.

The half life

Here, we consider a system containing many nuclei of the same species at some initial time. The number of any radioactive parent nuclei decreases with time since it emits radiation in the form of α, or β emissions. The decay of a particular nucleus cannot be predicted and is not affected by physical influences like temperature. The rate of isotope decay depends on two factors.

- The total number of undecayed nuclei present in the system. That is, on doubling the average number of undecayed nuclei must double the rate of decay.
- The stability of the isotope. Some isotopes decay more rapidly than

others. The rate of decay gives the number of nuclei that decay per second.

In general, the decay rate, called the activity, A, is given by

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

where

- The negative sign shows the decrease in the number of the radioactive nuclei with time,
- N is the number of undecayed nuclei at the subsequent time t,
- The decay constant λ of a radioactive nuclide is defined as its probability of decay per unit time; having SI unit s^{-1}. It is a positive rate also called the exponential decay constant, disintegration constant, rate constant, or transformation constant.

The SI unit of activity, A, is Becquerel (Bq); where 1Bq = 1 decay per second. We can also use the unit Curie, Ci, where 1 Ci = $3.7 \times 10^{10}\,Bq$.

The quantity of the parent radioactive nuclei is subject to exponential decay since it decreases at a rate proportional to its current value. This exponential decay law is given by

$$N = N_0 e^{-\lambda t}$$

where N_0 is the number of undecayed nuclei at an initial time t0 = 0.

In practice, the stability of radioactive nuclei against decay and the decay rate are most often estimated in terms of the half-life, $t_{1/2}$, rather than the decay constant λ. The half-life is defined as the time at which half of the original nuclei have decayed. Or, it can also be stated somewhat differently as the time after which one half of the original number of nuclei remains untransformed. Since nuclear decay is an example of a purely statistical process, a more precise definition of half-life is that each nucleus has a 50% chance of living for a time equal to one half-life, $t_{1/2}$. The half-life of a certain decay can be determined by using $N = N\left(t_{1/2}\right) = N_0/2$, in to exponential decay law as:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Scientists predicted that the half-lives of naturally occurring nuclear reactions fall in a very wide range - vary from fractions of milliseconds like Radium C (), to billions of years like U-235 (4.5 billion years). For some induced radioactive elements (initially stable made radioactive by exposure to specific radiation), the half life is a few millionths or even hundred millionths of a second.

Activity 7.6:

Perform experiments on Half life simulation using free applets available from the internet.

Key term:

The half life of a radioactive isotope is the amount of time it takes for one half of the radioactive isotope to decay.

Why do we use a term like half-life rather than lifetime?

The answer can be found by examining Figure 7.10, which shows how the number of radioactive nuclei in a sample of initial value N_0, decreases with time. The number of radionuclide's left at $t_{1/2}$ is $N_0/2$. Half of the remaining nuclei decay in the next half-life $(2t_{1/2})$, is $N_0/4$. Then, it becomes $N_0/8$ in the third half-life and so on. For n integral numbers of half-lives, the number of original nuclides left, N, can be calculated by

$$N = 2^{-n} N_0$$

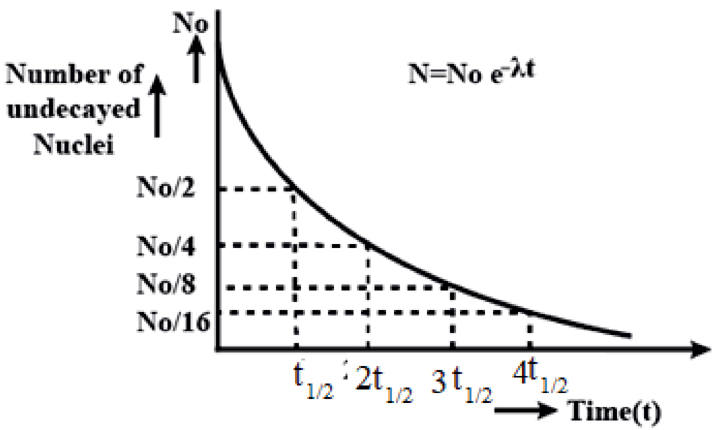


Figure 7.10: Number of undecayed nuclei vs time graph of a radioactive isotope

Examples 7.3

1) The half-life of radium is equal to 1590 years. What is the activity of 1 g of radium-226? Solution: $t_{1/2} = 1590 \text{ yrs} = 50176584000 \text{ s}$

The gram molecular weight of Ra-226 is M = 226.0254098 g/mol. There are $N_A = 6.023 \times 10^{23}$ atoms/mol of radium. Therefore, the number of atoms of 1g of Ra-226 would be:

$$N = \frac{N_a \times 1g}{M} = \frac{1g \times 6.023\times10^{23}\,\text{atoms / mol}}{226.0254098\ \text{g / mol}} = 2.664\times10^{21}\,\text{atoms}$$

The decay constant is,

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{50176584000\text{s}} = 1.381\times10^{-11}\text{s}^{-1}\ \lambda = 0.693/t1/2=13.81\ \text{pBq}$$

The desired activity is

$$A = \lambda N = 1.381\times10^{-11}Bq\times2.664\times10^{21} = 3.6793\times10^{10}Bq$$

$$A = \lambda N = \left(\frac{0.693}{t_{1/2}}\right)N = \left(\frac{0.693}{150176584000s}\right)\times\left(2.67\times10^{21}\right)$$

$A = 1.23 \times 10^{10}$ Bq

That is, the number of decays per second in one gram of radium is 12300 million.

2) The radioisotope strontium-90 has a half-life of 38.1 years. If a sample contains 400 mg of Sr-90, how many milligrams will remain when the age of the sample is 190.5 years?

Solution

$t_{½} = 38.1$ years, t = 190.5 years

From the equation $t_{1/2} = 0.693/\lambda$, we obtain

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{38.1\text{yr}} = 0.018189\ \text{yr}^{-1}$$

Then the mass remained after 190.5 years becomes

$$m = m_0e^{-\lambda t} = 400\ \text{mg}\times e^{-0.018189\,\text{yrs}^{-1}\times190.5\,\text{yrs}} = 12.509\text{mg}$$

Review Questions 7.2

1. In the radioactive decay of radon-222, both alpha and gamma radiation are emitted, with the latter having energy of 8.2×10^{-14} J per nucleus decayed:

$$^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{0}_{0}\gamma + {}^{4}_{2}\text{He}$$

2. How much energy does 1mol of Rn release in the form of gamma radiation?

3. What can be the reason that tritium (H-3) isotope is a radioisotope where there is no electrostatic repulsive force in its nucleus? How does this isotope decay?
4. Why does Uranium 235 decay via alpha decay (by the way of Thorium-231) into 231 Pa?
5. You know that excessive exposure to radiation may damage living tissues and organs, depending on the amount of radiation received (i.e. the dose). Write some factors on which the extent of the potential damage depends on?
6. It is known that radioactive decay is a statistical process. Is it possible to determine the constancy of $t_{1/2}$, or λ for a given radioactive element with one of few atomic nuclei?
7. Determine the reaction equations of the decays of U-238 by alpha emission and Th-234 by beta minus emission?
8. Does the probability per unit time that a radioactive nucleus will decay, increase, decrease or remain the same independent of time?
9. Compare Geiger counter with Scintillator.
10. Given alcohol, argon gas, oxygen and hydrogen gas, which one of these acts as ionizing gas in Geiger Muller counter?
11. What is the activity of a sample of 100 million atoms of carbon-14? The decay constant, λ, is $3.84 \times 10^{-12}\ s^{-1}$.
12. Here is a table showing the activity of a sample of technetium-99, a gamma emitter which is often used in medical investigations. Plot the activity vs time graph to show this data and work out the half life

Activity (Bq)	Time (hours)
4000	0
2000	6
1000	12
500	18
250	24
125	30
75	36

13. How do the electric charges of alpha, beta and gamma rays differ?
14. How does the source differ for a beam of gamma rays and a beam of X rays?
15. When beta emission occurs, what change takes place in an atomic nucleus?
16. Among the three radiations, alpha, beta and gamma radiations, which radiation is the most fatal to the human beings?
17. Why does neutron radiation attain the highest penetration depth compared to alpha and beta radiations?

7.3 Use of nuclear radiation

By the end of this section you should be able to:

- *investigate the benefits and hazards (nuclear waste) of nuclear energy production;*
- *discuss and appreciate the application of nuclear medicine*

? Brainstorming questions

1. Have you ever heard that goiter can be treated using radiations?
2. Have you ever heard about how ages of fossils such as Lucy/Dinknesh are determined? Outside of some harmful applications such as nuclear weaponry, there remains a wide array/range of ways in which radioactive material and the radiation it gives off remain useful in the daily lives of people all over the world. The nuclear technologies are in use to improve agriculture, industries, medicine, water and sanitation and power generation. Among these, only two uses, namely, medical application and radioactive dating are discussed below.

Medical applications of nuclear radiation

Nuclear medicine uses radioactive material inside the body to see how organs or tissue are functioning (for diagnosis) or to destroy damaged or diseased organs or tissue (for treatment).

For diagnosis, nuclear medicine can show how the organs or tissues are functioning. For most diagnostic procedures, a tracer, which contains the radioactive material, is injected, swallowed, or inhaled. Then, a radiation detector is used to see how much of the tracer is absorbed or how it reacts in the organ or tissue. The nuclear diagnostics are more often used in imaging .

Some radioactive nuclides in use for diagnosis include fluorine-18, gallium-67, krypton-81m, rubidium-82, nitrogen-13, technetium-99m, indium-111, iodine-123, xenon-133, and thallium-201.

To diagnose cancer, positron emitting nucleotides (PET) are used to scan for a short period. In this type of nuclear medicine, the tracer is used to show the natural activity of cells, providing more detailed information on how organs are working and if there is damage to the cells. Some common uses of PET scans include diagnosing heart disease, Alzheimer's disease, and brain disorders. PET scans are often combined with computed tomography (CT) scans or magnetic resonance imaging (MRI) which provides three-dimensional images of the organ.

Nuclear treatment in Ethiopia

Based on the basics of medical application, in Ethiopia, there are Nuclear Technology activities. Previously, the Black Lion Specialized Hospital (a teaching hospital under Addis Ababa University) uses radioisotope for the treatment of cancer.

For treatment (Nuclear radiation therapy):- Radiation therapy is a type of cancer treatment that uses high energy beams to destroy cancer cells and shrink tumors. Cancer cells grow and divide faster than most normal cells. The radiation works by making small breaks in the DNA inside cells. These breaks keep cancer cells from growing and dividing and cause them to die. The radiation may injure noncancerous cells, but most are able to recover. There are two broad types of radiation therapy that doctors use to treat cancers: internal and external.

- External beam radiation, or teletherapy: In this type of treatment, the energy beams come from a machine outside of the body. The beam is precisely aimed and it penetrates the body to reach the cancer site. For example, such treatment can be carried out using a gamma beam from a radioactive cobalt-60 source.
- Internal radiation therapy: Brachytherapy and Systemic radiation therapy: In Brachytherapy, a doctor places usually a gamma or beta emitter, in or near the cancer site. The implants come in different shapes, which include: tube, wire, capsule, seeds, and pellets. For example, iodine-131 is commonly used to treat thyroid cancer and non-malignant thyroid disorders. It is produced in wire form. In Systemic radiation therapy, the patient requires swallowing a radioactive substance, which travels throughout the body to find and kill the cancerous cells. Alternatively, a healthcare professional may inject the radioactive substance into a person's vein.

Activity 7.8:

Contact a knowledgeable person such as a radiologist in a local hospital for demonstration regarding the use of nuclear medicine for analysis or treatment. If a tour cannot be arranged your teacher may perhaps invite a specialist to speak to the class.

Radioactive dating

Radioactive dating or radioisotope dating is a technique which is used to date materials by use of naturally occurring radioactivity.

That is, in radioactive dating, scientists count the number of parent isotopes and daughter isotopes formed from the nuclear decay to determine how many half-lives have passed and provide a suggestion of the age of an object. Thus, once the isotopic abundances of each parent/daughter elements is determined using the mass spectrometer, the age (t), of the object can be calculated using the formula N=N0e^lambda*t.

Its most famous application is carbon-14 dating. Carbon-14 has a half-life of 5730 years. Radioactive carbon has the same chemistry as stable carbon, and so it mixes into the ecosphere, where it is consumed and becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon. Thus, if you know the number of carbon nuclei in an object (perhaps determined by mass and Avogadro's number), you multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei in the object. When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished as it decays. By comparing the abundance of $^{14}\mathrm{C}$ in an artifact with the normal abundance in living tissue, it is possible to determine the artifact's age (or time since death).

Carbon-14 dating can be used for biological tissues as old as 50 or 60 thousand years, but is most accurate for younger samples, since the abundance of $^{14}\mathrm{C}$ nuclei in them is greater. Very old biological materials contain no $^{14}\mathrm{C}$ at all. There are instances in which the date of an artifact can be determined by other means, such as historical knowledge or tree-ring counting. These cross-references have confirmed the validity of carbon-14 dating and permitted us to calibrate the technique as well. Carbon-14 dating revolutionized parts of archaeology.

Localized Applications of radioactive dating

Dinikenesh or Lucy was discovered in 1974 in Ethiopia, at Hadar, in Afar by paleoanthropologist Donald Johanson. After all her bone fragments were collected and reconstructed, Lucy was taken to USA so that her age is measured precisely. Which method was used to measure the age of Lucy? Can carbon dating be applicable?

Carbon-14 has the same chemistry as stable carbon, and so it mixes into the ecosphere, where it is consumed and becomes part of every living organism. When the organism dies, carbon exchange with the environment ceases, and as a result C-14 is not replenished as it decays. Since the ratio of C-14 to total carbon nuclei is very small (about 1.3×10^{-12}), C-14 concentration falls nearly to zero after about nine half-lives (about more than 50 thousand years). This implies that Lucy is too old for radiocarbon dating and hence other methods were employed to find her age. The appropriate method used was argon-argon (^{40}Ar-^{39}Ar) dating on tiny crystals in layers of volcanic ash sandwiching the sediments where Dinikenesh was found. In this measurement, her age was found to be 3.18 million years. Figure 7.11 shows several hundred fossils of Lucy 40% pieces of the skeleton of female found in Afar region in Ethiopia.

Figure 7.11: Several hundred fossils of Lucy with 40 percent pieces.

Review Questions 7.3

1. State some uses of radioactive isotopes.
2. Why might a radioactive source be injected into a patient?
3. What are radioactive tracers used for?
4. What is radioactive dating? Explain how it works.
5. Injected radioactive sources can be used as tracers to make soft tissues, such as blood vessels or the kidneys, show up through medical imaging processes. Which radiation type should the radioactive source used emit?
6. Depending on the object that scientists are trying to study, different radioactive dating techniques can be used. Why do scientists prefer to use:
   A.K-40 for dating of rocks that are older than 100,000?
   B. C-14 for dating of fossils and some rocks?
7. How can carbon dating be used to measure age of dead organisms?
8. List some commonly used element pairs to establish absolute ages.
9. Right after scanning using radiation for diagnosis, does the body remain radioactive? Is it harmful to approach the irradiated person?

7.4 Nuclear Reaction and Energy Production

By the end of this section you should be able to:

- *distinguish between the two types of nuclear reaction;*
- *appreciate the sun as a big nuclear reactor.*
- *discuss the use (energy production, production of important isotopes, etc.) and misuse (nuclear bomb) of artificial nuclear reaction.*

Nuclear reactions are processes in which one or more nuclides are produced from the collisions between two atomic nuclei or from the collisions between a nucleus and a subatomic particle. In general, there are two types of nuclear reactions, namely;

- fission reaction, and
- fusion reaction.

Nuclear fission reaction

Nuclear fission is the splitting of a heavy atomic nucleus such as uranium-235, into two fragments of roughly equal mass. Nuclear fission is a form of nuclear transmutation, meaning that the starting atoms are not the same elements as the resultant — or daughter — product atoms. In nuclear reactors, nuclear fission usually takes place by inducing neutrons into the parent nuclei. In neutron induced fission reactions, two additional neutrons are released per one incident neutron.

Nuclear fission is accompanied by the release of a large amount of energy. Part of this energy is converted into the kinetic energy of the fission fragments and the remaining energy is converted into different forms such as heat and sound. For example, consider the fission reaction equation on U-235, given by:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{144}_{56}\mathrm{Ba} + ^{89}_{36}\mathrm{Kr} + 3^{1}_{0}\mathrm{n} + 200\mathrm{MeV}$$

The reaction is described pictorially as shown in Figure 7.12.

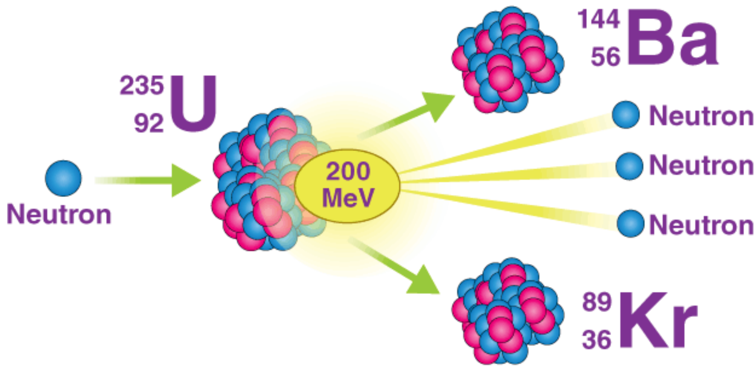


Figure 7.12: The atomic fission of ^{235}U.

In this reaction, the sum of the nuclear binding energy U-235 and the kinetic energy of the incident neutron (n-1) is equal to the sum of kinetic energy of the two daughter nuclides (Ba-144 and Kr-89); the kinetic energy of the three neutrons and the 200MeV energy, which is directly released

Key terms:

- *Nuclear fission is the splitting of a nucleus of an atom into two or more smaller nuclei.*
- *Nuclear fusion is a reaction in which two or more atomic nuclei are combined to form one or more different atomic nuclei and subatomic particles.*

Example 7.4

A nuclear fission reaction is given below. Complete the equation, find the value of X and determine the type of the element, Y.

$$^{235}U + {}^{1}_{0}n \rightarrow {}^{X}Y + {}^{139}Cs + 2\,{}^{1}_{0}n + \text{energy}$$

Solution:

The equation is balanced on both sides so the total mass number on the left side must be the same as the total mass number on the right side. Thus,

$235 + 1 = X + 139 + 2$, or $X = 95$, which corresponds to Rb-95.

Activity 7.9: Simulating nuclear reactions

Simulate nuclear reactions produced by high speed particles using marbles accelerated down sloping aluminum channel into saucer. Those in saucer are analogous to target nuclei. Include several marbles of differing sizes. Show effect of speed by launching from different height and angles and effect of mass and increased momentum of projectiles by using different marbles. Marble sufficient momentum can cause ejection of one or more marbles from saucer.

Discuss analogy to bombarding nuclei with particles of ever increasing mass proton, deuteron, and alpha.

Applications of fission reaction

Fission reaction has several applications. Its application in nuclear reactors is the concern of this section. There are two types of nuclear reactors: power reactor and research reactor.

Power reactors (PR) are used to generate electricity by the process of nuclear fission. In a chain nuclear reaction, undertaking in nuclear reactor, the kinetic energy of fission fragments and the energy released will be converted to heat. The heat produces steam which is used to drive the turbine of a generator to produce electricity.

Chain nuclear reaction:

In a chain reaction, neutrons released in fission produce an additional fission in at least one further nucleus. This nucleus in turn produces neutrons, and the process repeats. The process may be controlled (nuclear power) or uncontrolled (nuclear weapons). Chain reaction occurs only above a certain mass called critical mass.

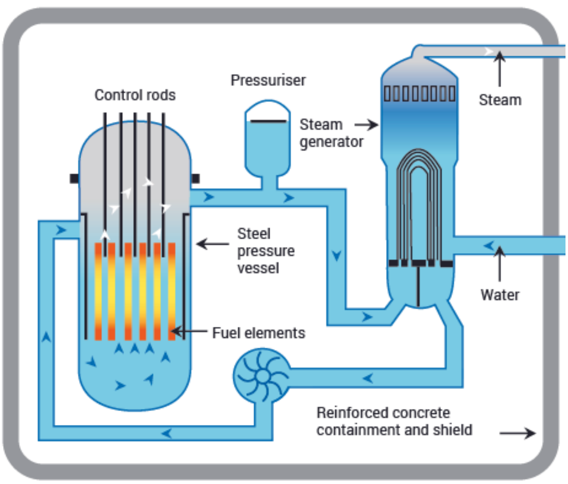


Figure 7.14: Scheme of a power reactor

There are several components common to most types of reactor:

- **Fuel:** Uranium is the basic fuel. Usually pellets of uranium oxide (UO_2) are arranged in tubes to form fuel rods. The rods are arranged into fuel assemblies in the reactor core.
- **Moderator:** Material in the core which slows down the neutrons released from fission so that they cause more fission. It is usually water, but may be heavy water or graphite.
- **Control rods or blades:** These are made with neutron-absorbing material such as cadmium, hafnium or boron, and are inserted or withdrawn from the core to control the rate of reaction, or to halt it.
- Coolant: A fluid circulating through the core so as to transfer the heat from it. In light water reactors the water moderator functions also as primary coolant.
- Pressure vessel or pressure tubes: Usually a robust steel vessel containing the reactor core and moderator/coolant, but it may be a series of tubes holding the fuel and conveying the coolant through the surrounding moderator.
- Steam generator: Part of the cooling system of pressurized water reactors where the high-pressure primary coolant bringing heat from the reactor is used to make steam for the turbine, in a secondary circuit.
- Containment: The structure around the reactor and associated steam generators which is designed to protect it from outside intrusion and to protect those outside from the effects of radiation in case of any serious malfunction inside. It is typically a meter-thick concrete and steel structure.

Research reactors (RR) are nuclear fission-based nuclear reactors that serve primarily as a neutron source. Research reactors are simpler than power reactors and operate at lower temperatures. They need far less fuel, and far less fission products build up as the fuel is used. They are also called non-power reactors, in contrast to power reactors. The neutrons produced by a research reactor are used for different purposes such as neutron scattering, analysis and testing of materials, production of radioisotopes, research and public outreach, and education.

Radioisotopes can be produced in research reactors by exposing suitable target materials to the intense neutron radiation for an appropriate time. The quality and specific activity of the radioisotopes produced depends on both the target and the irradiation conditions. A wide range of isotopes are made at reactors, from elements as light as carbon-14 to as heavy as mercury-203, with irradiations ranging from minutes to weeks.

For example, Mo-99—the parent to the widely used medical diagnostic radioisotope Tc-99m—is usually produced via neutron-induced fission of targets with U-235 using a 4- to 8-day irradiation time.

The problems posed by nuclear waste of reactors

Nuclear waste refers to the byproducts of nuclear reactors. One of the main difficulties is that the isotopes used in nuclear power stations typically have very long half lives. Plutonium-239 has a half-life of 24100 years; in contrast plutonium-238 has a half-life of 88 years. Spent nuclear fuel is the most important source of waste from nuclear power stations and is mainly unconverted uranium. About 3% of it is fission products from nuclear reactions. The actinides (uranium, plutonium, and curium) are responsible for the bulk of the long-term radioactivity, whereas the fission products are responsible for the bulk of the short-term radioactivity.

After about 5 percent of a nuclear fuel rod has reacted inside a nuclear reactor, it will no longer able to be used as fuel due to the build-up of fission products.

Scientists are experimenting on methods for reusing these rods in order to reduce waste and use the remaining actinides as fuel.

Activity 7.10 : Research nuclear power

1. Work in a small group to research one of the following topics. Students can be assigned to investigate and report
   - The fraction of energy generated from nuclear power in Africa and the rest of the world
   - Peaceful uses of Nuclear Radiation in Ethiopia and Africa
   - Nuclear Facilities in Africa

Present your findings to the rest of your class in a form of your choice.

7.4.1. Nuclear fusion reaction and its uses

Nuclear fusion reaction is a process of making a single heavy nucleus from the combination of two or more lighter nuclei. In most cases, fusion reaction releases more energy than fission reaction. The process releases energy because the total mass of the resulting single nucleus is less than the mass of the two original nuclei. The leftover mass becomes energy.

Use of nuclear fusion as source of energy in the sun

The principal source of energy in the sun is a net fusion reaction in which four hydrogen nuclei fuses and produce a helium nucleus and two positrons. This is a net reaction of a more complicated series of events:

$$4\,^{1}_{1}H \rightarrow \,^{4}_{2}He + 2\,^{0}_{+1}\beta$$

The mass of the resulting helium nucleus has a mass that is 0.7% less than that of the four hydrogen nuclei. This lost mass (the mass defect) is converted into energy during the fusion. The energy produced in this reaction is about 3.6×10^{11} kJ of energy per mole of He-4 produced. This energy is greater by 20 fold than the energy produced by the nuclear fission of one mole of U-235, and over 3 million times larger than the energy produced by (chemical) combustion of one mole of octane.

The heavy isotopes of hydrogen, a deuteron,   and a triton, , also undergo fusion at extremely high temperatures. They form a helium nucleus and a neutron:

$$^{2}_{1}H + \,^{3}_{2}H \rightarrow \,^{4}_{2}He + \,^{1}_{0}n$$

This change proceeds with a mass loss of 0.0188 amu, corresponding to the release of   of He-4 formed. The very high temperature is necessary to give the nuclei enough kinetic energy to overcome the very strong repulsive forces resulting from the positive charges on their nuclei so they can collide.

Figure 7.13 shows the cyclic sun fusion reaction to form a very strong release in the electromagnetic radiation.

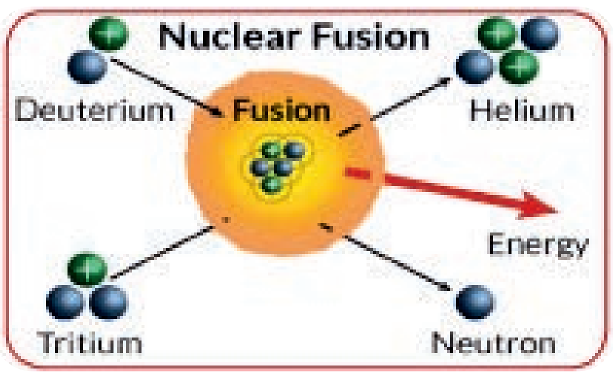


Figure 7.13: Nuclear fusion reaction process of the H isotopes.

Hydrogen bomb: One of the misuses of nuclear fusion reaction is its application for hydrogen bomb, which has higher destructive power and greater efficiencies than atomic bombs. Fusion weapons are also referred to as thermonuclear bombs.

Activity 7.10:

Observe simulation of Nuclear Collisions using free applets available from the internet. Demonstrate nuclear reactions using virtual laboratory simulation software's like PhET.

Review Questions 7.4

1. List uses of applications of nuclear reaction
2. What are the basic differences between nuclear fission and fusion reaction processes?
3. Complete the following nuclear reactions:

   (a) ${}^{9}_{4}Be + {}^{1}_{1}H \rightarrow {}^{6}_{3}Li + \_\_$

   (b) ${}^{10}_{5}B + {}^{4}_{2}He \rightarrow {}^{13}_{7}N + \_\_$
4. Describe the nature of research reactors by comparing it with the power reactors.
5. Why is Technetium-99 considered as the most important radioactive isotope for medical applications?
6. Is nuclear energy from radioisotopes in use for space travel?
7. Is there any radiation hazard from nuclear wastes of nuclear reactors? If so, how?
8. Caesium-137 is a by-product of nuclear fission within a nuclear reactor.

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{137}_{55}Cs + {}^{95}_{x}Rb + y\,{}^{1}_{0}n $$

   a) Copy and complete this equation which describes the production of ${}^{137}_{55}Cs$ by finding x and y.

   b) The half-life of Cesium is 30 years. When the fuel rods are removed from a nuclear reactor core, the total activity of caesium-137 is 5.8×10^{15} Bq. After how many years will this have fallen to 1.6×10^{6} Bq?

   c) Comment on the problems of storage of the fuel rods over this time period.
9. Does Fusion produce radioactive nuclear waste the same way fission does?
10. Is there chain reaction in nuclear fusion? Can fusion cause a nuclear accident?
11. Can fusion reactors be used to produce weapons?
12. In the fusion reaction below, determine the mass defect.

$$ 4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{+1}\beta + energy $$

7.5 Safety Rules Against Hazards of Nuclear Radiation

By the end of this section you should be able to:

- *implement safety rules against hazards of nuclear radiation.*

Nuclear Safety rules

Nuclear safety is defined by the International Atomic Energy Agency (IAEA) as "The achievement of proper operating conditions, prevention of accidents or mitigation of accident consequences, resulting in protection of workers, the public and the environment from undue radiation hazards". The IAEA defines nuclear security as "The prevention and detection of and response to, theft, sabotage, unauthorized access, illegal transfer or other malicious acts involving nuclear materials, other radioactive substances or their associated facilities"This covers nuclear power plants and all other nuclear facilities, the transportation of nuclear materials, and the use and storage of nuclear materials for medical, power, industry, and military uses.

The nuclear power industry has improved the safety and performance of reactors and has proposed new and safer reactor designs. However, a perfect safety cannot be guaranteed. Potential sources of problems include human errors and external events that have a greater impact than anticipated: The designers of reactors at Fukushima in Japan did not anticipate that a tsunami generated by an earthquake would disable the backup systems that were supposed to stabilize the reactor after the earthquake. Catastrophic scenarios involving terrorist attacks, insider sabotage, and cyber-attacks are also conceivable.

Nuclear safety therefore covers at minimum:

- Extraction, transportation, storage, processing, and disposal of fissionable materials
- Safety of nuclear power generators
- Control and safe management of nuclear weapons, nuclear material capable of use as a weapon, and other radioactive materials
- Safe handling, accountability and use in industrial, medical and research contexts
- Disposal of nuclear waste
- Limitations on exposure to radiation

Protecting yourself from radiation

Radiation is part of our life. Background radiation, coming primarily from natural minerals, is around us all the time. Fortunately, there are very few situations where an average person is exposed to uncontrolled sources of radiation above background. Nevertheless, it is wise to be prepared and know what to do if such a situation arises.

One of the best ways to be prepared is to understand the radiation protection principles of time, distance and shielding.

- **Time:** For people who are exposed to radiation in addition to natural background radiation, limiting or minimizing the exposure time reduces the dose from the radiation source.
- **Distance:** Just as the heat from a fire reduces as you move further away, the dose of radiation decreases dramatically as you increase your distance from the source.
- **Shielding:** Barriers of lead, concrete, or water provide protection from penetrating gamma rays and X-rays. This is why certain radioactive materials are stored under water or in concrete or lead-lined rooms, and why dentists place a lead blanket on patients receiving X-rays of their teeth. Therefore, inserting the proper shield between you and a radiation source will greatly reduce or eliminate the dose you receive.

Safety precautions when using radioactive sources in schools

Radioactive sources which are used in school are usually very weak. They can only be used in the presence of an authorized teacher. They are kept in a sealed container except when they are being used in an experiment or demonstration. The containers can be designed based on the appropriate shields. They are immediately returned to the container when the experiment or demonstration is finished. When using the radioactive source it should be:

- handled with tongs or forceps, never with bare hands. Moreover, hands must be washed after the experiment and definitely before eating.
- kept at arm's length, pointing away from the body.
- always kept as far as possible from the eyes.

- What to do in the event of a nuclear accident?
- What are the three main safety guidelines for nuclear reactors?

Small group work:

Gather information about the possible sources of nuclear radiation from the internet.

- Enquire about radiation hazards in your locality by having a meeting with the nearby Radiation Protection Authority and what could be done to protect yourself and others from radiation hazards.
- If there is no nearby authority, let you send your questions to an accessible body of authority by electronic media.
- Let you organize a report about what measures you need to take as school community and present it to the whole class.

Expert Guest:

Your teacher will invite you a medical personnel from the nearby medical establishment who is knowledgeable about nuclear medicine. Ask the guest to explain

- how and why nuclear medicine is practiced, where the medicine comes from and how it is transported; handled, stored.
- what measures should be implemented during use and disposal of these medicines. To actively engage with the guest, you need prior reading.
- Discuss about hazards of nuclear radiations and safety with your teacher.

Review questions 7.5

1. How does radiation affect the public? How can you help the public to protect from radiation hazards?
2. How do you need to be careful around radiation?
3. How can exposure to radiation be minimized?
4. How is a person contaminated by radiation such as from bomb get decontaminated?
5. Is nuclear waste from nuclear reactors harmful to humans and the environment? If so how can we protect the hazard?
6. What barriers provide protection from radiation produced by a nuclear power plant?

UNIT SUMMARY

- The chemical shorthand to describe atom X, where Z is the atomic number, and A = Z +N, is ${}^{A}_{Z}X$.
- The atom and the nucleus are assumed spherical and the atomic radius is estimated by:

$$R = R_0 A^{1/3} = \left(1.2\times10^{-15}\,\text{m}\right) A^{1/3}$$

where A is the mass number of the atom; R_0 is the radius of the nucleus which is roughly equal to $R_0 = 1.2\text{fm}$ and 1 femtometer (1 fm) = $10^{-15}\,\text{m}$.

- A member of a family of an element with the same number of protons but different number of neutrons is called an isotope. Some examples of isotopes of elements:
- The strong nuclear force is a very short-range attractive force that acts between nucleons: protons and neutrons. The strong nuclear force is strong enough to withstand the electrical repulsion up within a distance of slightly more than the radius of a nucleon, 10–15 m . Both nucleons are affected by the nuclear force almost identically.
- The weak nuclear force acts inside of individual nucleons, which means that it is even shorter ranged than the strong nuclear force. The weak nuclear force is a force that causes decay of an atom by beta emission.
- Nuclear binding energy is the energy that holds nucleons together. Another equivalent definition of binding energy is the minimum energy that is required to disassemble the nucleus of an atom into its constituent nucleons.
- The mass of an atomic nucleus is less than the sum of the individual masses of the free constituent protons and neutrons. The energy equivalent to this difference in mass can be calculated by the Einstein equation,

$$BE = \Delta mc^2$$

where BE is the nuclear binding energy, and c is the speed of light. This 'missing mass' is known as the mass defect, and represents the energy that was released when the nucleus was formed.

- The mass defect of an atom can be calculated by:

$$\text{Äm} = Zm_p + (A - Z)m_n - M$$

where Zm_p is the total mass of the protons; $(A - Z)m_n$ is the total mass of the neutrons, and M is the mass of the nucleus. Using this into the former equation, we obtain:

$$BE = \Delta mc^2 = [Zm_p + (A - Z)m_n - M]c^2$$

- Nuclear energy is usually expressed in atomic energy unit (aeu), where:

$$1\text{aeu} = c^2 \times 1\,\text{amu} = \left(9\times10^{16}\text{m}^2/\text{s}^2\right)\times\left(1.602\times10^{-27}\,\text{kg}\right) = 1.44\times10^{-10}\text{J}$$

Or, in megaelectron volt (MeV), where

$$1\text{MeV} = 1.602\times10^{-13}\ \text{J, and } 1\text{aeu} = 931.1\text{MeV}$$

- When mass is in amu/u, the nuclear binding energy in MeV becomes,

$$BE = [Zm_p + (A-Z)m_n - M] \times 931.1 MeV$$

- Binding energy per nucleon (BEN) is the average energy required to remove an individual nucleon from a nucleus. It is given by

$$BEN = \frac{BE}{A}$$

- Radioactivity is the phenomenon of the spontaneous disintegration of unstable atomic nuclei to atomic nuclei to form more energetically stable atomic nuclei. Radioactive decay is a highly exoergic, statistically random, process that occurs with a small amount of mass being converted to energy.
- Representations of nuclear decay reaction of element with chemical symbol X by alpha (α α), or beta (β^-, or β^+) emission to give a daughter element Y are:

$$\alpha - \text{emission}: \; {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$$\beta^- - \text{emission}: \; {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \overline{\upsilon}$$

$$\beta^+ - \text{emission}: \; {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta$$

- Gamma rays are high energy, high frequency, electromagnetic radiations. Gamma radiation usually accompanies alpha or beta decay.
- Radiations damage cells by ionizing atoms or molecules by removing electrons from them.
- Absorbed dose describes the amount of energy deposited per unit mass in an object or person. The units for absorbed dose are gray (Gy, international unit) and rad; where 1Gy = 1J/kg and 1rad = 0.01 Gy = 0.01 J/kg.
- Radiation doses above 3 Gy (300 rad) can be fatal and doses above 6 Gy (600 rad) are almost certain to be fatal, with death occurring within several months (in shorter times at higher doses).
- A radiation detector is a device that measures the ionization of radiations (i.e., creating electrons and positively charged ions). Scintilator, Gaseous ionization detectors and Geiger counter.
- The rate of isotope decay is called the activity, A, and is given by

$$A = \frac{\Delta N(t)}{\Delta t} = -\lambda N(t)$$

- where the negative sign shows the decrease in the number of the radioactive nuclei with time, N(t) is the number of undecayed nuclei at the subsequent time t, the decay constant λ of a radioactive nuclide is defined as its probability of decay per unit time; having SI unit s–1. It is a positive rate also called the exponential decay constant, disintegration constant, rate constant, or transformation constant.
- The SI unit of activity, A, is Becquerel (Bq); where 1Bq = 1 decay per second. We can also use the unit Curie, Ci, where 1 Ci = $3.7 \times 10^{10}\, Bq$.

- If N_0 is quantity at time t =0, the quantity of the parent radioactive nuclei is subject to exponential decay given by $N(t) = N_0 e^{-\lambda t}$
- The half-life is defined as the time at which half of the original nuclei have decayed. Or, it can also be stated somewhat differently as the time after which one half of the original number of nuclei remains untransformed.
- The half-life is related to the decay constant by

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- Fission occurs when a neutron slams into a larger atom, forcing it to excite and split into two smaller atoms—also known as fission products.
- Nuclear fusion reaction is a process of making a single heavy nucleus from the combination of two or more lighter nuclei.
- A nuclear reactor is a device used to initiate and control a fission nuclear chain reaction or nuclear fusion reactions.

END OF UNIT QUESTIONS 7

1. Discuss (a) Rutherford and (b) Bohr model of an atom.
2. What are the radii of U-238 and U-235?
3. What do you mean by reduced mass? How is this mass related with the nuclear binding energy?
4. State some uses of radioactive isotopes.
5. State one risk or danger associated with radioactivity.
6. What are the basic differences between nuclear fission and fusion reaction processes?
7. Write the nuclear reaction equation for K-242 that decays by beta minus emission. Do the same for Pu-239 decays by alpha emission.
8. Compare alpha and beta emission interms of their nature, relative ionizing effect, relative penetrating power, the effect of magnetic and electric field on the radiation, and the common detectors.
9. What are research reactors (RR)? How are RR important for the in agriculture and medical applications? How are the research reactors (RR) different from the power reactors (PR)
10. When a patient undergoes radiotherapy, how does medical staff protect themslves from radiation?
11. What effects can high doses of radiation have on the human eyes?
12. Iodine-131, in the form of iodine chloride is used to treat thyroid cancer. Why does the patient radioactive substance orally to treat the cancer?